

## Adjustment by Sliding Mode of a Synchronous Motor Speed

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**Abstract:** The electric drive at variable speed is inevitably requested by external perturbation and it is the seat of its parametric variations. These effects, influence considerably on its behavior. However, this study presents a contribution aiming to improve the performances of an electric drive thanks to the use of the algorithms of the Sliding Mode Control (SMC). The results of digital simulation obtained, illustrate good performances of this technique.

**Key words:** Sliding mode, synchronous motor, direct torque control, inverter of voltage

### INTRODUCTION

In variable speed domain, the theory of Sliding Mode Control (SMC) (Buhler, 1986) come from the family of variable structure control has been used for the autopilotage of the synchronous motor. However, the SMC still has some weak point such as sensitivity to parameter variation and chattering of state path caused by discontinuous input and the torque oscillations. In this study, a sliding mode control which can be characterized by high accuracy, fast response and robustness is applied to speed control of Synchronous Motor of Permanent Magnate (SMPM).

We present solutions to limit these drawbacks by using a nonlinear limiting device. The content of the article, present initially the synthesis of the speed controller in sliding mode applied to a synchronous motor. Then solutions for the attenuation of the torque oscillations are developed. Lastly, a qualitative study of performances is carried out. We will start in the first time, to present the elements modelling of the synchronous motor followed by variable structure control. The machine and the inverter which feeds it are briefly described. Finally, the last part is devoted to the perturbation of speed control of the synchronous motor by the variable structure control and the evaluation of its performances according to system parameters variation. The simulation results in steady and transient state are show are compared for will validate the study suggested.

### MODEL OF THE SYNCHRONOUS MACHINE

The studied machine is a synchronous motor of permanent magnate, three-phase, balanced and with smooth rotor, whose magnets are laid out on surface of the rotor, the neutral is insulated and the instantaneous

sum of the currents of phase is null. Consequently, even if a possible homopolar FEM exists, it cannot contribute, in some form that it is, to the production of the electromagnetic torque of the machine (Bouzekri, 1995). Under these conditions, it is possible to separate the active FEM from those which are not it. This separation allows a simplification of the models of the machine and, especially, a better adaptation to a possible automation of calculation.

By taking account of the usual simplifying hypothesis (Barret, 1985) the model of the machine is represented by the following system of equations:

$$\frac{d}{dt} \begin{bmatrix} i_{ds} \\ i_{qs} \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L} i_{ds} + p i_{qs} \Omega \\ \frac{-R_s}{L} i_{qs} + p i_{ds} \Omega - \frac{p k_t}{L} \Omega \\ \frac{p k_t}{j} i_{qs} - \frac{1}{j} T_r \\ p \Omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_{ds} \\ U_{qs} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ \Omega \\ \theta \end{bmatrix} \quad (2)$$

- $U_{ds}, U_{qs}$  : Instantaneous voltages in the plan (d,q).
- $i_{ds}, i_{qs}$  : Instantaneous currents in the plan (d,q).
- $\Omega$  : Rotor angular speed expressed in electrical radians.
- $\theta$  : Rotor position.
- $R_s$  : Stator resistance of phase.
- $L$  : Cyclic inductance.
- $k_t$  : Permanent rotor flux.
- $p$  : Numbers of pair of poles.

**The direct torque control:** In a three-phase inverter of tow levels voltage (Faiz *et al.*, 2003) the voltage measured between the out put of each branch and the neutral point can have two values,  $U_c$  or  $V_{k0}$  with:

$$V_{k0} = S_k \cdot U_c \quad (3)$$

Where,  $S_k$  the signal controls k connects and  $U_c$  the voltage rectified at the entry of the inverter.

The operation can be described as follow Fig. 1.

With:

$S_k = 1$ : The switch top is closed and the switch of bottom is open.

$S_k = 0$ : The switch top is opened and the switch of bottom is closed.

