

Evolutionary Design for Estimation of Crop Protection Protocols

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Abstract: A new approach to estimating a key component of popular crop protection protocols is developed. The approach, featuring sequential sampling based on an evolutionary sample design, is presented and a numerical example is given to illustrate its use. Practical applicability in a research environment is considered. Economic implications are indicated along with directions for further research.

Key words: Evolutionary design, estimation, crop protection, economic implications

INTRODUCTION

The most prominent protocol of Integrated Pest Management (IPM) involves the decision making parameter known as the economic or action threshold (Davis and Tisdell, 2002). The economic threshold was designed to prevent pesticide treatments which are not economical and thereby achieve the dual purpose of reducing pesticide use and farmers' costs. Economic thresholds have been developed for many crop-pest systems and are frequently part of crop protection protocols promoted to farmers by agricultural information agencies. For example, survey data reveal the success of these efforts in terms of their impact on crop protection in the United States (U. S. Department of Agriculture, 1994).

In view of the popularity of the economic threshold and the existence of strategies with similar character in other areas of management, the purpose of this study is to provide a new method and data collection procedure for estimating the economic threshold. The method offers potentially 2 advantages over traditional approaches:

- The economic threshold is estimated directly while unnecessary estimation of other parameters in the agro-ecosystem is avoided.
- Specific assumptions concerning parametric forms for dosage/response and damage relationships as typically specified for economic threshold estimation are not required.

In particular, avoidance of the need to estimate dosage/response and damage relationships is potentially important. These relationships have often presented

difficulties for applications and have proved both troublesome and contentious in some cases (Pannell, 1990; Fox and Weersink, 1995). Even estimation of what are believed to be theoretically sound damage relationships alone has caused problems. For example, Cousens (1985) important model of crop-weed competition often exhibits extraordinary sensitivity to experimental design. Fortunately, the character of the economic threshold facilitates direct estimation without resorting to specific parametric forms for dosage/response and damage relationships if data collection is directed toward this purpose.

THE ECONOMIC THRESHOLD

The economic threshold refers to a pest population level related to a fixed pesticide treatment rate. The farm-level decision rule associated with the economic threshold is to apply a predetermined amount of pesticide, D , if monitoring reveals a pest population that exceeds a threshold level, T . Hence, the economic threshold is a rule of thumb with an if-then-else character; viz., if the pest population exceeds T , then treat with amount, D ; else, do not treat. From a manager's viewpoint, D and T are easily understood parameters which guide pesticide treatment decisions.

For example, the predetermined amount of pesticide, D , is frequently a pesticide label dosage rate while the threshold pest population level, T , is often obtained from publications of agricultural information agencies. As with other rules of thumb, the economic threshold is intended to facilitate rational management while recognizing that practical decision making must often be done without the luxury of lead time or extensive analysis.

Table 1: States, actions and conditional expected payoffs defined by the economic threshold

State	$\epsilon \leq T$	$\epsilon > T$
Probability	$\int_a^T g(\epsilon) d\epsilon$	$\int_T^b g(\epsilon) d\epsilon$
Action		
$x = 0$	$\frac{\int_a^T v(0, \epsilon) g(\epsilon) d\epsilon}{\int_a^T g(\epsilon) d\epsilon}$	$\frac{\int_T^b v(0, \epsilon) g(\epsilon) d\epsilon}{\int_T^b g(\epsilon) d\epsilon}$
$x = D$	$\frac{\int_a^T v(D, \epsilon) g(\epsilon) d\epsilon}{\int_a^T g(\epsilon) d\epsilon}$	$\frac{\int_T^b v(D, \epsilon) g(\epsilon) d\epsilon}{\int_T^b g(\epsilon) d\epsilon}$

From an analytical perspective, the decision parameter, T , associated with the economic threshold corresponds to a partitioning of a state space into a two state-two action game against nature in which nature pursues a mixed strategy (Table 1). Under the economic threshold concept, managers implement an if-then-else strategy under perfect information. In the table, the state space is depicted by population values, ϵ , while the action space is depicted by pesticide treatment rates, x . Payoffs, $v(x, \epsilon)$, are depicted as depending on action-state pairs while nature's mixed strategy is characterized by $g(\epsilon)$. Payoffs corresponding to the action-state pairs in the game are expected payoffs conditional on the partition defining the game. For example, using conditional probability formulas, the first payoff element shown in the Table 1,

$$\frac{\int_a^T v(0, \epsilon) g(\epsilon) d\epsilon}{\int_a^T g(\epsilon) d\epsilon}$$

is the expected payoff given that the pest population is below the threshold and treatment decisions are guided by the threshold. The parameter comprising the economic threshold defines the optimal partition; i.e., the game against nature which is expected to be most advantageous for managers to play.

