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# Matrix Inversion and Optical Flow Computation

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**Abstract:** The optical flow field is an approximation to the 2D motion field, which is produced by projecting the 3D scene velocity into the image. In order to be useful for tasks such as surface structure recovery or the estimation of viewer or object motion, the optical flow has to be dense and accurate. Optical flow computation is an important and challenging problem in the analysis of image sequences. It is a difficult and computationally expensive task and is an ill-posed problem, which expresses itself as the aperture problem. However, optical flow vectors can be estimated by using regularization methods, in which additional constraints functions are introduced. In this study we propose to improve optical flow estimation by including colour information as constraints functions in the optimization process. The proposed technique based an a simple matrix inversion using colour information as constraints functions in the optimization process and it has shown encouraging results.

**Key words:** Optical flow, motion estimation, colour information

## INTRODUCTION

A fundamental problem in processing sequences is the computation of optical flow. This flow is a 2D vector field resulting from a perspective projection on the image plane of the 3D velocity field of a moving scene. Optical flow is a convenient and useful way for image motion representation and 3D interpretation. It often plays a key role in varieties of motion estimation techniques and has been used in many computer vision applications. Optical flow may be used to perform motion detection, autonomous navigation (knowledge of local motion of the environment relative to the observer system simplifies the calculation time-to-collision and focus of-expansion for example), scene segmentation (segmenting scene into moving and static objects), surveillance system (motion can be an important source for a surveillance system when objects of interest can be detected and tracked using the optical flow vector to define the future trajectories), motion compensation for encoding sequences and stereo disparity measurement[1-3]. Optical flow estimation and computation methods can be classified into three main categories: differential approaches, block-matching approaches and frequential approaches[1]. Despite more than two decades of research, the proposed methods for optical flow estimation are relatively inaccurate and non-robust. Many methods for the estimation of optical flow have been proposed[1,2,4,5-14].

**Optical flow constraint equation:** Optical flow can be computed from a sequence by making assumptions about the variations of the scene brightness. One such assumption[4], known as the brightness constancy assumption, is represented by the following Eq:

$$I(x,y,t) = I(x+\delta x, y+\delta y, t+\delta t) \quad (1)$$

Where I(x, y, t) represents the luminance function at pixel (x, y) at time t and ($\delta x$, $\delta y$) is the displacement occurring at pixel (x, y) during $\delta t$.

We perform a Taylor development limited to the first order and we get:

$$I(x,y,t) = I(x,y,t) + \frac{\delta I}{\delta x}\delta x + \frac{\delta I}{\delta y}\delta y + \frac{\delta I}{\delta t} \quad (2)$$

Cancelling I(x, y, t) on both sides and dividing by $\partial t$ ($\partial t \rightarrow 0$) we obtain:

$$I_x u + I_y v + I_t = 0 \quad (3)$$

Where:
$I_x$, $I_y$ and $I_t$ are first partial derivatives of I respectively with respect to x, y and t and u and v are the optical flow components respectively in the x and y directions.

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Equation 3 is called optical flow constraint equation. It provides only the normal velocity component (aperture problem). The system is undetermined because we only have one equation for two unknowns. To overcome this problem, it is necessary to add an additional constraint.

Horn[4] adds a smoothness constraint based on the assumption that neighboring points of the same object will have similar velocities. This involves the computation of secondorder differentials. The goal is to minimize the sum of the total errors throughout the whole image:

$$a^2(I_x + I_y + I_t)^2 + (\nabla^2 u + \nabla^2 v)^2 \tag{4}$$

The first term expresses the error due to the change in image intensity and the second term relates to the change in velocity.

An alternative formulation due to Lucas and Kanade,[15] tries to minimize the error for a certain region R. It is categorized as the local methods by[1].

The error expression is:

$$min \sum_{X \in \Omega} W^2(x)[\nabla I(x,t).v + I_t(x,t)]^2 \tag{5}$$

In a considered neighbourhood, we have:

$$A^T W^2 A v = A^T W^2 b \tag{6}$$

Where for n pixels $x_i \in \Omega$:

$$\begin{aligned} A &= [\nabla I(x_1), \cdots, \nabla I(x_n)]^T \\ W &= diag[W(x_1), \ldots W(x_n)] \\ b &= [I_t(x_1), \ldots I_t(x_n)] \end{aligned} \tag{7}$$

They apply the second order differential as the weight W(x) to reduce the effect of those points where F(x) is far from linear. They also suggest that an iterative estimation of velocity and a coarse-fine searching can produce more accurate result.

Horn's method assumes temporal sampling rate is high enough for small spatial neighbourhood to work well, Lucas and Kanade's method is big improvement because it can use arbitrary region size which really helps when displacement is large. In order to obtain accurate first and second order differential, the input has to be highly over-sampled and the intensity through out the image is near linear[1].

