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# The Distribution of Loads in Polar Coordinates

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**Abstract:** This study presents the Raphson-Newton method in polar co-ordinates for the study of the distribution of loads which is necessary for the evaluation continues of the current performance of the system and for the analysis of the influence of the variations to be envisaged for the development of the systems in case or the request of the loads increases.

**Key words:** Load flow, Newton Raphson, power flow, evalution, variations

## INTRODUCTION

Load flow calculations provide power flows and voltages for a specified power system subject to the regulating capability of generators, condensers and tap changing under load transformers as well as specified net interchange between individual operating systems. This information is essential for the continuous evaluation of the current performance of a power system and for analyzing the effectiveness of alternative plans for system expansion to meet increased load demand (Dimitrovski and Tomsovic, 2004; Gilbert *et al.*, 1998). These analyses require the calculation of numerous load flows for both normal and emergency operating conditions.

The load flow problem consists of the calculations of power flows and voltages of network for specified terminal or bus conditions.

## EQUATIONS OF LOADS DISTRIBUTION

The load flow problem can be solved by the Newton-Raphson method using a set of nonlinear equations. For n bus system:

$$S_i^* = P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \tag{1}$$

Specified real and reactive powers in terms of bus voltages:

$$\begin{aligned} S_1^* &= P_1 - jQ_1 = I_1 V_1^* = (Y_{11}V_1 + Y_{12}V_2)V_1^* \\ S_2^* &= P_2 - jQ_2 = I_2 V_2^* = (Y_{21}V_1 + Y_{22}V_2)V_2^* \end{aligned} \tag{2}$$

In polar coordinates:

$$\begin{aligned} V_i &= |V_i| e^{\delta i} \\ Y_{ij} &= |Y_{ij}| e^{\gamma ij} \end{aligned} \tag{3}$$

$$S_i^* = P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$= \sum_{k=1}^{n} |Y_{ik}||V_i||V_k| e^{j[\delta_k - \delta_i + \gamma_{ik}]} \tag{4}$$

$$i = 1, ...., n$$

$$P_i = \sum_{k=1}^{n} |Y_{ik}||V_i||V_k| \cos(\delta_k - \delta_i + \gamma_{ik})$$

$$Q_i = \sum_{k=1}^{n} |Y_{ik}||V_i||V_k| \sin(\delta_k - \delta_i + \gamma_{ik}) \tag{5}$$

For two bus systems

$$\begin{aligned} &P_1 = P_{G1} - P_{D1} = |Y_{11}| V_1^2 \cos\gamma_{11} + |Y_{12}||V_1||V_2| \\ &\cos(\delta_2 - \delta_1 + \gamma_{12}) \equiv F_{1P} \\ &P_2 = P_{G2} - P_{D2} = |Y_{21}||V_2||V_1| \cos(\delta_1 - \delta_2 + \gamma_{21}) + \\ &|Y_{22}| V_2^2 \cos\gamma_{22} \equiv F_{2P} \\ &Q_1 = Q_{G1} - Q_{D1} = -|Y_{11}| V_1^2 \sin\gamma_{11} - |Y_{12}||V_1||V_2| \\ &\sin(\delta_2 - \delta_1 + \gamma_{12}) \equiv F_{1q} \\ &Q_2 = Q_{G2} - Q_{D2} = |Y_{21}||V_2||V_1| \sin(\delta_1 - \delta_2 + \gamma_{21}) - \\ &|Y_{22}| V_2^2 \sin\gamma_{22} \equiv F_{1q} \end{aligned} \tag{6}$$

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$$Q_1 = Q_{G1} - Q_{D1}$$

$$= -|Y_{11}|V_1^2 \sin\gamma_{11}$$

$$-|Y_{12}||V_1||V_2|\sin(\delta_2 - \delta_1 + \gamma_{12}) \equiv F_{1q} \quad (7)$$

$$Q_2 = Q_{G2} - Q_{D2}$$

$$= |Y_{21}||V_2||V_1|\sin(\delta_1 - \delta_2 + \gamma_{21})$$

$$-|Y_{22}|V_2^2 \sin\gamma_{22} \equiv F_{1q}$$

Note:

- The Eq. 6 are algebraic equations in permanent mode.
- These equations are nonlinear equations.
- The balance of the active powers gives:

$$P_{G1} + P_{G2} = P_{D1} + P_{D2} + F_{1P} + F_{2P} = P_{D1} + P_{D2} + P_L \quad (8)$$

Where $P_L$: Represent the losses of powers.