Given enough information, the decision parameter, T , associated with the economic threshold can be determined analytically. Assume a payoff function $v(x, \epsilon)$ and a probability density function for pest level, $g(\epsilon)$. Given the predetermined treatment rate, D , the following optimization model determines the decision parameter, T , in order to maximize the value of the game given nature's mixed strategy and the parametric form of a manager's strategy:

$$\begin{aligned} \text{Maximize } E[v(D, T)] = \\ \int_a^T v(0, \epsilon) g(\epsilon) d\epsilon + \int_T^b v(D, \epsilon) g(\epsilon) d\epsilon \end{aligned} \quad (1)$$

Assuming differentiability of $E[v(D, T)]$, the necessary condition for solution of (1) is

$$\frac{\partial E[v(D, T)]}{\partial T} = (v(0, T) - v(D, T))g(T) = 0 \quad (2)$$

Given (2), a sufficient condition for solution of (1) is

$$\begin{aligned} \frac{\partial^2 E[v(D, T)]}{\partial T^2} = (v(0, T) - v(D, T))g'(T) \\ + \left(\frac{\partial v(0, T)}{\partial T} - \frac{\partial v(D, T)}{\partial T} \right) g(T) < 0 \end{aligned} \quad (3)$$

Note that from (2) and (3), specific expressions for $v(x, \epsilon)$ and $g(\epsilon)$ can yield a specific value for the solution to (1).

Note also that (1) corresponds to the value of the game associated with Table 1 under perfect information. A traditional approach to determining the solution to (1) is to use experimental observations on treatment and pest population to estimate parametric forms for $v(x, \epsilon)$ and $g(\epsilon)$. Often, geometric and/or arithmetic series dominate experimental design and typically, questions relying on analysis of variance for answers are perhaps more appropriately pursued with the resulting data than is estimation of the economic threshold. The parametric form of $v(x, \epsilon)$ typically involves relationships commonly referred to as dosage/response and damage relationships. Solving (1) with the specific parametric expressions provides the economic threshold decision parameter, T . An empirical example of this approach, including specific parametric forms for $v(x, \epsilon)$ and $g(\epsilon)$, is contained in Hall (1988). A second parametric approach, apparently rarely used, to finding the solution to (1) can be based on Bayesian methods and sequential sampling.

While either of the 2 traditional approaches to estimating the economic threshold can provide valid results, both involve estimation of parameters not typically used by managers such as those contained in dosage/response and damage relationships and also require specific parametric forms in order to be implemented. The latter in particular invites specification error to adversely influence findings and tends to limit investigations to functional specifications known to be tractable. The study begins to detail a new approach to estimating the decision parameter, T , directly along with

a sample design based on the outcome of an evolutionary algorithm to implement the approach. The method is based on the optimality conditions in (2) and (3), some general properties of $v(x, \epsilon)$ and sequential sampling based on processing of logical (true/false) operations by an evolutionary algorithm.

LOGICAL PAYOFF RESTRICTIONS

This study presents the foundation for a new method of estimating the key parameter in many crop protection protocols known as the economic threshold. The method involves sequential sampling based on processing of logical operations by an evolutionary algorithm to identify the economic threshold decision parameter directly while avoiding estimation of any underlying structural parameters associated with the agro-ecosystem. It should be emphasized, that the method is intended for use in a research environment where experimentation to generate and collect data for estimation is possible.

The following assumptions are made concerning nature's mixed strategy and the payoff function depicted in Table 1.

First, it is assumed that $g(T) > 0$. This assumption means that nature's mixed strategy will not involve a vanishing probability at the optimal partition of the state space. This assumption does not appear to be restrictive. In fact, there does not seem to be any reason to expect that a population level, almost surely never observed, will play a significant role in a management protocol. Second, it is assumed that the payoff function satisfies the single crossing property in its arguments (Milgrom and Shannon, 1994). This assumption requires that for $x' > x''$ and $\epsilon' > \epsilon''$, $v(x', \epsilon') > v(x'', \epsilon'') \Rightarrow v(x', \epsilon') > v(x'', \epsilon')$ using the term "better" to refer to a larger payoff, satisfaction of the single crossing property means that if a larger pesticide treatment rate is better than a smaller one at a particular population level, then it will also be better than the smaller treatment rate at an even larger population level. This assumption seems plausible for most situations and it is noteworthy that payoff functions in the literature, including studies by Hall (1988) and Liu *et al.* (1999), have involved payoffs satisfying the single crossing property. These assumptions imply some important restrictions on the payoff function that can be used in identifying the solution to (1) with an appropriate sample design. As shown in Appendix A, these assumptions imply that $v(D, T) > v(0, \epsilon)$ for all $\epsilon > T$ and $v(D, T) < v(0, \epsilon)$ for all $\epsilon < T$.

