

Interactive Search Based Stochastic Multiobjective Thermal Power Dispatch

¹S. Jarnail Dhillon, ²J.S. Dhillon and ³D.P. Kothari

¹Department of Electrical Engineering, G.Z.S. College of Engineering and Technology,
Bathinda, 151001, Punjab, India

²Department of Electrical and Instrumentation Engineering Sant Longowal,
Institute of Engineering and Technology Longowal, 48106, Punjab, India

³Centre for Energy Studies, Indian Institute of Technology, Delhi, New Delhi, 110016, India

Abstract: This study deals with decision making methodology based on fuzzy set theory to determine the optimal generation schedule of multi-objective problem with due consideration of uncertainties in system input data and system load. The stochastic models are converted into their deterministic equivalent by taking their expected values. To determine trade-off relationship between conflicting objectives in the non-inferior domain, the weighting method is exploited. In this method, the multiobjective problem is first converted into a scalar optimization problem and then weights are searched in a systematic manner. A new interactive search technique based on binary successive approximation method is devised to search weights assigned to the objectives and incremental cost to obtain the non-inferior solution. Binary coded strings are used to represent weights assigned to the objectives as well as the incremental cost and the continuous values are obtained to represent a point in the search space through mapping. Once the trade-off has been obtained, fuzzy set theory helps the Decision Maker (DM) to choose the optimal operating point over the trade-off curve and adjust the generation schedule in the most preferred economic manner. This method has shown improvement because the weights are searched for more significant digits in fixed number of iterations. The validity of the proposed method has been examined on a sample system and the results are compared with the efficient method to solve the scalar objective problem and weight pattern is searched by evolutionary search method.

Key words: Stochastic multi-objective optimization, economic load dispatch, fuzzy set, evolutionary method, successive approximation method

INTRODUCTION

In a large number of real-life problems, a decision-maker is faced with multiple goals. The levels of attainment of these goals are to be expressed in the form of qualitative performance criteria, some of which can be selected as optimization objectives. Normally, the decision making input system data were assumed to be well behaved and deterministic. But in practical situations the input system data cannot be predicted and estimated with hundred percent certainties. It is bound to vary depending upon the uncertainties, load changes, load forecasting errors, ageing of equipment, measurement errors etc. So the single datum used in the generation scheduling procedure can be incorrect in real life circumstances. Due to these variations, the optimum solution found out using deterministic data cannot result into practically optimum solution. It is worthwhile to assume the system data as variable and uncertain for more

realistic approach (Gent and Lamont, 1971; Dhillon *et al.*, 1993, 1985, 2002; Viviani and Heydt, 1981).

However, with the increasing concern given recently to environmental considerations, a revised power dispatch is required that meets the demand for power while accounting for both cost and emission. An excellent review by Chowdhary and Rahman (1980) updates the developments in the area. The earlier proposals for minimum emission dispatch (Gent and Lamont, 1971) or reduced area wide emission (Delson, 1974) were usually rather simple, adding a single constraint to the problem. The cost and emission functions are conflicting functions in that minimizing pollution maximizes cost and vice versa. So, multiple criteria must be considered simultaneously to attain meaningful, practical, optimal schedule of operation. Nanda *et al.* (1988) has proposed a goal programming technique to solve the optimal load dispatch problem for thermal generating units running with natural gas and fuel oil. Yokoyama *et al.* (1988) have proposed an efficient

algorithm to obtain the optimal power flow in a power system operation and planning phases by solving a multi-objective optimization problem. Kermanshahi *et al.*, Lakhwinder (1990) presented a decision making methodology to determine the optimal generation dispatch and environmental marginal cost for power system operation with multiple conflicting objectives. The economic emission load dispatch problem has been solved through different methods (Kotheri and Dhillon, 2004, Huang and Huang, 2003).

Therefore, in this study the authors formulate multiobjective generation scheduling problem as a stochastic multi-objective problem with explicit recognition of uncertainties in the system production cost coefficients, emission coefficients and system load, which are treated as random variables. The objectives are clubbed in a single objective with the help of the weighting method. The successive approximation method is proposed to search the optimal weight pattern in the non-inferior domain. Further for a known weight combination, the generation schedule is also obtained by successive approximation method in which incremental cost, λ_p is represented by binary coded string. Fuzzy methodology has been exploited for solving a decision making problem involving multiplicity of objectives and selection criterion for best compromised solution. The objectives are quantified by eliciting the corresponding membership function. The shape of fuzzy membership function may be decided by the DM and generally depends upon the type of the problem. In this study, the membership function of the objectives is defined in a subjective manner by considering the rate of increase of membership satisfaction function. The best compromised solution is one, which provides maximum satisfaction level from the participating objectives/goals during the search of weights.