- The balance of the reactive powers gives:

$$Q_{G1} + Q_{G2} = Q_{D1} + Q_{D2} + F_{1q} + F_{2q} = Q_{D1} + Q_{D2} + Q_L \quad (9)$$

Where $Q_L$: Represent the reactive power absorptive by inductances of lines.

If $Q_i$<0: It is said that it is generated by the load capacities of lines.

- $F_{1P}, F_{2P}, F_{1q}, F_{2q}$ : Are functions of the voltages and phases.

Therefore:

$$P_L = P_L(|V_1|,|V_2|,\delta_1,\delta_2)$$
$$Q_L = Q_L(|V_1|,|V_2|,\delta_1,\delta_2) \quad (10)$$

- The Eq. 6 lead to the difference in angle $(\delta_1 - \delta_2)$ and not the two angles separately.
- According to these equations there are 12 variables and 4 equations.

## SOLUTION OF THE LOAD DISTRIBUTION PROBLEM

- It is necessary to estimate them and them $Q_{Di}$.
- We can specify the variables of control $P_{Gi}$ et $Q_{Gi}$.
- The states variables remain unknown.
- The other problem which remainder is that of the angles which are given in the form of difference and $(\delta_i - \delta_j)$ no separate.

The stages to follow to find the solution of the problem of the load flow for a system of two bus is as follows:

- One fixes $\delta_1$ = 0.
- The total number of unknown factors is 5. $(|V_1|,|V_2|,\delta_2,P_{G1},Q_{G1})$ Reduced the number of variables of states to 3. $(|V_1|,|V_2|,\delta_2)$
- One can specify the tension $|V_1|$ in bus; one will thus have like reference $(|V_1|,\delta_1)$. Then the number of unknown factors becomes 4: $(|V_2|,\delta_2,P_{G1},Q_{G1})$.

## NEWTON-RAPHSON ALGORITHM APPLIES TO THE LOAD FLOW IN POLAR COORDINATES

I If one applies the method of Newton-Raphson to the powers of the Eq (10), (Roebuck and Saylor, 1991),one obtains by considering that the bus of reference is the play of bar 1

$$P_i = F_{ip}^{[0]} + \left(\frac{\partial F_{ip}}{\partial \delta_2}\right)^{[0]} \Delta\delta_2^{[0]} + \cdots\cdots + \left(\frac{\partial F_{ip}}{\partial \delta_n}\right)^{[0]} \Delta\delta_n^{[0]}$$
$$+ \left(\frac{\partial F_{ip}}{\partial |V_2|}\right)^{[0]} \Delta V_2^{[0]} + \cdots\cdots + \left(\frac{\partial F_{ip}}{\partial |V_n|}\right)^{[0]} \Delta V_n^{[0]} \quad (11)$$

$$Q_i = F_{iq}^{[0]} + \left(\frac{\partial F_{iq}}{\partial \delta_2}\right)^{[0]} \Delta\delta_2^{[0]} + \cdots\cdots + \left(\frac{\partial F_{iq}}{\partial \delta_n}\right)^{[0]} \Delta\delta_n^{[0]}$$
$$+ \left(\frac{\partial F_{iq}}{\partial |V_2|}\right)^{[0]} \Delta V_2^{[0]} + \cdots\cdots + \left(\frac{\partial F_{iq}}{\partial |V_n|}\right)^{[0]} \Delta V_n^{[0]}$$

$$Q_i = F_{iq}^{[0]} + \left(\frac{\partial F_{iq}}{\partial \delta_2}\right)^{[0]} \Delta\delta_2^{[0]} + \cdots\cdots + \left(\frac{\partial F_{iq}}{\partial \delta_n}\right)^{[0]} \Delta\delta_n^{[0]}$$

$$+\left(\frac{\partial F_{iq}}{\partial |V_2|}\right)^{[0]} \Delta V_2^{[0]} + \cdots\cdots + \left(\frac{\partial F_{iq}}{\partial |V_n|}\right)^{[0]} \Delta V_n^{[0]}$$