These solutions are applied only to the grey sequences where the pixels intensity is represented by a single function which values are in [0:255].

**Use of colour information as additional constraint:** The brightness assumption implies that the (R, G, B) components of each image remain unchanged during the motion undergone within a small temporal neighbourhood[7]. Therefore, R, G and B images can be used in a similar way as the luminance function: they have to satisfy the optical flow constraint equation. Markandey and Flinchbaugh[6] have proposed a multispectral approach for optical flow computation. Their two-sensors proposal is based on solving a system of two linear equations having both optical flow components as unknowns. The equations are deduced from the standard optical flow constraint (3). In their experiments, they used colour TV camera data and a combination of infrared and visible images. Finally, they used two channels to resolve the ill-posed problem[13].

Golland and Bruckstein[8] follow the same algebraic method. They compare a straightforward 3-channels approach using RGB data with two 2-channel methods, the first based on normalized RGB values and the second based on a special hue-saturation definition. The standard optical flow constraint may be applied to each one of the RGB quantities, providing an over determined system of linear Eq.[13]:

$$\begin{cases} R_x u + R_y v + R_t = 0 \\ G_x u + G_y v + G_t = 0 \\ B_x u + B_y v + B_t = 0 \end{cases} \tag{8}$$

Then the pseudo-inverse computation gives the following solution for the system:

$$V = (A^T.A)^{-1}.A^T.b \tag{9}$$

Where:

$$A = \begin{bmatrix} R_x & R_y \\ G_x & G_y \\ B_x & B_y \end{bmatrix} ..;..b = \begin{bmatrix} -R_t \\ -G_t \\ -B_t \end{bmatrix} ..and..V = \begin{bmatrix} u \\ v \end{bmatrix} ..optical.flow.vector. \tag{10}$$

This assumes that the matrix ($A^TA$) is non-singular. By definition this matrix is singular if its rank is equal to 1, i.e. its columns or lines are linearly dependent, which mean that the first order spatial derivatives of the colour components (R, G, B) are dependent. Since the sensitivity functions Dr(λ), Dg(λ) and Db(λ) of the light detectors are linearly independent, the first derivatives of the R, G, B functions will also be independent for images sequence with colour changing in two different directions. But if the colour is a uniform distribution, the (R, G, B) functions are linearly dependent or if the colours of the considered region change in one direction only, the gradient vectors of (R, G, B) are parallel so that the spatial derivatives are dependent and the matrix ($A^TA$) is singular.

To improve this problem, the idea is the use of two independent functions for colour characterization so that their gradient directions are not parallel. The ideal case is obtained when the gradient directions of the two chosen functions are normal. One possible solution is the use of two different colour systems: the normalized RGB system, denoted rgb system and the HSV system[6,8,13].

The rgb system is computed in the following way:

$$\begin{cases} r = \dfrac{R}{R+G+B} \\ g = \dfrac{G}{R+G+B} .....\text{where}: ...r+g+b=1 \\ b = \dfrac{B}{R+G+B} \end{cases} \quad (11)$$

It is clear that any pair of (r, g, b) forms a system of two independent functions. If we are taking the r and g components, the optical flow computation system to be solved is given by Eq. 5, where:

$$A = \begin{bmatrix} r_x & r_y \\ g_x & g_y \end{bmatrix} ..;...b = \begin{bmatrix} -r_t \\ -g_t \end{bmatrix} ..\text{and}.. \quad V = \begin{bmatrix} u \\ v \end{bmatrix} ..\text{optical.flow.vector.} \quad (12)$$

If we consider the HSV system, we will have a similar system involving H and S instead of r and g.

**Proposed method for optical flow computation:** It was shown that a colour sequence could be straightforwardly considered as a set of three different sequences produced by three types of light sensors with different sensitivity functions in response to the same input sequence[6,8]. So we propose to use the same formulation as those proposed by Horn and Schunck for the luminance function and to apply it to the three colour components.