STOCHASTIC MULTIOBJECTIVE OPTIMIZATION PROBLEM FORMULATION

In the multiobjective problem formulation, five important non-commensurable objectives in electrical thermal power system are undertaken. These are economy, environmental impacts because of emissions. The multi-objective optimization problem is defined as:

$$\text{Minimize cost } J_1 = \sum_{i=1}^N (a_{i1}P_i^2 + b_{i1}P_i + c_{i1}) \text{ Rs } h^{-1} \quad (1a)$$

$$\text{Minimize } NO_x \text{ emission } J_2 = \sum_{i=1}^N (a_{i2}P_i^2 + b_{i2}P_i + c_{i2}) \text{ kg } h^{-1} \quad (1b)$$

$$\text{Minimize } CO_2 \text{ emission } J_3 = \sum_{i=1}^N (a_{i3}P_i^2 + b_{i3}P_i + c_{i3}) \text{ kg } h^{-1} \quad (1c)$$

$$\text{Minimize } SO_2 \text{ emission } J_4 = \sum_{i=1}^N (a_{i4}P_i^2 + b_{i4}P_i + c_{i4}) \text{ kg } h^{-1} \quad (1d)$$

Subject to (i) power balance equation

$$\sum_{i=1}^N P_i = P_D + P_L \quad (1e)$$

$$\text{Power limits } P_i^L \leq P_i \leq P_i^U \quad i = 1, 2, \dots, N \quad (1f)$$

where

a_{i1} , b_{i1} and c_{i1} are cost coefficients of i th unit
 a_{i2} , b_{i2} and c_{i2} are NO_x emission coefficients of i th unit
 a_{i3} , b_{i3} and c_{i3} are SO_x emission coefficients of i th unit
 a_{i4} , b_{i4} and c_{i4} are CO_x emission coefficients of i th unit
 P_i is real power generation of i th unit.
 P_D is the power demand.

P_i^L and P_i^U are the lower and upper limits of real power, respectively.

N is the number of generators

P_L is the transmission loss and is expressed through the simplified well known loss formula expression as a quadratic function:

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N B_{i0} P_i + B_{00} P_0 \quad (1g)$$

The stochastic model of multi-objective problem is formulated by considering cost coefficients, emission coefficients and load demand as random variables. Then the generator output automatically becomes random. Random variables are considered as normally distributed and statistically dependent to each other. By taking expectations, the stochastic model can be converted into its deterministic equivalent. The expected value of a function can be obtained by expanding the function, employing Taylor's series, about the mean. Deterministic equivalent of stochastic multi-objective optimization problem is stated as:

Minimize expected cost

$$\begin{aligned} \bar{J}_1 = & \sum_{i=1}^N \bar{a}_{i1} \bar{P}_i^2 + \bar{b}_{i1} \bar{P}_i + \bar{c}_{i1} + \bar{a}_{i1} \text{var}(P_i) \\ & + 2\bar{P}_i \text{cov}(a_{i1}, P_i) + \text{cov}(b_{i1}, P_i) \end{aligned} \quad (2a)$$

Minimize expected NO_x emission

$$\bar{J}_2 = \sum_{i=1}^N \bar{a}_{i2} \bar{P}_i^2 + \bar{b}_{i2} \bar{P}_i + \bar{c}_{i2} + \bar{a}_{i2} \text{var}(P_i) + 2\bar{P}_i \text{cov}(a_{i2}, P_i) + \text{cov}(b_{i2}, P_i) \quad (2b)$$

Minimize expected CO₂ emission

$$\bar{J}_3 = \sum_{i=1}^N \bar{a}_{i3} \bar{P}_i^2 + \bar{b}_{i3} \bar{P}_i + \bar{c}_{i3} + \bar{a}_{i3} \text{var}(P_i) + 2\bar{P}_i \text{cov}(a_{i3}, P_i) + \text{cov}(b_{i3}, P_i) \quad (2c)$$

Minimize expected SO₂ emission

$$\bar{J}_4 = \sum_{i=1}^N \bar{a}_{i4} \bar{P}_i^2 + \bar{b}_{i4} \bar{P}_i + \bar{c}_{i4} + \bar{a}_{i4} \text{var}(P_i) + 2\bar{P}_i \text{cov}(a_{i4}, P_i) + \text{cov}(b_{i4}, P_i) \quad (2d)$$