Admitting that the point (n) is a neutral, the voltage of the neutral line can be evaluated as follow:

$$\begin{cases} V_{1n} = \frac{1}{3} U_c (2S_A - S_B - S_C) \\ V_{2n} = \frac{1}{3} U_c (-S_A + 2S_B - S_C) \\ V_{3n} = \frac{1}{3} U_c (-S_A - S_B + 2S_C) \end{cases} \quad (4)$$

The application of the Clarke frame allows the establishment of (1, 2, 3  $\rightarrow \alpha \beta$ ).

$$\begin{cases} V_\alpha = U_c (S_A - \frac{1}{2}(S_B + S_C)) \\ V_\beta = \sqrt{\frac{3}{2}} U_c (S_B - S_C) \end{cases} \quad (5)$$

Therefore, the space voltage vector  $\bar{V}_s$  might be written as follow.

$$\bar{V}_s = V_\alpha + jV_\beta = \sqrt{\frac{2}{3}} (V_{1n} + V_{2n} e^{j\frac{2\pi}{3}} + V_{3n} e^{-j\frac{2\pi}{3}}) \quad (6)$$

In this study, the flux estimate, is carried out by the integration of the stator voltage (Faiz *et al.*, 2003; Bolea *et al.*, 1994).

The selection of the voltage vector depends on the position of stator flux.

$$\begin{cases} \hat{\phi}_{\alpha s} = \int_0^t (V_{\alpha s} - R_s I_{\alpha s}) dt \\ \hat{\phi}_{\beta s} = \int_0^t (V_{\beta s} - R_s I_{\beta s}) dt \end{cases} \quad (7)$$

$$\bar{\phi}_s = \sqrt{\hat{\phi}_{\alpha s}^2 + \hat{\phi}_{\beta s}^2} \quad (8)$$

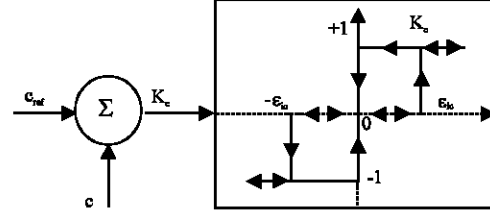


Fig. 1: Hysteresis bands three levels

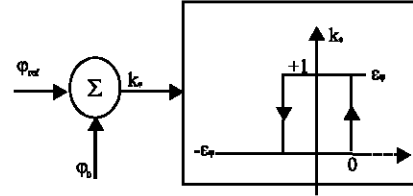


Fig. 2: Hysteresis bands tow levels

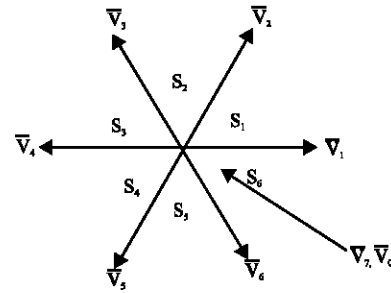


Fig. 3: Evolution of the flux vector in the plan (a,b) devoded on 6 sectors

The space in which located the flux vector  $\bar{\phi}_s$  is determined from the components  $\hat{\phi}_{\alpha s}$  and  $\hat{\phi}_{\beta s}$ .

The electromagnetic torque is given by:

$$\hat{T}_e = p.(\hat{\phi}_{\alpha s} I_{\beta s} - \hat{\phi}_{\beta s} I_{\alpha s}) \quad (9)$$

**Mode of electromagnetic torque control:** The value precalculated of electromagnetic torque is compared with that of its reference. An hysteresis bands on three levels represented in the Fig. 1. The out put of hysteresis bands is a function of the error value (Bahi, 1986).

$k_c = 1$  torque grows,  $k_c = 0$  torque decrease and  $k_c = -1$  stable torque.

In a similar way, measured flux will be compared with flux of reference, but by using a hysteresis bands on two levels, to see Fig. 2.

$K_\phi = 1$  flux grows;  $K_\phi = 0$  flux decrease.

The out put of the hysteresis bands must indicate the direction of evolution of the module of  $\bar{\phi}_s$ , in order to select the corresponding voltage vector (Fig. 3).