To summarize, the optimality conditions in conjunction with the assumptions about $v(x, \epsilon)$ imply the following restrictions on the solution to (1):

$$v(D, T) = v(0, T) \quad (4)$$

$$v(D, T) > v(0, \epsilon) \text{ for all } \epsilon > T \quad (5)$$

$$v(D, T) < v(0, \epsilon) \text{ for all } \epsilon < T \quad (6)$$

The logical payoff restrictions (4) - (6) provide a basis for discovering the solution to (1). A recently developed advance in evolutionary computational methods (Karavas and Moffitt, 2004) facilitates parsimonious sequential sampling and rapid discovery of the economic threshold using (4)-(6). Performance is demonstrated in a numerical example presented in a later study.

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Appendix A: This Appendix provides a proof that conditions (2), (3), satisfaction of the single crossing property by the payoff function and monotonicity of the payoff function in ϵ imply that $v(D, T) > v(0, \epsilon)$ for all $\epsilon > T$ and $v(D, T) < v(0, \epsilon)$ for all $\epsilon < T$. From (2) and (3), it is apparent that

$$\frac{\partial v(0, T)}{\partial T} < \frac{\partial v(0, T)}{\partial T}$$

T at the optimal strategy. This inequality implies that $v(D, T)$ is steeper in a neighborhood of T than is $v(0, T)$; hence, $v(D, T+\Delta) > v(0, T+\Delta)$ and $v(D, T-\Delta) < v(0, T-\Delta)$ for small positive Δ . By the single crossing property, $v(D, T+\Delta) > v(0, T+\Delta) \Rightarrow v(D, \epsilon) > v(0, \epsilon)$ for all $\epsilon > T+\Delta$. By monotonicity, $v(D, T+\Delta) > v(0, \epsilon)$ for all $\epsilon > T+\Delta$. Allowing Δ to approach zero gives the desired result. Similar reasoning shows the second part.

PROCESSING OF LOGICAL OPERATIONS

A general overview of a new method of estimating the decision parameter associated with the economic threshold using an evolutionary algorithm for sample design is as follows. By their nature, evolutionary algorithms search for the fittest member of a population by creating an initial population, evaluating the fitness of each population member, using the characteristics of the fittest members to create a second more fit population and so on, until the characteristics of the fittest population member are identified. If fitness of a potential value of the economic

threshold is measured as consistency with the logical payoff restrictions (4)-(6), then an evolutionary algorithm can be used to create a sample (population) of candidate threshold values, evaluate the fitness of each candidate value, use the fittest values of the sample to create a second sample and so on, until the fittest threshold value is identified. In the new method, an evolutionary algorithm is used not only to derive an estimate of the economic threshold from data but also to indicate data samples needed sequentially to estimate the threshold value. A more specific overview of a new method of estimating the decision parameter associated with the economic threshold is as follows:

- Call a potential solution to (1), T_k .
- Create, via an evolutionary algorithm, a population of potential solutions for evaluation.
- Evaluate each potential solution through comparison of (5) and (6) to a database of untreated observations in order to determine statements which are contradictory.
- Use a monetary measure of the total extent of logical contradictions of each potential solution to gauge fitness.
- Combine the fittest potential solutions to generate a new population of potential solutions which is then utilized as in step (c).

Continue this process until the solution is found. Note that a key feature of the method is a sequence of samples based on processing of logical operations to achieve fitness; hence, the experimental design used to generate data is determined by an evolutionary algorithm aimed at directly estimating the economic threshold.

The specifics of steps (c) and (d) in the overview are as follows. A binary coded form (a chromosome in the language of evolutionary algorithms) of a potential solution is denoted by T_k . Then $f(D, T_k)$ is defined as a monetary measure of the extent of the contradiction of T_k with (5) plus a monetary measure of the extent of the contradiction of T_k with (6):

$$f(D, T_k) = \sum_{i=1}^n \{ [\max(v(D, T_k), v(0, \epsilon_i)) - v(D, T_k)] \cdot \left[\frac{\max(T_k, \epsilon_i) - T_k}{\epsilon_i - T_k} + [\max(v(D, T_k), v(0, \epsilon_i)) - v(0, \epsilon_i)] \cdot \left[\frac{\max(T_k, \epsilon_i) - \epsilon_i}{T_k - \epsilon_i} \right] \right] \} \quad (7)$$

Where, the summation in (7) is over a database of distinct observations, $i = 1, 2, \dots, n$, for which $v(0, \epsilon_i)$ is

available. Hence, the fitness of (D, T_k) is tested against a sample of n untreated payoffs in search of contradictions. Equation 7 consists of two terms where each term is a product of two elements shown in brackets. The first bracketed element in each of the terms is a monetary measure of the extent of contradiction of (D, T_k) with the corresponding where the summation in (7) is over a database of distinct observations, $i = 1, 2, \dots, n$, for which $v(0, \epsilon_i)$ is available.