Minimize expected variance of power

$$\bar{J}_5 = \left(\sum_{i=1}^N \text{var}(P_i) + \sum_{i=1}^{N-1} \sum_{j=i+1}^N 2 \text{cov}(P_i, P_j) \right) \quad (2e)$$

$$\text{Subject to} \quad \sum_{i=1}^N \bar{P}_i = \bar{P}_D + \bar{P}_L \quad (2f)$$

$$\bar{P}_i^L \leq \bar{P}_i \leq \bar{P}_i^U, \quad i = 1, 2, \dots, N \quad (2g)$$

where \bar{P}_i is the expected real power generation ith of generator,

\bar{a}_{i1} , \bar{b}_{i1} and \bar{c}_{i1} are the expected cost coefficients of ith unit.

\bar{a}_{i2} , \bar{b}_{i2} and \bar{c}_{i2} are the expected NO_x emission coefficients of ith unit

\bar{a}_{i3} , \bar{b}_{i3} and \bar{c}_{i3} are the expected SO₂ emission coefficients of ith unit

\bar{a}_{i4} , \bar{b}_{i4} and \bar{c}_{i4} are the expected CO₂ emission coefficients of ith unit

\bar{P}_D is the expected power demand.

\bar{P}_i^L and \bar{P}_i^U are the expected lower and upper limits of real power, respectively.

N is the number of generators

\bar{P}_L is the expected transmission loss and is given as:

$$\begin{aligned} \bar{P}_L = & \sum_{i=1}^N \sum_{j=1}^N \bar{P}_i \bar{B}_{ij} \bar{P}_j + \sum_{i=1}^N \bar{B}_{ii} \text{var} P_i \\ & + \sum_{i=1}^{N-1} \sum_{j=i+1}^N 2.0 \bar{B}_{ij} \text{cov}(P_i, P_j) + \sum_{i=1}^N 2.0 \bar{P}_i \text{cov}(P_i, B_{ii}) \\ & + \sum_{i=1}^N \sum_{j=1}^N 2.0 \bar{P}_j \text{cov}(P_i, B_{ij}) + \sum_{i=1}^N \bar{B}_{io} \bar{P}_i + \bar{B}_{oo} \bar{P}_o \end{aligned} \quad (2h)$$

In this study, variance and covariance are replaced by the coefficients of variation and correlation, respectively. In general variance and covariance are defined as:

$$\text{var}(x) = C^2(x) \bar{x}^2 \quad (3a)$$

$$\text{cov}(x, y) = R(x, y) C(x) C(y) \bar{x} \bar{y} \quad (3b)$$

where $C(x)$ and $C(y)$ are the coefficients of variation and \bar{x} and \bar{y} are the expected values of variables x and y , respectively. $R(x, y)$ is correlation coefficient and varies from -1 to 1. The zero value of coefficient of variation implies no randomness, in other words, the complete certainty about the value of random variables. Using (3a) and (3b), the multi-objective optimization problem defined by (2a-2h) can be rewritten as:

$$\text{Minimize} \quad [\bar{J}_1, \bar{J}_2, \bar{J}_3, \bar{J}_4, \bar{J}_5]^T \quad (4a)$$

$$\text{Subject to} \quad \sum_{i=1}^N \bar{P}_i = \bar{P}_D + \bar{P}_L \quad (4b)$$

$$\bar{P}_i^L \leq \bar{P}_i \leq \bar{P}_i^U, \quad i = 1, 2, \dots, N \quad (4c)$$

where

$$\bar{J}_j = \sum_{i=1}^N (\bar{A}_{ij} \bar{P}_i^2 + \bar{B}_{ij} \bar{P}_i + \bar{C}_{ij}) \quad j = 1, 2, 3, 4.$$

with

$$\bar{A}_{ij} = [1 + C^2(P_i) + 2R(a_{ij}, P_i) C(a_{ij}) C(P_i)] \bar{a}_{ij} \quad (4d)$$

$$\bar{B}_{ij} = [1 + R(b_{ij}, P_i) C(b_{ij}) C(P_i)] \bar{b}_{ij}$$

$$\bar{C}_{ij} = \bar{c}_{ij}$$

$$\bar{J}_5 = \sum_{i=1}^N \sum_{j=1}^N P_i T_{ij} P_j \quad (4d)$$

with

$$\begin{aligned} T_{ii} &= \sum_{i=1}^N C^2(P_i) \bar{P}_i^2 \\ T_{ij} &= \sum_{i=1}^N R(P_i, P_j) C(P_i) C(P_j) \bar{P}_i \bar{P}_j ; i \neq j \\ \bar{P}_L &= \sum_{i=1}^{NG} \sum_{j=1}^{NG} \bar{P}_i U_{ij} \bar{P}_j + \sum_{i=1}^N \bar{B}_{io} \bar{P}_i + \bar{B}_{oo} \bar{P}_o \end{aligned} \quad (4e)$$