Table 1: Sequences of switching

| Sector (S) |            | 1   | 2   | 3   | 4   | 5   | 6   |
|------------|------------|-----|-----|-----|-----|-----|-----|
| $k_{q=1}$  | $k_{c=1}$  | 110 | 010 | 011 | 001 | 101 | 100 |
|            | $k_{c=0}$  | 111 | 000 | 111 | 000 | 111 | 000 |
|            | $k_{c=-1}$ | 101 | 100 | 110 | 010 | 011 | 001 |
| $k_{q=0}$  | $k_{c=1}$  | 010 | 011 | 001 | 101 | 100 | 110 |
|            | $k_{c=0}$  | 000 | 111 | 000 | 111 | 000 | 111 |
|            | $k_{c=-1}$ | 001 | 101 | 100 | 110 | 010 | 011 |

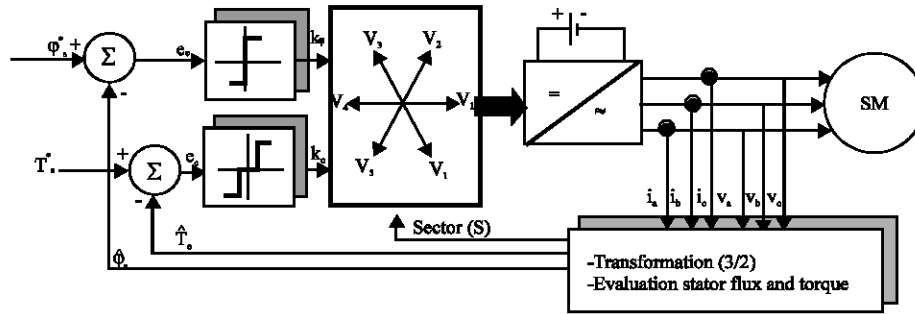


Fig. 4: Diagramme of the control system by DTC

The Table 1, shows the switching proposed by the technique of the Direct Torque Control (DTC) is as follows:

With DTC, the stator flux and torque can be controlled simultaneously. The control of the torque is ensured by a switching between the at-rest states (the voltages applied at the poles of the machine are null, stator flux is fixed) and the states active (the machine is supplied, stator flux evolves). An increase in the reference torque led to an acceleration of flux, so, an increase in the sliding and torque  $\hat{T}_e$ . Conversely, in the reference torque involves a deceleration of stator flux, so a minimize in the sliding and torque  $\hat{T}_e$  (Fig. 4).

### CONTROL BY SLIDING MODE

In order to reduce the influence of the parametric variations on the behavior of the system, we use algorithms of variable structure control for the speed control of synchronous motor of permanent magnet supplied with an inverter of voltage. This structure allowed to improve the robustness of the control desired in spite of the perturbation. The results of simulation obtained testify to the good performances of this technique of control. The structure of direct flux and torque control is then summarized in Fig. 5 (Bolea *et al.*, 1994; Etien *et al.*, 2002; Silva *et al.*, 2002).

The variable structure control is by nature a nonlinear system. The principal characteristic of the systems with variable structure is that their control law changes in a discontinuous way (Lachtar, 2006).

The modeling of variable structure control led to differential equations of the form:

$$\dot{x} = f(t, x) \quad (10)$$

Where,  $x$  is a vector of dimension  $n$ :  $x = (x_1, x_2, \dots, x_n)$  and  $f(t, x)$  are continuous functions per pieces, presenting discontinuities on a surface  $S$  which can be expressed the hypersurface  $S(x) = 0$ , of dimension  $(n-1)$  and that space divided into two parts according to the sign of  $S(x)$  positive or negative.

When the trajectory of phase remains on surface  $S(x)$ , the system is known as in sliding mode limited and that until it arrives in a state of balance. The condition for obtaining the sliding mode is as follow.

$$S(x) \cdot \dot{S}(x) < 0 \quad (11)$$

While this last condition is checked, the dynamics of the system as well as its stability are independent of the function  $f(t, x)$ , it's depends only on the parameters on the selected hypersurface. This explains the invariance of these control laws compared to the perturbation acting on the controlled part.