Hence, the fitness of (D, T_k) is tested against a sample of n untreated payoffs in search of contradictions. Eq. 7 consists of two terms where each term is a product of two elements shown in brackets. The first bracketed element in each of the terms is a monetary measure of the extent of contradiction of (D, T_k) with the corresponding condition from (5)-(6). The second element in each term is an indicator of the condition on which the contradiction is predicated. Appropriate evaluation of the indicators requires that numerators in ratios be evaluated first and the ratio set to zero if the numerator is zero.

Note that (5) requires that T_k satisfy $v(D, T_k) > v(0, \epsilon)$ for all $\epsilon > T_k$. Hence, for an observed value of $\epsilon > T_k$, say ϵ_i , the simultaneous inequalities $v(D, T_k) < v(0, \epsilon_i)$ and $\epsilon_i > T_k$ signal a contradiction with the necessary condition (5).

The difference, $v(0, \epsilon_i) - v(D, T_k)$, provides a natural measure of the extent of the contradiction of T_k with (5) and is expressed in monetary units. On the other hand, if $\epsilon_i > T_k$ and $v(D, T_k) > v(0, \epsilon_i)$, then T_k does not contradict the necessary condition (5) and, of course, an appropriate measure of the extent of the contradiction with (5) should be zero. Observe that the expression, $\max(v(D, T_k), v(0, \epsilon_i)) - v(D, T_k)$, provides a consistent measure of the contradiction with (5) for $\epsilon_i > T_k$ where, $\max(\cdot)$ is the larger of its arguments. This is the case since, this expression takes on the value $v(0, \epsilon_i) - v(D, T_k)$ whenever, $v(D, T_k) < v(0, \epsilon_i)$ and is zero otherwise. Similar reasoning reveals that the expression, $\max(v(D, T_k), v(0, \epsilon_i)) - v(0, \epsilon_i)$, provides a consistent measure of the contradiction of T_k with (6) for $\epsilon_i < T_k$.

The first element in the product in the first term of the summand of (7), $[\max(v(D, T_k), v(0, \epsilon_i)) - v(D, T_k)]$, is the extent of the contradiction of (D, T_k) with (5) when $\epsilon_i > T_k$. The second element in the product in the first term of the summand of (7),

$$\left[\frac{\max(T_k, \epsilon_i) - T_k}{\epsilon_i - T_k} \right]$$

ensures that the condition, $\epsilon_i > T_k$, which must be satisfied in order for $[\max(v(D, T_k), v(0, \epsilon_i)) - v(D, T_k)]$ to represent a contradiction with (5), holds. To see this, observe that

$$\left[\frac{\max(T_k, \epsilon_i) - T_k}{\epsilon_i - T_k} \right]$$

is one if $\epsilon_i > T_k$ and is zero otherwise. Similar reasoning reveals the product in the second term of (7) represents a contradiction of T_k with (6). Finally, $f(D, T)_k$ is nonnegative since both elements in both of its terms are non-negative.

The significance of the fitness expression (7) for successful estimation of the economic threshold based on fitness is derived from the following proposition.

Proposition: Suppose, a database of untreated observations, ϵ_i , evenly spaced by design over a range which encompasses both the economic threshold and any value of T_k whose fitness will be evaluated. The solution to (1), say (D, T) , minimizes (7) and if the number of distinct observations, n , for which $v(0, \epsilon_i)$ is available grows large, then the T_k which, minimizes (7) also solves (1); i.e., $(D, T) \in \underset{(T_k)}{\operatorname{argmin}} (D, T_k)$ and $\lim_{n \rightarrow \infty} \underset{(T_k)}{\operatorname{argmin}} (D, T_k)$

$$(D, T_k) = \underset{(T)}{\operatorname{argmin}} E[v(D, T)].$$

Proof: See Appendix B.

The proposition assures that the solution to (1), which defines the economic threshold, necessarily minimizes (7) and that the minimum of (7) is the solution to (1) if the sample is large. While it is important not to generalize from a single numerical example, a good solution to (1) is found in the next section through pursuit of the minimum of (7) using a sequence of relatively small databases.