The powers of balances are defined:

$$\Delta P_i = P_i - F_{ip}^{[0]}$$

$$\Delta Q_i = Q_i - F_{iq}^{[0]} \tag{12}$$

The two systems of Eq. (11) and (12) combined give:

$$\begin{bmatrix} \Delta P_2^{[0]} \\ \Delta P_n^{[0]} \\ \hline \Delta Q_2^{[0]} \\ \Delta Q_n^{[0]} \end{bmatrix} = \left[\begin{array}{cc|cc} \frac{\partial F_{2p}^{[0]}}{\partial \delta_2} & \frac{\partial F_{2p}^{[0]}}{\partial \delta_n} & \frac{\partial F_{2p}^{[0]}}{\partial |V_2|} & \frac{\partial F_{2p}^{[0]}}{\partial |V_n|} \\ \frac{\partial F_{np}^{[0]}}{\partial \delta_2} & \frac{\partial F_{np}^{[0]}}{\partial \delta_n} & \frac{\partial F_{np}^{[0]}}{\partial |V_2|} & \frac{\partial F_{np}^{[0]}}{\partial |V_n|} \\ \hline \frac{\partial F_{2q}^{[0]}}{\partial \delta_2} & \frac{\partial F_{2q}^{[0]}}{\partial \delta_n} & \frac{\partial F_{2q}^{[0]}}{\partial |V_2|} & \frac{\partial F_{2q}^{[0]}}{\partial |V_n|} \\ \frac{\partial F_{nq}^{[0]}}{\partial \delta_2} & \frac{\partial F_{nq}^{[0]}}{\partial \delta_n} & \frac{\partial F_{nq}^{[0]}}{\partial |V_2|} & \frac{\partial F_{nq}^{[0]}}{\partial |V_n|} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{[0]} \\ \Delta \delta_n^{[0]} \\ \hline \Delta V_2^{[0]} \\ \Delta V_n^{[0]} \end{bmatrix} \tag{13}$$

By using another notation, one will have:

$$\begin{bmatrix} \underline{\Delta P}^{[0]} \\ \underline{\Delta Q}^{[0]} \end{bmatrix} = \left[J^{[0]}\right] \begin{bmatrix} \underline{\Delta \delta}^{[0]} \\ \underline{\Delta V}^{[0]} \end{bmatrix} \Leftrightarrow \begin{bmatrix} \underline{\Delta \delta}^{[0]} \\ \underline{\Delta V}^{[0]} \end{bmatrix} = \left[J^{[0]}\right]^{-1} \begin{bmatrix} \underline{\Delta P}^{[0]} \\ \underline{\Delta Q}^{[0]} \end{bmatrix}$$

from where

$$\begin{bmatrix} \underline{\delta}^{[1]} \\ \underline{V}^{[1]} \end{bmatrix} = \begin{bmatrix} \underline{\delta}^{[0]} \\ \underline{V}^{[0]} \end{bmatrix} + \left[J^{[0]}\right]^{-1} \begin{bmatrix} \underline{\Delta P}^{[0]} \\ \underline{\Delta Q}^{[0]} \end{bmatrix}$$

and

$$\begin{bmatrix} \underline{\delta}^{[2]} \\ \underline{V}^{[2]} \end{bmatrix} = \begin{bmatrix} \underline{\delta}^{[1]} \\ \underline{V}^{[1]} \end{bmatrix} + \left[J^{[1]}\right]^{-1} \begin{bmatrix} \underline{\Delta P}^{[1]} \\ \underline{\Delta Q}^{[1]} \end{bmatrix}$$

for iteration (I+1)

$$\begin{bmatrix} \underline{\delta}^{[I+1]} \\ \underline{V}^{[I+1]} \end{bmatrix} = \begin{bmatrix} \underline{\delta}^{[I]} \\ \underline{V}^{[I]} \end{bmatrix} + \left[J^{[I]}\right]^{-1} \begin{bmatrix} \underline{\Delta P}^{[I]} \\ \underline{\Delta Q}^{[I]} \end{bmatrix} \tag{14}$$

**EXAMPLE OF LOAD FLOW CALCULATION**

The description above applies to a load bus, where the active power and reactive power flow are specified and the voltage magnitude and angle is to be calculated, (Dias and Hawary, 1987; Prabha, 1993).

1: Floating bus 2: generator bus 3: load bus Nodal admittance matrix:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -j9 & j4 & j5 \\ j4 & -j14 & j10 \\ j5 & j10 & -j15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Values for $P_2$, $P_3$, $Q_3$ are given, so the iterative solution centres on these quantities.