with

$$\begin{aligned} U_{ii} &= 1.0 + C(P_i)^2 + 2.0 R(P_i, B_{ii}) C(P_i) C(B_{ii}) \\ U_{ij} &= 1.0 + R(P_i, P_j) C(P_i) C(P_j) + 2.0 R(P_i, B_{ij}) \\ &\quad C(P_i) C(B_{ij}) \bar{P}_i B_{ij} P_j ; i \neq j \end{aligned}$$

SOLUTION APPROACH

To generate the non-inferior solutions of the multi-objective problem, the weighting method is used. In this method, the multi-objective optimization problem is converted into a scalar optimization problem as:

$$\text{Minimize } \sum_{n=1}^5 w_n \bar{J}_n \quad (5a)$$

$$\text{Subject to } \sum_{i=1}^N \bar{P}_i = \bar{P}_D + \bar{P}_L \quad (5b)$$

$$\bar{P}_i^L \leq \bar{P}_i \leq \bar{P}_i^U, \quad i = 1, 2, \dots, N \quad (5c)$$

$$\sum_{n=1}^5 w_n = 1, w_n \geq 0 \quad (5d)$$

To solve the scalar optimization problem, the Lagrangian function is defined as:

$$L(\bar{P}_i, \lambda_p) = \sum_{n=1}^5 w_n \bar{J}_n + \sum_{i=1}^N \lambda_p (\bar{P}_D + \bar{P}_L - \sum_{i=1}^N \bar{P}_i) \quad (6)$$

where λ_p is the Lagrangian multiplier.

The necessary conditions to minimize the unconstrained Lagrangian function are:

$$\frac{\partial L}{\partial \bar{P}_i} = \sum_{n=1}^5 w_n \frac{\partial \bar{J}_n}{\partial \bar{P}_i} + \lambda_p \left(\frac{\partial \bar{P}_L}{\partial \bar{P}_i} - 1 \right) = 0, \quad i = 1, 2, \dots, N \quad (7a)$$

$$\frac{\partial L}{\partial \lambda_p} = \bar{P}_D + \bar{P}_L - \sum_{i=1}^N \bar{P}_i = 0 \quad (7b)$$

The above equations can be rewritten as:

$$\sum_{j=1}^N X_{ij} \bar{P}_j = Y_i \quad i = 1, 2, \dots, N \quad (8)$$

$$\text{where } X_{ii} = \sum_{k=1}^4 2 w_k \bar{A}_{ik} + 2 (w_5 T_{ii} + \lambda_p U_{ii})$$

$$X_{ij} = \sum_{k=1}^N 2 (w_5 T_{ij} + \lambda_p U_{ij}) \quad ; i \neq j$$

$$Y_i = \lambda_p (1 - \bar{B}_{io}) - \sum_{k=1}^4 w_k \bar{B}_k$$

As λ_p is known during the search, \bar{P}_i is obtained by solving above simultaneous equations using Gauss Elimination method. The search of λ_p is terminated when (7b) is satisfied.

DECISION MAKING

Considering the imprecise nature of the DM's judgment, it is natural to assume that the DM may have fuzzy or imprecise goals for each objective function. The fuzzy sets are defined by equations called membership functions. These functions represent the degree of membership in certain fuzzy sets using values from 0 to 1. The membership value 0 indicates incompatibility with the sets, while 1 means full compatibility. By taking account of the minimum and maximum values of each objective function together with the rate of increase of membership satisfaction, the DM must determine the membership function $\mu(J_i)$ in a subjective manner. It is assumed that $\mu(J_i)$ is a strictly monotonic linear decreasing and continuous function and is defined as:

$$\mu(J_i) = \begin{cases} 1 & ; J_i \leq J_i^{\min} \\ \frac{J_i^{\max} - J_i}{J_i^{\max} - J_i^{\min}} & ; J_i^{\min} \leq J_i \leq J_i^{\max} \\ 0 & ; J_i \geq J_i^{\max} \end{cases} \quad (9)$$

where J_i^{\min} and J_i^{\max} are the minimum and maximum values of i th objective function in which the solution is expected. The value of membership function suggests

how far (in the scale from 0 to 1) a non-inferior solution has satisfied the \bar{J}_i objective. The decision regarding the best solution is made by the selection of minimax of membership function as defined below (Tapia and Murtagh, 1991).

$$\mu^D = \text{Max} \left[\text{Min} \left\{ \mu(J_j)^k ; j = 1, 2, \dots, 5 \right\} ; k = 1, 2, \dots, 2^L + 1 \right] \quad (10)$$

The function μ_D in (10) can be treated as a membership function for non-dominated solutions. The solution which attains highest membership μ_D^k in the fuzzy set so obtained can be chosen as best solution or the one having highest cardinal priority ranking.