Appendix B: This Appendix provides a proof of the proposition contained in the text. Suppose that a database of distinct observations, ϵ_i , $i = 1, 2, \dots, n$, including $v(0, \epsilon_i)$ is available. The ϵ_i are assumed to be evenly spaced over a range which encompasses both the economic threshold and any value of T_k whose fitness will be evaluated. Renummer and rearrange the database if necessary to give, $\epsilon_1 < \epsilon_2 < \dots < \epsilon_n$, i.e., place the untreated observations into ascending order. Assume that, n is chosen large enough initially to ensure that for T_k below the solution to (1), $\epsilon_i - \epsilon_{i-1} < \epsilon_i - \Delta_1$ where, Δ_1 solves $v(0, \epsilon_i) - v(D, \Delta_1) = 0$ and for T_k above the solution to (1), $\epsilon_i - \epsilon_{i-1} < \epsilon_i - \Delta_2$ where Δ_2 solves $v(D, \Delta_2) - v(0, \epsilon_i) = 0$. This assumption on n simplifies the proof of the second part of the proposition by eliminating the need to deal with otherwise

uninteresting gaps in the fitness function (7). Now consider the fitness of T_k , $f(D, T_k)$, shown as (7) in the text and denote the solution to (1) as (D, T) .

To see that $f(D, T_k) = 0$ if $(D, T_k) = (D, T)$, recall that, under the assumptions contained in the text,

$$\underset{(T)}{\operatorname{argmax}}$$

$E[v(D, T)]$ satisfies $v(D, T) = v(0, T)$, $v(D, T) > v(0, \epsilon)$ for all $\epsilon > T$ and $v(D, T) < v(0, \epsilon)$ for all $\epsilon < T$. Now consider the fitness of the solution to (1), $f(D, T)$, in light of these requirements. Observe that for all i such that $\epsilon_i = T$, both elements of both terms of $f(D, T)$ are zero. For all i such that $\epsilon_i > T$, the first element in the first term, $\max(v(D, T), v(0, \epsilon_i)) - v(D, T)$, is zero since $v(D, T) > v(0, \epsilon_i)$. Also, the second element in the second term,

$$\frac{\max(T, \epsilon_i) - \epsilon_i}{T - \epsilon_i}$$

is zero since $\epsilon_i > T$. For all i such that $\epsilon_i < T$, the second element in the first term,

$$\frac{\max(T, \epsilon_i) - T}{\epsilon_i - T}$$

is zero since $\epsilon_i < T$. Also, the first element in the second term, $\max(v(D, T), v(0, \epsilon_i)) - v(D, T)$, is zero since $v(D, T) < v(0, \epsilon_i)$. Hence, from the properties of the solution to (1), $f(D, T) = 0$. Moreover, since $f(D, T_k)$ is nonnegative, it is clear that no (D, T_k) can provide a fitter solution to (7) than does the solution to (1).

To see that $f(D, T_k) = 0$ only if $(D, T_k) = (D, T)$ as the number of distinct observations in the untreated database grows large, consider again the fitness of T_k , $f(D, T_k)$, shown as (7) in the text. Suppose $f(D, T_k) = f(D, T) = 0$.

From (7), this can only occur if at least one of the elements in the product $[\max(v(D, T_k), v(0, \epsilon_i)) - v(D, T_k)] \cdot [\max(T_k, \epsilon_i) - T_k]$

$$\frac{\max(T, \epsilon_i) - T_k}{\epsilon_i - T_k}$$

and at least one of the elements in the product $[\max(v(D, T_k), v(0, \epsilon_i)) - v(0, \epsilon_i)]$.

$$\frac{\max(T_k, \epsilon_i) - \epsilon_i}{T_k - \epsilon_i}$$

are zero for all ϵ_i . If $T_k = T$, then both elements in both products are zero for all ϵ_i , as described earlier. If $T_k < \epsilon_i$

$< T < \epsilon_i + 1$, then the second element in the second product is zero so the second product is zero; however, both of the elements in the first product are positive yielding a positive fitness. If $T_k > \epsilon_i + 1 > T > \epsilon_i$, then the second element in the first product is zero so the first product is zero; however, both of the elements in the second product are positive yielding a positive fitness since untreated profit is higher than treated before the optimal threshold and treated profit is higher than untreated after. Note however, that for $T_k = T + \Delta$ where Δ is such that $\epsilon_i < T + \Delta < \epsilon_i + 1$ at least one of the terms in both products will be zero giving a fitness of zero despite $T_k \neq T$. However, as ϵ_i grows large $\epsilon_i + 1 - \epsilon_i \rightarrow 0$ $\forall T_k \rightarrow T$ zero for all ϵ_i . If $T_k = T$, then both elements in both products are zero for all ϵ_i , as described earlier. If $T_k < \epsilon_i < T < \epsilon_i + 1$, then the second element in the second product is zero so the second product is zero; however, both of the elements in the first product are positive yielding a positive fitness. If $T_k > \epsilon_i + 1 > T > \epsilon_i$, can be made arbitrarily small, requiring that D be made arbitrarily small as well, giving the result.