The bus voltages are:

$$V_1 = 0.1 < 0^0$$
$$V_2 = 1.1 < \delta_2,$$
$$V_3 = |V_3| < \delta_3$$

so the solution variables are $\delta_2 = |V_3| < \delta_3$

Express the set values in terms of the variables:

$$S_2 = V_2.I_2^* = 1.1\angle\delta_2\{j4.V_1 - j14V_2 - j10.V_3\}^*$$
$$= 4.4\angle(\delta_2 - 90^\circ) + 16.9\angle(90^\circ) + 11|V_3|\angle(\delta_2 - \delta_3 - 90^\circ)$$
$$\Rightarrow P_2 = 4.4\cos(\delta_2 - 90^\circ) + 11|V_3|\cos(\delta_2 - \delta_3 - 90^0)$$

$$S_3 = V_3.I_3^* = |V_3|\angle\delta_3\{j5 + j11 - j15.V_3\}^*$$
$$P_3 = 5.0|V_3|\cos(\delta_3 - 90^\circ) + 11|V_3|\cos(\delta_3 - \delta_2 - 90^\circ)$$
$$\Rightarrow$$
$$Q_3 = 5.0|V_3|\sin(\delta_3 - 90^\circ) + 11|V_3|\sin(\delta_3 - \delta_2 - 90^\circ) + 15|V_3|^2$$

Iterative solution: starting with initial estimates of $\delta_2, |V_3|, \delta_3$

- Calculate power and reactive power errors:

$$\Delta P_2 = P_{2S} - P_2$$
$$\Delta P_3 = P_{3S} - P_3$$
$$\Delta Q_3 = Q_{3S} - Q_3$$

- Jacobian elements

$$\frac{\partial P_2}{\partial \delta_2} = -4.4\sin\left(\delta_2 - 90^\circ\right) - 11V_3\sin\left(\delta_2 - \delta_3 - 90^\circ\right)$$

$$\frac{\partial P_2}{\partial \delta_3} = 11V_3\sin\left(\delta_2 - \delta_3 - 90^\circ\right)$$

$$\frac{\partial P_2}{\partial V_3} = 11\cos\left(\delta_2 - \delta_3 - 90^\circ\right) \quad \text{etc.}$$

- Invert the Jacobian and hence calculate the corrections to the estimates $\Delta\delta_2, \Delta\delta_3, \Delta V_3$
- Form new estimates

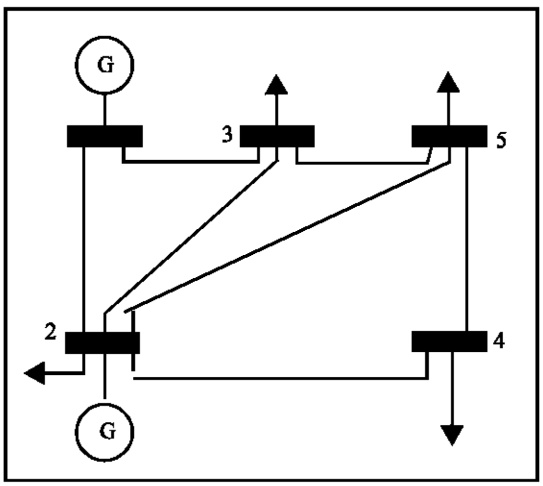


Fig. 1: Scheduled generation and assumed bus voltages for sample system

$$\delta_2 \to \delta_2 + \Delta\delta_2; \delta_2 \to \delta_3 + \Delta\delta_3; V_3 \to V_3 + \Delta V_3$$

and repeat from stage 1 (Table 1). Sample results

$$P_1 = 0.5\text{pu} \quad P_{13} = 0.483\text{pu}$$

$$P_{12} = 0.017\text{pu} \quad P_{23} = 1.017\text{pu}$$

## ILLUSTRATIVE EXAMPLES

A 5-bus system shown in Fig.1 is test to illustrate the procedure of proposed method. In this system, bus 3,4 and 5 are PQ bus, bus 2 is PV bus and bus 1 is slack bus (Table 2-5).