ALGORITHM FOR HEURISTIC SEARCH OF INCREMENTAL COST

In the proposed method the number of binary bits to represent the incremental cost and weights has been selected as thirty and six, respectively to get accurate results. The successive approximation strategy to search the incremental cost, λ is elaborated here. The stepwise procedure is outlined below:

- Read NB, number of binary digits to represent, λ .
- Set binary digit counter, $i = 1$
- $N = 2^{NB-i}$
- Increment i ; $i = i + 1$
- If ($i = NB$) then Goto 10
- Determine N_1 and N_2 as
 $N_1 = N + 2^{NB-1}$
 $N_2 = N - 2^{NB-1}$
- Determine λ_1 and λ_2 as

$$\lambda_1 = \lambda^{\min} + \frac{N_1}{2^{NB} - 1} (\lambda^{\max} - \lambda^{\min}) \quad (11)$$

- Determine P_i^1 ; $i = 1, 2, \dots, N$ from (8) using Gauss Elimination method

$$\text{Determine } \Delta P_D^1 = \left| P_D + P_L - \sum_{i=1}^N P_i^1 \right| \quad (12)$$

$$\lambda_2 = \lambda^{\min} + \frac{N_2}{2^{NB} - 1} (\lambda^{\max} - \lambda^{\min}) \quad (13)$$

- Determine P_i^2 ; $i = 1, 2, \dots, N$ from (8) using Gauss Elimination method

$$\text{Determine } \Delta P_D^2 = \left| P_D + P_L - \sum_{i=1}^N P_i^2 \right| \quad (14)$$

- If ($\Delta P_D^1 < \Delta P_D^2$) THEN set $N = N_1$ and $\Delta P_D = \Delta P_D^1$
 ELSE set $N = N_2$ and $\Delta P_D = \Delta P_D^2$

- If ($\Delta P_D \leq \epsilon$) then continue ELSE GOTO 4
- Stop.

Algorithm for heuristic search of weights: Heuristic evolutionary method is proposed to search the optimal weight combination. In this method $(2^L + 1)$ weight combinations are simulated at 2^L corner points of an L-dimensional hypercube centered on initial point w_i^c . $(2^L + 1)$ non-inferior solutions are generated and membership function is obtained using (9). The best or preferred non-inferior solution is obtained using (10). To continue the iterative process, another hypercube is formed around the preferred point. Successive approximation strategy to search the weights is elaborated here. The weights are generated as given below:

$$\alpha_i^j = \alpha_i^c + \gamma_i^j ; i = 1, 2, \dots, L ; j = 1, 2, \dots, 2^L \quad (15)$$

where $\gamma_i^j = \pm 2^{NW-k}$

with NW is the number of binary bits used to represent weights.

α_i^j is the scalar weights, α_i^c is the initial value of weights.

Weights β_i^j , are mapped in the range of 0-100 Eq. 16.

$$\beta_i^j = \beta_i^{\min} + \frac{\alpha_i^j}{2^{NW-1}} (\beta_i^{\max} - \beta_i^{\min}) ; i = 1, 2, \dots, L \quad (16)$$

$$; j = 1, 2, \dots, 2^{L-1}$$

The normalized weights, w_i^j are obtained as:

$$w_i^j = \frac{\beta_i^j}{\sum_{i=1}^L \beta_i^j} ; j = 1, 2, \dots, 2^{L-1} \quad (17)$$

where β_i^{\min} and β_i^{\max} are the minimum and maximum value of the weights, β_i^j , respectively (0-100). γ is the distance of the corners of the hypercube from the point around which hypercube is generated. A matrix has been generated from possible combinations of binary bits. 0 bit is replaced by $-\gamma$ and 1 bit is replaced by $+\gamma$. As an illustration the generation of weight combinations for three objectives has been shown in Table 1. For three objectives 2^3 (eight) different possible weight combinations can be obtained. In general the different possible weight combinations are 2^L .