A NUMERICAL EXAMPLE

A numerical example is used to illustrate the estimation approach described in the previous study. A model is specified in order to generate data which are associated with an example where the optimal decision parameters are known. Estimation based on fitness (7) is then used to identify the solution to (1) using only the data which have been generated. Hence, the functional forms and parameters which generate the example data are neither estimated nor used in estimation. Ultimately, the approach identifies a sequence of experiments needed to best "zero in" on the economic threshold based on fitness (7). In this example, the following expression, based on functional forms contained in Liu *et al.* (1999) simulates the outcomes of experiments; i.e., is used for $v(x, \epsilon)$.

$$V(x, \epsilon) = p[y_0 - a\epsilon \exp(-bx)] - cx \quad (8)$$

Where, p is the price of the agricultural product, y_0 is yield per acre without pest damage, a is yield loss per pest unit, ϵ is number of pests per acre, $\exp(-bx)$ is the proportion of surviving pests with parameter b and pesticide treatment rate, x and the total cost of pesticide materials and application is given by $c x$. Satisfaction of the single crossing property by (8) is demonstrated in Appendix C; Therefore, the proposition shown in the preceding section is applicable. The following values are assumed for the economic and technical parameters: $p = 3$, $y_0 = 100$, $a = .4$, $b = 1$, $c = 50$, $D = .3$ and $\epsilon \sim N(50, 10)$. For these values of the economic and technical

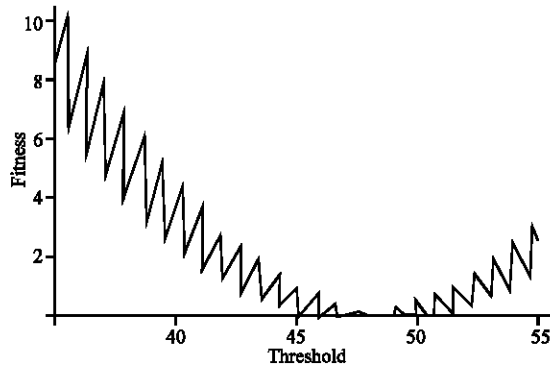
parameters, the optimal value of T , determined by solving (2), is $T = 48.2287$. Expected net revenue, $E[v(D, T)]$ evaluated using (1), is 241.536. Note again that the structure given in (8) along with numerical parameter values associated with (8) would not be known or estimated in practice rather only payoffs generated by experimentation would be observed. The expression (8) and numerical parameter values are used to simulate experimental results for the purpose of this example.

Processing of logical operations for fitness indicates the sequence of experiments that are needed. A range of 25 evenly spaced observations, ϵ_i , are used in conjunction with (8) to generate an initial database consisting of observations on $v(0, \epsilon_i)$; i.e., payoff without pesticide treatment (practical aspects of selecting such a range and number of observations are discussed in the next section) and referred to subsequently as the untreated database. The observations on x_i , ϵ_i and v_i associated with the initial untreated database are shown in Table 2. Note that the last column Processing of logical operations for fitness indicates the sequence of experiments that are needed. A range of 25 evenly spaced observations, ϵ_i , are used in conjunction with (8) to generate an initial database consisting of observations on $v(0, \epsilon_i)$; i.e., payoff without pesticide treatment (practical aspects of selecting such a range and number of observations are discussed in the next study) and referred to subsequently as the untreated database. The observations on x_i , ϵ_i and v_i associated with the initial untreated database are shown in Table 2. Note that the last column of Table 2 shows observed net revenue per acre when the pest population associated with a row in the table is left untreated. Observed net revenue per acre is simulated for purposes of this example using (8) and the parameter values shown earlier; hence, for the case of observation 1 in Table 2, observed net revenue per acre is 257.40 when a pest population of 35.5 is left untreated ($257.40 = 3 [100 - .4(35.5) \exp(-1(0))] - 50(0)$).