**Synthesis of the variable-structure control for control speed:** The diagram of control speed is shown as follow (Fig. 6).

The synthesis of the law of variable structure control for the control speed of the synchronous motor considered is made starting from mechanical equation:

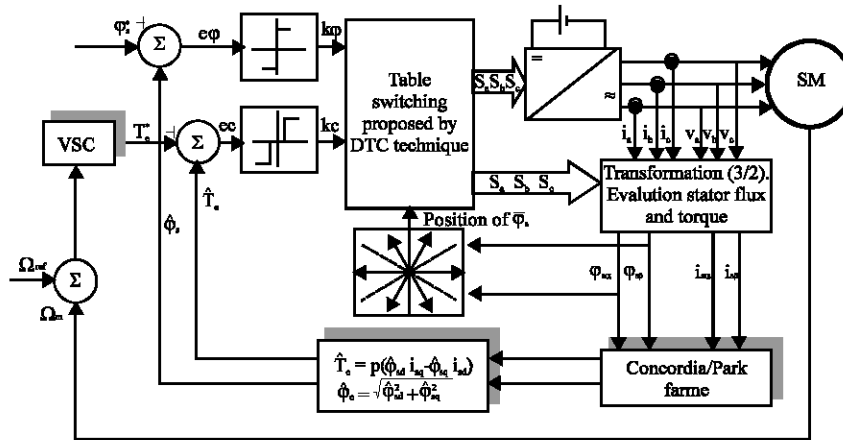


Fig. 5: Diagramme of the control system by a controller in sliding mode

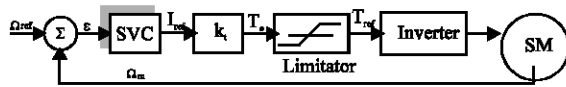


Fig. 6: Diagramme functional of control speed

$$J \frac{d\Omega}{dt} + f \cdot \Omega = T_e - T_r \quad (12)$$

Where:

- $T_r$  : The torque load.
- $f$  : The coefficient of viscous frictions.
- $J$  : Moment of inertia.

The electromagnetic torque can be described by following equation.

$$T_e = p \cdot k_t \cdot I_s \quad (13)$$

Where,  $I_s$  is stator current following the axis q.

We use a control variable structure with speed which generates the current of reference ( $I_{ref}$ ). The direct torque control of the synchronous motor is done by the imposition of the stator currents.

The mechanical equation linking the speed and the stator current is first order. By consequence we choose for the control speed a variety of order zero.

$$S_\Omega = K_\Omega \cdot e_\Omega \quad (14)$$

Where the error between the reference speed and measured speed is  $e_\Omega = \Omega_{ref} - \Omega_m$  and  $K_\Omega$  is a positive coefficient

The control used of type:

$$u = u_{eq} + u_n \quad (15)$$

The equivalent control is determined starting from the condition:

$$S_\Omega = \frac{dS_\Omega}{dt} = 0 \quad (16)$$

$$\frac{dS_\Omega}{dt} = \frac{de_\Omega}{dt} = \frac{d\Omega_{ref}}{dt} - \frac{d\Omega}{dt} = \frac{d\Omega_m}{dt} - \frac{p \cdot k_t}{J} \cdot (u_{eq} + u_n) + \frac{f}{J} \cdot \Omega_m + \frac{C_r}{J} = 0 \quad (17)$$

$$\frac{dS_\Omega}{dt} = -\frac{p \cdot k_t}{J} \cdot (u_{eq} + u_n) + \frac{f}{J} \cdot \Omega_m + \frac{C_r}{J} = 0$$

Given that  $u_n = 0$ , the expression of the equivalent control becomes:

$$u_{eq} = \frac{f \cdot \Omega_m + T_r}{p \cdot k_t} \quad (18)$$

The discontinuous control is selected so as to ensure the condition of sliding:

$$S_\Omega \cdot \frac{dS_\Omega}{dt} < 0 \quad (19)$$

Where:

$$\frac{dS_\Omega}{dt} = -\frac{p \cdot k_t \cdot u_n}{J} \quad (20)$$

The expression of  $dS_\Omega/dt$  is obtained by substituting the value of  $u_{eq}$  given by (17) in (18).