To implement the heuristic evolutionary search, the stepwise algorithm is outlined as below:

Table 1: Generation of weight combinations at hypercube corners (Three objectives)

Hypercube corners	Possible combinations of three binary bits b_2, b_1, b_0	Distance of hypercube corners from centre point a'_1, a'_2, a'_3	Possible generated weights at the hypercube corners
1	000	$- \gamma \ - \gamma \ - \gamma$	$a'_1 - \gamma \ a'_2 - \gamma \ a'_3 - \gamma$
2	001	$- \gamma \ - \gamma \ + \gamma$	$a'_1 - \gamma \ a'_2 - \gamma \ a'_3 + \gamma$
3	010	$- \gamma \ + \gamma \ - \gamma$	$a'_2 - \gamma \ a'_1 + \gamma \ a'_3 - \gamma$
4	011	$- \gamma \ + \gamma \ + \gamma$	$a'_1 - \gamma \ a'_2 + \gamma \ a'_3 + \gamma$
5	100	$+ \gamma \ - \gamma \ - \gamma$	$a'_1 + \gamma \ a'_2 - \gamma \ a'_3 - \gamma$
6	101	$+ \gamma \ - \gamma \ + \gamma$	$a'_1 + \gamma \ a'_2 - \gamma \ a'_3 + \gamma$
7	110	$+ \gamma \ + \gamma \ - \gamma$	$a'_1 + \gamma \ a'_2 + \gamma \ a'_3 - \gamma$
8	111	$+ \gamma \ + \gamma \ + \gamma$	$a'_1 + \gamma \ a'_2 + \gamma \ a'_3 + \gamma$

- Input the data.
- Find the minimum and maximum values of objectives; J_i^{\min} and J_i^{\max} $i = 1, 2, \dots, L$.
- Set the initial centre $\alpha_i^c = 2^{NW-1}$ where NW is the number of bits to represent weights.
- Set the initial value of membership function $\mu^P = 0$.
- Initialize iteration counter, $r = 0$.
- Increment iteration counter, $r = r+1$.
- Generate 2^L weight combinations at the edges of hypercube as given by (11).
- Initialize iteration counter, $k = 0$.
- Increment iteration counter, $k = k+1$.
- Generate the non-inferior solutions for k th weight combination by implementing the algorithm.
- Find membership function of the objectives from $\mu(J_i)^k$; $i = 1, 2, \dots, L$ (9).
- Find the intersection of the membership function, $\mu_k^{\min} = \text{Min}\{\mu(J_i)^k ; i = 1, 2, \dots, L\}$
- If $(k \leq 2^L + 1)$, then go to step 9.
- Find maximum satisfied membership function, $\mu^D = \text{Max}\{\mu_k^{\min} ; k = 1, 2, \dots, 2^L + 1\}$
- Choose weight combination α_i^{co} having maximum satisfied membership function μ^D as a centre of hypercube.
- If $(r \leq NW)$ then go to step 18, else continue.
- If $(\mu^D \leq \mu^P)$ then $\mu^P = \mu^D$ and $\alpha_i^c = \alpha_i^{\text{co}} ; i = 1, 2, \dots, L$, else go to step 6.
- Stop.

TEST SYSTEM AND RESULTS

The validity of the proposed method is illustrated on a six-generator system. The fuel cost, NO_x emission, CO_2 emission and SO_2 emission coefficients are taken from reference (Dhillon and Kothari, 2000) along with expected transmission loss coefficients. The generation schedule has been obtained for power demand of 1800 MW.

Economic load dispatch: When weight w_1 is set to 1 and rest of weights are set to zero in problem defined by (5), then it becomes economic dispatch problem. The

random variables are considered independent and uncorrelated to each other. The proposed method has been applied to search the incremental cost λ . A comparison of the results of the proposed method with the results of the efficient method as shown in Table 2 and 3 reveals that this method gives equally good results for economic load dispatch problem.

Multi-objective generation scheduling: Four objectives, economy, NO_x , SO_2 and CO_2 emissions are considered which have weights w_1, w_2, w_3 and w_4 , respectively and weight w_5 is set to zero. The random variables are considered independent and uncorrelated to each other. In the iteration, the weights are searched for $2^L + 1$ combinations as per the algorithm. The total number of iterations required to determine the overall best solution is $2 \times (NW - 1)$. The results obtained are shown in Table 4, the best result is obtained when k is five. The results obtained by the proposed method are compared with the results obtained by the evolutionary search technique (Brar *et al.*, 2002) and are shown in Table 5 and 6. It is evident from the comparison of the results that the results obtained by the proposed method are comparable with the results of evolutionary search method for multi-objective generation scheduling problem. The applicability of the proposed method has been shown for multi-objective generation scheduling problem considering four objectives. This method can also be applied when the number of objectives is less or more.