Figure 1 shows a graph of the fitness function (7) utilizing the untreated database (Table 2) for a large number of potential threshold values (T_k). It seems intuitive that threshold values further removed from the true threshold value will conflict with more points in the untreated database and will therefore generate larger fitness values from (7). This intuition is borne out by the u-shape of fitness shown in Fig. 1 and is the explanation for the basic u-shape of the graph of the fitness function generally. The "jagged" appearance of the u-shaped graph of fitness follows from the number of observations (25) and spacing of points in the untreated database. With a sufficiently large number of observations, a monotonic relationship between proximity of the threshold to the true

Table 2: Initial database on treatment (x_i), Pest Level (ϵ_i) and Payoff (v_i)

Observation	Treatment	Pest level	Payoff
1	0	35.5	257.40
2	0	36.3	256.44
3	0	37.1	255.48
4	0	37.9	254.52
5	0	38.7	253.56
6	0	39.5	252.60
7	0	40.3	251.64
8	0	41.1	250.68
9	0	41.9	249.72
10	0	42.7	248.76
11	0	43.5	247.80
12	0	44.3	246.84
13	0	45.1	245.88
14	0	45.9	244.92
15	0	46.7	243.96
16	0	47.5	243.00
17	0	48.3	242.04
18	0	49.1	241.08
19	0	49.9	240.12
20	0	50.7	239.16
21	0	51.5	238.20
22	0	52.3	237.24
23	0	53.1	236.28
24	0	53.9	235.32
25	0	54.7	234.36

Fig. 1: Graph of fitness ($f(D, T_k)$) given the initial untreated database

threshold and fitness is revealed. The slanted lines connecting the vertical portions of the graph result from the "missing" untreated values between observations in the untreated database and so do not fully reveal the underlying mono-tonic relationship. The distance between vertical portions of the graph reflects the spacing of points in the untreated database and reveals threshold values where a change in the number of points which conflict with the threshold occurs. Finer spacing of the untreated database reduces the "jagged" appearance. In the current example, an untreated database with approximately 200 points produces a smooth appearing graph when using the same scale shown in the figure. The graph reveals an approximate value of the fittest threshold; hence, given experimental observations for a large number of potential thresholds (T_k), it would be straightforward to estimate the true threshold.

Table 3: Population of Potential Thresholds (T_k) and Fitness ($f(D, T_k)$) by Iteration

Iteration		1	2	3	4
1	Threshold	50.10	44.49	50.03	53.05
	Fitness	0.3402	0.4377	0.4029	0.6003
2	Threshold	50.00	44.44	44.44	50.06
	Fitness	0.4309	0.3841	0.3841	0.3753
3	Threshold	44.44	46.00	44.44	48.44
	Fitness	0.3841	0.2977	0.3841	0.0000

In order that estimation of the threshold be practical, it is important to limit the number of experimental observations that are needed. Sequential sampling directed by an evolutionary algorithm provides a means of generating the needed information with a sequence of relatively small databases. Table 3 shows iterations and the fitness of each potential threshold as determined from (7). Note that each of the columns labeled 1, 2, 3 and 4 in Table 3 shows a potential threshold estimate and its fitness by iteration. A fitness value of zero indicates no contradictions arise when confronted with the untreated database. As is evident from the table, only three iterations with four potential threshold values per iteration are needed to provide a reasonably good estimate (48.44) of the economic threshold for this example problem. Expected net revenue, $E[v(D, T)]$ evaluated using (1) and this estimate of the economic threshold is 241.535 and differs by only a tenth of a cent from the Table 3 shows iterations and the fitness of each potential threshold as determined from (7). Note that each of the columns labeled 1, 2, 3 and 4 in Table 3 shows a potential threshold estimate and its fitness by iteration. A fitness value of zero indicates no contradictions arise when confronted with the untreated database. As is evident from the table, only three iterations with four potential threshold values per iteration are needed to provide a reasonably good estimate (48.44) of the economic threshold for this example problem. Expected net revenue, $E[v(D, T)]$ evaluated using (1) and this estimate of the economic threshold is 241.535 and differs by only a tenth of a cent from the optimal value. Of course, a larger, more refined untreated database could provide greater precision if desired in this example and outside this example as well if permitted by the relative cost of experimentation. In this example, a good estimate of the economic threshold was achieved with a relatively small amount of data. The underlying reason for the apparent efficiency may be that processing of logical operations to achieve fitness focuses data collection on the task of estimating the economic threshold rather than on myriad empirical issues that might be addressed by a generic, less focused sample design. The precision of the economic threshold estimate obtained in this example is especially noteworthy given that specification of parametric forms for dosage/response and damage relationships, often only vaguely known, was avoided.

Appendix C: This Appendix demonstrates satisfaction of the single crossing property by the payoff function used in the numerical example contained in the text.

Satisfaction of the single crossing property by the payoff function, $v(x, \epsilon)$, requires that for $x' > x''$, and $\epsilon' > \epsilon$, $v(x', \epsilon') > v(x'', \epsilon'') \Rightarrow v(x', \epsilon') > v(x'', \epsilon')$.