Thus, the algorithm:

$$\begin{cases} \text{if } S_\Omega > 0 \text{ then } u_n < 0 \\ \text{if } S_\Omega < 0 \text{ then } u_n > 0 \end{cases} \quad (21)$$

We will consider a discontinuous controller of type signs "Sgn". The influence of the varieties of sliding on the response is studied. Into their basic equations, terms relating to acceleration and to the load are introduced.

**Basic discontinuous controller:** Several choices for the discontinuous controller ( $u_n$ ) can be made. The simplest consists in expressing the discontinuous controller  $u_n = [u_1, u_2, \dots, u_m]$  with the function signs compared with:

$S = [S_1, S_2, \dots, S_m]$ :

$$\begin{cases} \text{Sgn}(S_\Omega) = +1 & \text{if } S_\Omega > 0 \\ \text{Sgn}(S_\Omega) = -1 & \text{if } S_\Omega < 0 \end{cases} \quad (22)$$

Thus  $u_n$  expresses itself as follow:

$$u_n = K \cdot \text{Sgn}(S_\Omega) \quad (23)$$

Where,  $K$  is a positive coefficient.

This first choice of the discontinuous function is represented in Fig. 7.

If the coefficient  $K$  is very small, the response time will be long, if  $K$  is very high, the response time will be fast but undesirable oscillations may well appear (usually called chattering) on the response in steady-state operation.

The variety of basic sliding for the control speed as follow:

$$S_\Omega = K \cdot e_\Omega \quad (24)$$

Where,  $E_\Omega = \Omega_{\text{ref}} - \Omega_m$  is the error between the reference speed and measured speed.  $K$  is an adaptive positive coefficient according to  $\Omega_{\text{ref}}$ .

The controller is of type (15):

$$\begin{cases} \text{if } S_\Omega > 0 & \text{then } \text{Sgn}(S_\Omega) = +1 \\ \text{if } S_\Omega < 0 & \text{then } \text{Sgn}(S_\Omega) = -1 \end{cases} \quad (25)$$

The result is extracted from the tests obtained for variable-structure control for a control speed set up on the synchronous motor and which will be presented in Fig. 8.

The Fig. 8, shows the response of speed, controller  $u_n$ , flux estimated and real one and torque estimated and real one in the case of a loadless starting for a level speed of  $100 \text{ rd s}^{-1}$  followed by a level of resistive torque value  $5 \text{ Nm}$  for  $t = 0.015 \text{ s}$  and  $10 \text{ Nm}$  for  $t = 0.03 \text{ s}$ .

These operating conditions were retained for all the tests relating to the control speed exposed in this part. The response in speed obtained with variable-structure

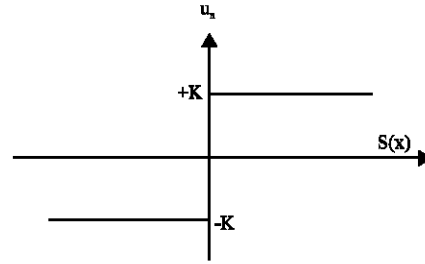


Fig. 7: Function signs

control for a loadless starting is very fast (lower than  $0.013 \text{ s}$ ). We observe that the error speed caused by the perturbation of the load is very quickly compensated. The response in torque is practically instantaneous. The high frequency oscillations and of high amplitude that noticed on the torque are due to the discontinuous part of the control which takes significant values. This is awkward because that way induces oscillations on the response speed.

With the aim of reducing high frequency oscillations (undesirable on the response), we will present some classic solutions which consist in imposing a variation of the value of the controller  $u_n$  according to the distance between the variable of state and the sliding surface (Silva *et al.*, 2002). Some of these methods introduce thresholds (dead space) on the switching of the function signs, which can be seen like a band surrounding surface of switching.