Stochastic multi-objective generation scheduling: The economy, environmental impacts because of NO_x , SO_2 and CO_2 emissions and variance of power are the five objectives considered which have weights, w_1, w_2, w_3, w_4 and w_5 , respectively. The cost and emission coefficients are treated as random variables. The minimum and maximum values of the objectives are calculated by giving full weightage to one objective at a time and no weightage to the other objectives. Three different cases are considered to realize the effect of variance and covariance of the random variables to each other (pair wise).

Table 2: Comparison of results

Method	J_1 Rs h^{-1}	P_L MW	λ Rs MWh^{-1}	ΔP_n MW
Proposed	18721.390	130.14780	10.670020	0.9155273E-06
Efficient	18721.390	130.14830	10.670010	0.9536743E-05

Table 3: Generation schedule corresponding to Table 1

Method	P_1 MW	P_2 MW	P_3 MW	P_4 MW	P_5 MW	P_6 MW
Proposed	251.6940	303.7786	503.4812	372.3225	301.4699	197.4014
Efficient	251.4015	303.7784	503.4813	372.3207	301.4699	197.4015

Table 4: Weight search by successive approximation method

K	w_1	w_2	w_3	w_4	J_1 (Rs h^{-1})	J_2 (kg h^{-1})	J_3 (kg h^{-1})	J_4 (kg h^{-1})	μ_n
1	0.30000	0.30000	0.30000	0.1000	18768.010	2300.254	11252.240	58196.590	0.210587
2	0.25000	0.35000	0.35000	0.0500	18757.890	2257.068	11245.680	58474.890	0.358728
3	0.23684	0.39474	0.34211	0.02632	18752.990	2212.061	11242.120	59011.270	0.513120
4	0.22973	0.41892	0.33784	0.01351	18754.450	2174.947	11242.350	59784.460	0.640435
5	0.23333	0.42000	0.34000	0.00667	18758.640	2152.628	11244.330	60575.370	0.716997

Table 5: Comparison of results

Method	w_1	w_2	w_3	w_4	J_1 (Rs h^{-1})	J_2 (kg h^{-1})	J_3 (kg h^{-1})	J_4 (kg h^{-1})	μ_n
Proposed	0.23333	0.42000	0.34000	0.00667	18758.640	2152.628	11244.330	60575.370	0.716997
Evolutionary	0.02500	0.82500	0.02500	0.12500	18776.190	2262.629	11256.790	58358.340	0.339653

Table 6: Comparison of results

Method	P_1 MW	P_2 MW	P_3 MW	P_4 MW	P_5 MW	P_6 MW	P_L MW	λ Rs/MWh
Proposed	229.6640	273.0310	497.4341	350.8827	375.4346	209.0196	135.4665	6.198579
Evolutionary	242.8604	308.4247	419.2461	369.2791	368.3573	231.8503	140.0172	12.37195

Table 6: Best weight combination under different cases

Case No.	w_1	w_2	w_3	w_4	μ_n
I	0.31638	0.22599	0.35593	0.06215	0.03955
II	0.17838	0.34054	0.27568	0.01622	0.18919
III	0.20000	0.38182	0.30909	0.01818	0.09091

Table 7: Best solution for the best weight combination under different cases

Case No.	J_1 (Rs h^{-1})	J_2 (kg h^{-1})	J_3 (kg h^{-1})	J_4 (kg h^{-1})	(MW ²)	P_L (MW)	λ (Rs h^{-1})	ΔP_n (MW)	μ_n
I	18790.430	2327.169	11265.480	59354.600	6547.993	139.607	11.607	0.0000916	0.9928
II	18753.380	2209.751	11242.050	59685.080	7823.121	135.095	7.565	0.0000153	0.5506
III	18803.690	2215.737	11272.210	59905.490	5520.864	136.301	7.121	0.0000916	0.5634

Table 8: Generation schedule for different cases

Case No	P_1 MW	P_2 MW	P_3 MW	P_4 MW	P_5 MW	P_6 MW
I	249.049	314.759	425.480	373.750	346.768	229.483
II	241.719	284.104	473.489	359.909	355.706	220.168
III	239.642	286.546	467.026	358.719	364.141	220.227

Case I: All the variables are independent to each other.