In the first stage we have to minimize a function containing the three colour components, each component satisfying the optical flow constraint equation without any smoothness term:

$$\min_{u,v} \begin{pmatrix} F = .\left(R_x.u + R_y.v + R_t\right)^2 + .\left(G_x.u + G_y.v + G_t\right)^2 \\ +.\left(B_x.u + B_y.v + B_t\right)^2 \\ = e_R^2 + e_G^2 + e_B^2 \end{pmatrix} \quad (13)$$

The problem will be posed as finding (u, v) optical flow components minimising F. By deriving F respectively to u and v and equalizing to zero we have the simple solution by using the inverse method to compute the optical flow components:

$$V = A^{-1}.b \quad (14)$$

Where:

$$A = \begin{bmatrix} R_x^2 + G_x^2 + B_x^2 & R_xR_y + G_xG_y + B_xB_y \\ R_xR_y + G_xG_y + B_xB_y & R_y^2 + G_y^2 + B_y^2 \end{bmatrix} \quad ;..b = -\begin{bmatrix} R_xR_t + G_xG_t + B_xB_t \\ R_yR_t + G_yG_t + B_yB_t \end{bmatrix}. \quad (15)$$

In the second stage we add a local (on a small region around each pixel) smoothness term on the magnitude of optical flow vector with a weight $\alpha$. The motion of any object between two following times ($t_0$ and $t_0 + \partial t$ where $\partial t \to 0$) is supposed to be very small and it can be used as a small displacement in any direction. So Eq. 9 with the smoothness term will be:

$$\underset{U,V}{\text{MIN}} \begin{pmatrix} F = (R_xu + R_yv + R_t)^2 + (G_xu + G_yv + G_t)^2 \\ +(B_xu + B_yv + B_t)^2 + \dfrac{1}{2}\alpha^2 \|V\|^2 \\ = \varepsilon_R^2 + \varepsilon_G^2 + \varepsilon_B^2 + \varepsilon_s^2 \end{pmatrix} \quad (16)$$

The same solution is found when adding the smoothness term in the function F to minimize. This solution is obtained by using only the matrix inversion method; we have the following matrix for this study:

$$A = \begin{bmatrix} R_x^2 + G_x^2 + B_x^2 + \alpha^2 & R_xR_y + G_xG_y + B_xB_y \\ R_xR_y + G_xG_y + B_xB_y & R_y^2 + G_y^2 + B_y^2 + \alpha^2 \end{bmatrix} \quad ;..b = -\begin{bmatrix} R_xR_t + G_xG_t + B_xB_t \\ R_yR_t + G_yG_t + B_yB_t \end{bmatrix}. \quad (17)$$

We do not use iterative method to compute the optical flow components and the proposed method is only based on the function optimisation and matrix inversion.

## RESULTS

In the implementation of all studied methods, the images of R, G and B, r and g and H and S are obtained from the brightness function of images sequence (R, G, B). The first order derivatives of the sequence functions are computed by using the (1/12)(-1, 8, 0, -8, 1) kernel. We used a 5X5 neighbourhood, where each line was a copy of the estimation kernel mentioned above. For the computation of temporal derivatives, a 3X3X2 spatiotemporal neighbourhood was used.

- For the four test sequence, grey scale sequence, we present the results with Horn Schunck and Lucas Kanade Fig. 1-3.
- For the Tennis sequence, grey scale sequence we applied the Horn-Schunck algorithm with it's two versions Fig. 4 and 5.
- For the synthetic sequence, colour sequence, we applied the proposed method with it's versions Fig. 6 and 7.

Fig. 1: Four sequences test. a) SRI : Camera right translation 2 pixels/frame. b) NASA: Camera moves 1 pixel/frame. c) Rubik cube: Cube and its support rotation 0.2 to 2.0 pixels/frame. d)Hamburg taxi: Four moving objects-taxi, car, van, and pedestrian par 1, 3, 3, 0.3 pixels/frame

Fig. 2 : Horn-schunck results ($\lambda = 2$, $\sigma = 3$)

Fig. 3: Lucas-Kanade results ( $\Omega = 5*5$ et $\lambda = 1.0$) results ($\lambda = 2$, $\sigma = 3$)

Fig. 4 : Tennis sequence

Fig. 5: Optical flow a) Horn Schunck b) Horn Schunck with smoothness term without smoothness term CPU time 1.922s CPU time 1.812s

Fig. 6: Synthetic sequence a) Using rgb space. b) Using HSV space. c) Using RGB space. d)Using RGB space with smoothness term ($\lambda$=3)

Fig. 7 : Optical flow results with the proposed method

Fig. 8: Perversi sequence

Fig. 9: Optical flow results a) Our method b) Horn Shunck CPU time 2.313s CPU time 2.531s for 5 iterations

- For the Perversi sequence, colour real sequence, we used the combination of R,G and B to form the intensity function for applying Horn Schunck algorithm with smoothness term compared with the RGB proposed method with smoothness term. Fig. 8, 9.

## CONCLUSION

The proposed method is fast in computing and has a simple implementation. But it requires the presence of significant gradients of the colour functions. If the gradient magnitude of these functions is too small ($\approx 0$), our method as any gradient-based method would fail to give reliable results. This implies that all differential methods are not reliable when the scene contains objects with uniform colour.

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