Table 1: Sample results

| $\delta_2 = (0)$ | $\delta_3 = (0)$ | $V_3$ |
|---|---|---|
| 10 | -20 | 0.9 |
| -2.8 | 4.8 | 1.24 |
| -0.34 | -3.82 | 1.09 |
| -0.23 | -5.21 | 1.05 |
| -0.22 | -5.28 | 1.05 |
| 0.22 | -5.8 | 1.05 |

from which the power flow can be calculated

Table 2: Scheduled generation and assumed bus voltages for sample system.

| Bus code P | Assumed voltage | Generation Mw | Generation Mvar | Load Mw | Load Mvar |
|---|---|---|---|---|---|
| 1 | 1.06+j0.0 | 0 | 0 | 0 | 0 |
| 2 | 1.00+j0.0 | 40 | 30 | 20 | 10 |
| 3 | 1.00+j0.0 | 0 | 0 | 45 | 15 |
| 4 | 1.00+j0.0 | 0 | 0 | 40 | 5 |
| | 1.00+j0.0 | 0 | 0 | 60 | 10 |

Table 3: Bus voltages

| Iteration | Bus2 $\|V_2\|$ | Bus2 $\delta_2 = (\text{Rd})$ | Bus3 $\|V_3\|$ | Bus3 $\delta_3 = (\text{Rd})$ | Bus4 $\|V_4\|$ | Bus4 $\delta_4 = (\text{Rd})$ | Bus5 $\|V_5\|$ | Bus5 $\delta_5 = (\text{Rd})$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.0 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 1 | 1.0562 | -2.75889 | 1.03579 | -5.05317 | 1.03595 | -5.3986 | 1.03270 | -6.27305 |
| 2 | 1.04755 | -2.80341 | 1.02433 | -4.99892 | 1.02373 | -5.33167 | 1.01814 | -6.15473 |
| 3 | 1.04755 | -0.00078 | 1.02433 | 0.00095 | 1.02373 | 0.00117 | 1.01814 | 0.00207 |

Table 4: Changes in Bus powers

| Iteration | Bus2 $\Delta P_2$ | Bus2 $\Delta Q_2$ | Bus3 $\Delta P_3$ | Bus3 $\Delta Q_3$ | Bus4 $\Delta P_4$ | Bus4 $\Delta Q_4$ | Bus5 $\Delta P_5$ | Bus5 $\Delta Q_5$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.50000 | 1.18500 | -0.37500 | 0.103000 | -0.4000 | 0.00500 | -0.60000 | -0.06000 |
| 2 | -0.09342 | -0.03857 | -0.00102 | -0.003586 | 0.01172 | -0.03868 | 0.002244 | -0.06563 |
| 3 | -0.00323 | 0.00040 | 0.00018 | -0.000530 | 0.00064 | -0.00069 | 0.00103 | -0.00132 |

Table 5: Bus power

| Iteration | Bus2 $P_2$ | Bus2 $Q_2$ | Bus3 $P_3$ | Bus3 $Q_3$ | Bus4 $P_4$ | Bus4 $Q_4$ | Bus5 $P_5$ | Bus5 $Q_5$ |
|---|---|---|---|---|---|---|---|---|
| 1 | -0.30000 | -0.98500 | -0.07500 | -0.28000 | 0.00000 | -0.05500 | 0.00000 | -0.04000 |
| 2 | 0.293392 | 0.23857 | -0.44898 | -0.11414 | -0.141172 | -0.01132 | -0.62244 | -0.03437 |
| 3 | 0.203230 | 0.19996 | -0.45018 | -0.14947 | -0.40064 | -0.04931 | -0.60103 | -0.09868 |

## CONCLUSION

This study presents an alternative to the way the load flow equations are currently solved. Instead of combining the nodal equations and the bus constraints into a single set of 2 nonlinear equations, the NR method is applied to the two primitive sets of equations, (Gomz and Romos, 2002). The enlarged model, in which current injections are retained in the state vector, leads to a very simple solution methodology if polar coordinates are adopted. A straightforward approach to dealing with PV buses is also proposed. Experiments confirm that, depending on the number of PV buses, the computational effort per iteration ranges between 50 and 80% of that required by other formulations. Not only comes this saving from the simplicity of the Jacobian terms, as in other polar-based methods, but from the mismatch vector computation as well, particularly when many zero-injection buses are present, (Tinney and Hard, 1967). While the convergence rate of the proposed method, when transmission networks are solved, is similar to that of existing implementations, a noticeable improvement is obtained when dealing with distribution networks.

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