For $v(x, \epsilon) = p[y_0 - a \exp(-bx)] - c x$, the single crossing property requires that

$p[y_0 - a \exp(-bx')] - cx' > p[y_0 - a \exp(-bx'')] - cx''$.
 $\Rightarrow p[y_0 - a \exp(-bx')] - cx' > p[y_0 - a \exp(-bx'')] - cx''$.
 The first inequality implies that $p a \exp(-bx'') - \exp(-bx')$
 $> c(x' - x'')$ which implies that $p a \exp(-bx'') - \exp(-bx') > c(x' - x'')$ and thus, gives the second inequality.

PRACTICAL APPLICABILITY

This section focuses on practical considerations related to the estimation approach developed and illustrated earlier. Three important issues are examined in turn: The approach's requirement that control be exerted over a pest population level in generating data, accommodation of heterogeneity of biophysical relationships and/or other characteristics across farms and the need to develop an untreated database.

The need for control over the level of a pest population under experimental conditions is not uncommon and certainly not peculiar to this new approach. For example, populations of arthropods and weeds are controlled in experiments using techniques such as those involving cages and weeding/seeding since such control is often helpful in generating appropriate experimental data. Adequate precision is provided by such controls. There do not seem to be any additional requirements of the approach developed earlier that would preclude application of such experimental methods to generate the desired sequential samples. Heterogeneity across farms typically necessitates compromise in estimation of the economic threshold regardless of whether the evolutionary or traditional parametric approaches are used. Efforts by Hall (1988) in a parametric context are particularly revealing of the issue. Expressly for reasons of real-world practicality, Hall (1988) extended the notion of the economic threshold to the regional (multi-farm) level making the following remark:

"... a farmer might read an Agricultural Extension bulletin that recommends a pesticide application if the number of pests per square foot, estimated by a standard procedure, exceeds the economic threshold reported in the Agricultural Extension bulletin. Budget constraints don't permit designing experiments and performing calculations for every farmer. If the threshold is to be formally calculated rather than picked out of thin air, the best we can hope for is recommendations applicable to all

farmers in a region. The economic threshold for prescriptive purposes could be calculated in a fashion that optimizes profit to the growing region of farmers for whom the threshold is recommended."

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Hall (1988) maintains correctly that inter-farm differences require that research-based threshold recommendations inevitably involve recommendations believed to be good for the farmers using them as a group and must be based on representative conditions. Hence, while the evolutionary approach may enable avoidance of the need for specific functional specifications, it certainly does not resolve the general dilemma posed by heterogeneity and will require compromises with respect to heterogeneity in application similar to traditional approaches.

Finally, as is clear from the numerical example in the preceding section, application of the evolutionary design requires development of an untreated database. The latter must be based on a range of untreated population levels that includes the true threshold value as well as any population levels that will be evaluated during the course of estimation. Since the true threshold value is unknown, the untreated database must be based on a range that is believed to include it. Once a range has been selected, the desired precision for the threshold estimate can be used in conjunction with the range width to determine the number of untreated observations that are needed. Clearly, the more prior information there is about the threshold value, the narrower the range can be and the lower the resources devoted to estimation can be. If a range is specified that does not include the true threshold value, then it is likely that poor fitness will be observed and expansion of the untreated database will be warranted.

CONCLUSION

A new approach to estimating popular crop protection protocols was presented. The approach to collecting data for use with the approach is based on

sequential sampling driven by processing of logical operations by an evolutionary algorithm to achieve a fit estimate of the economic threshold. The sequential sample design differs significantly from that underlying traditional approaches to estimating the economic threshold; however, it enables the economic threshold to be estimated without requiring specific parametric forms for dosage/response and damage relationships. The approach is designed for implementation in a research environment where experimentation can be pursued sequentially to generate information. A numerical example illustrates the approach and reveals its effective-ness.

The effectiveness of the approach in the numerical example may suggest that other management decision problems where protocols are characterized by rules of thumb may also be addressable by suitable modification of the fitness concept though this remains to be seen. Even so, a number of issues related to use of the method in practice need additional research to arrive at protocols that can be counted on. In particular, applicability of the approach will require additional consideration of fitness in the context of a changing economic and biophysical environment and the manner in which efficiency in pooling of data generation can be achieved.

While untreated observations are often part of experiments in a research environment, such observations are rarely regarded as a significant part of data development for estimation of parametric forms (Moffitt, 2001). Results here suggest that unless untreated observations are featured as a significant part of data development, estimation of popular crop protection protocols will probably rely on specific forms for payoffs and nature's mixed strategy. Because of the nature of popular crop protection protocols, untreated payoffs contain substantial information for decision which may be leveraged by evolutionary processing of logical operations to permit estimation.

Finally, the definition of fitness utilized here proved effective in the numerical example presented. Whether

alternative definitions could prove more effective is a direction which may prove useful in further related research.

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