$$C(a_{ij}) = C(b_{ij}) = C(P_i) = 0.1$$

$$R(P_i, P_j) = R(a_{ij}, P_j) = R(b_{ij}, P_i) = 0.0$$

Case II: The cost and emission variables are independent but power generations are dependent to each other.

$$C(a_{ij}) = C(b_{ij}) = 0.1, C(P_i) = 0.05$$

$$R(a_{ij}, P_i) = R(b_{ij}, P_i) = 0.0, R(P_i, P_j) = 0.8$$

Case III: All the variables are correlated to each other.

$$C(a_{ij}) = C(b_{ij}) = 0.1, C(P_i) = 0.05$$

$$R(P_i, P_j) = R(a_{ij}, P_j) = R(b_{ij}, P_i) = 0.5$$

The best solutions are obtained for the above mentioned cases in which diverse values of coefficients of variation and correlation coefficients are considered. The results corresponding to the different cases have been tabulated in Table 6-8. From the results it can be seen that there is significant variation in the objectives and generation schedules under different cases when diverse values of coefficients of variation and correlation coefficients are considered.

The variation of variance of power and expected cost with respect to coefficient of variation of power and correlation coefficient of powers has been shown in Fig. 1 and 2, respectively. From Fig. 1, it is evident that as the coefficient of variation of power, $C(P_i)$ is increased there is considerable increase in the values of expected cost and variance of power. Also, it is clear from Fig. 2 that with the variation of Correlation Coefficient (CC) there is considerable variation in expected cost and variance of power. The expected cost increases as CC is changed from negative to positive value whereas the

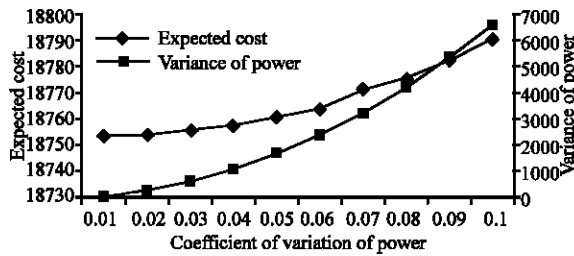


Fig. 1: Variation of expected cost and variance of power with the variation of coefficient of variation of power

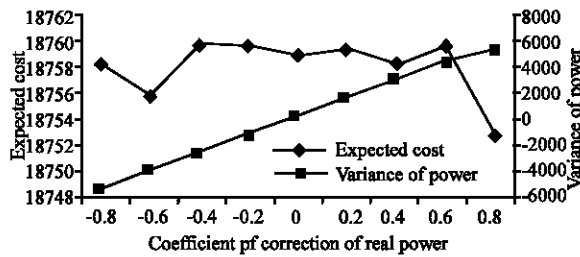


Fig. 2: Variation of expected cost and variance of power with the variation of coefficient of correlation of real power

variance of power increases almost linearly as CC is varied from negative to positive value. Due to these variations, there is a need to determine the optimum solution by taking into account the statistical variation of system parameters.

CONCLUSION

The operating cost and gaseous emissions are considered as objectives to be minimized simultaneously. The objectives are quantified by eliciting the corresponding membership function. In multi-objective framework it is realized that cost objective and emission objective are conflicting objectives. The solution set of the formulated problem is non-inferior due to contradictions among objectives taken and has been generated through weighting method. When the weight combinations are simulated by giving suitable variation, the number of non-inferior solutions increases exponentially with the increase in the number of objectives. So the process of generation of non-inferior surface becomes very time consuming. Another limitation with the simulated weight problem is that the DM may not be provided with the weight set that corresponds to actual best solution. In order to overcome the limitation of the interactive method it is proposed to search the optimal pattern with the help of successive approximation

method. In this method the solution is guaranteed within the fixed number of iterations. The accuracy of this method does not depend on initial guess whereas the accuracy of other methods is a function of initial guess. The weighting pattern that attains maximum satisfaction level from the membership function of the participating objectives have been designated the best achieved solution. The comparison of results reveals that the proposed search method gives the comparable results in terms of achieved satisfaction level in comparison with the efficient method and evolutionary search method. Further the proposed method provides the facility to consider the inaccuracies and uncertainties in the multi-objective generation scheduling problem. The practical utility of the stochastic formulation is illustrated through numerical example in diverse cases. Because of the tremendous amount of fuel cost and pollutant emission in thermal plants, a small percentage of saving or achievement in any objective can be considered significant. Thus it fully justifies the need for more accurate analysis and consideration of randomness in variables.

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