**Controller continue with integral component:** The high frequency oscillations which appear on the response in sliding mode can be avoided by returning continues the discontinuous controller  $u_n$  (23), by replacing the function signs by the function continues close:

$$u_n = K \cdot \frac{S_\Omega}{|S_\Omega| + \lambda} \quad (26)$$

Where,  $\lambda$  is a parameter defining the degree of attenuation of the oscillations. When  $\lambda$  tends towards to 0. We tend towards the same discontinuous controller defined by (23).

The function continuous  $u_n$  is illustrated on Fig. 9.

To increase the precision of the response of the system, we can use a continuous control including an integral component which becomes active when the point is close to surface. In general, the integral compensator decreases the error in steady-state operation, but it is often undesirable for the abrupt transient states, because it causes additional oscillations on the response. The controller  $u_n$  in this case becomes:

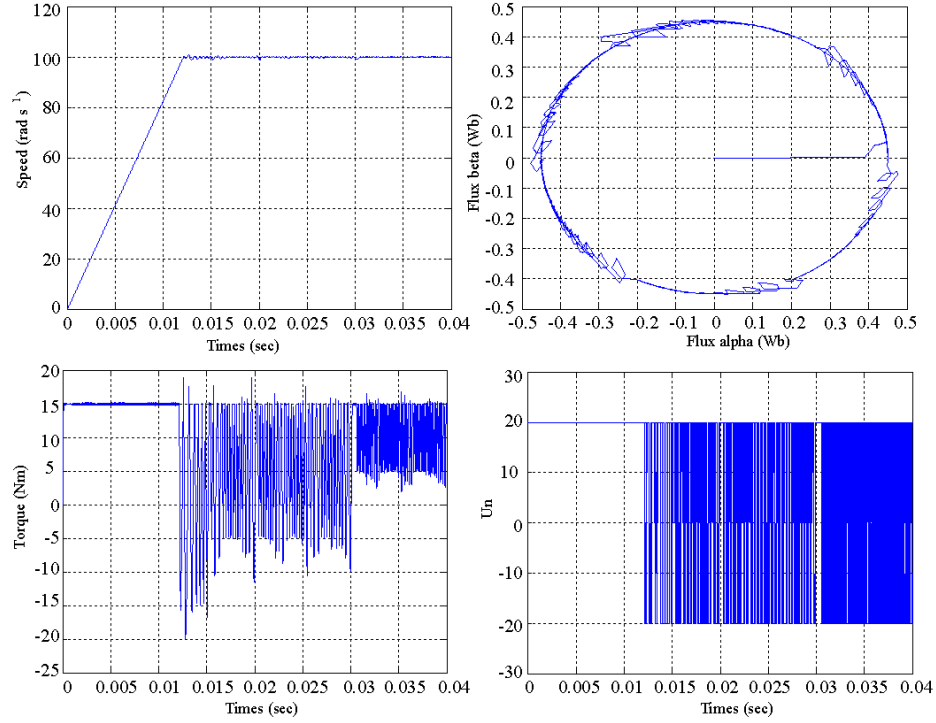


Fig. 8: Simulation with controller without integrate term

$$u_n = K \cdot \frac{S_\Omega}{|S_\Omega| + \lambda} + \eta \quad (27)$$

With:

$$\begin{aligned} \lambda &= \lambda_0 + |\gamma| \\ \begin{cases} \text{if } S_\Omega < \epsilon \text{ or } S_\Omega > -\epsilon & \text{then } \gamma = \gamma_0 \\ \int S_\Omega dt; \quad \eta = \eta_0 \int S_\Omega dt \\ \text{if } S_\Omega > \epsilon \text{ or } S_\Omega < -\epsilon & \text{then } \gamma = 0; \eta = 0 \end{cases} \quad (28) \end{aligned}$$

Where,  $\lambda_0$ ,  $\gamma_0$ ,  $\eta_0$ ,  $\epsilon$  are adaptive constants or parameters according to the references.

Using this controller, we divide space where the trajectories of phase in two parts evolve, the one with  $\lambda \rightarrow 0$  and  $|S_\Omega| > \epsilon$  with a controller of type (23) and the other where  $|S_\Omega| < \epsilon$  with a controller of type (27).

The parameter  $\epsilon$  is determined according to the concrete system and its characteristics. It is used for put on or put off the integral action during certain operations.

This type of controller is difficult to put into practice because there is a great number of parameter to determine.

The determination of the coefficients  $\lambda$ ,  $\gamma$ ,  $\eta$  and  $\epsilon$  result from the experience accumulated during the digital simulations and the determination of the quoted parameters is not exprimable in analytical form. In general,

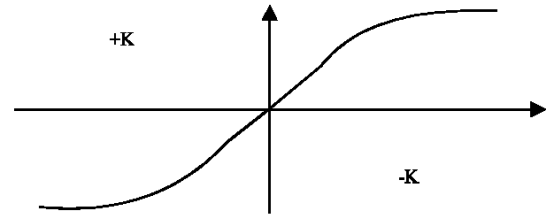


Fig. 9: Variable structure control made

the parameter  $\lambda$  is about the maximum error for control concerned. The parameter  $\epsilon$ , defines the conditions so that the integral correction is active or not and its definition corresponds to the aim of control (rapidity, precision, robustness). If a high rapidity is wished, we can activate the integral correction that in end of the transient state. The values of the parameters  $\lambda$  and  $\eta$  are determined according to the choice of the compromise rapidity, precision and robustness by digital simulation (Fig. 10).

We notice a clear improvement of the response in speed and in torque.

The frequency of switching of the controller is increased considerably and the oscillations on the response are eliminated.

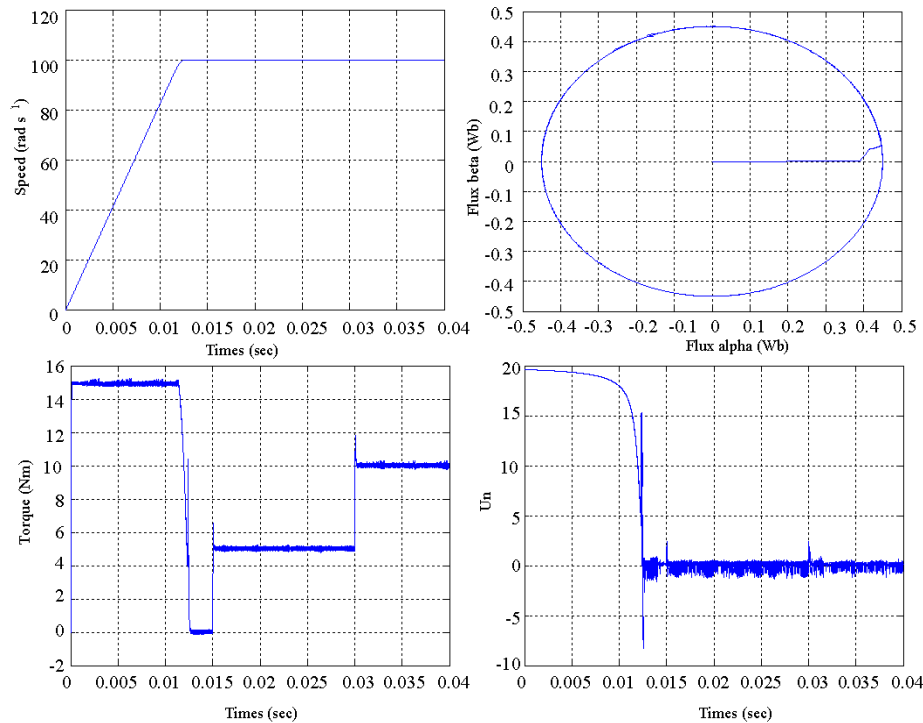


Fig. 10: Simulation with control with integral with softening

## CONCLUSION

The performances offered by the application of the adjustment to variable-structure control are promising as long as with the application for the synchronous motor uses in the variable speed installations. The possibility of the choice of the sliding surface makes the adjustment softer and improves torque quality developed.

This technique of adjustment in sliding mode finds its interest for the installations or the model and not easily identifiable exactly.

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