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Simulation of non Stationary Biphasic Flow in the Natural Channels with a Profile Variable in Long

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Abstract: The study deals with an approximate numerical method used to resolve non stationary biphasic flow equations emphasising on its prevalence with regard to the other electrical and hydraulic analogy methods. Considering the studied field of inquiry the approximate numerical resolution of the integral or differential equations of the non stationary flow remains the only possible way to determine all the unknown functions such as: y (x,t), z (x,t), Q (x,t).

Key words: Non stationary flow, control flow, numerical approach, solid discharge, natural equilibrium, floods, silting

INTRODUCTION

Non permanent current equations in open air, written in integral or differential form, are difficult to resolve. The closed form solution seems even more impossible to find because of their non linearity on one hand and of the complexity of certains dependent variables functions on the other hand. That is why a choice must be done either using the approximate numerical resolution or realizing analogical models which allow obtaining quickly the solution of problems arising from flow in grid or dendritic hydraulic networks (Cunge *et al*., 1980). The hydraulic or electrical analogy methods are less recommended, this is due to the application of new modelling systems made up of processors allowing the setting up and the checking of data such as accumulation volumes in the flood areas, the hydraulic network topology, through section, model construction programs, calculation and post processor programs, which can analyse the results obtained and also introduce the boundary conditions, a task which becomes extremely simple (Carlier, 1986). The control flow of the natural channels is the permanent concern of the engineers hydraulic and fluid mechanic (Hug, 1975; Rouse, 1938; Ippen, 1949) who seek to contain, as far as possible, the natural phenomena, in present case the floods, by work of re-calibering, reshaping, rescinding of the curves and damming up in order to control or at least to slow down the various processes of production and transfer of the sediments in the natural channels. These processes, which are generated by waves of flows which capacity of destruction or drive related to the horizontal speed of propagation is proven, are sometimes irreversible.

FLOW GRADUALLY VARIED IN A CHANNEL WITH VARIABLE SLOPE

The gradually varied flow in natural channels with removable bottoms presents a major importance for the analysis of the phenomena of erosion and deposit phenomena in the levels of these channels under of the influence of raising backwash curves created by the hydrotechnical structures. To study this flow, it is necessary to consider that the longitudinal profile of the channel itself is variable during all the period of transformation of the bed.

To solve this problem, it is necessary to take into account the system of the three following equations:

- Dynamic equation:

$$\frac{\partial y}{\partial x} = -\frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} - \frac{Q^2}{K^2} \quad (1)$$

- Equation of formation of the bed:

$$\frac{\partial Qs}{\partial x} = -\gamma'\frac{\partial z}{\partial t}b \quad (2)$$

- Equation of constancy of the flow: (Q = Cte).

This last condition is of primary importance; indeed, the flow varies in function of time according to a certain diagram. By taking Q=const., we must replace this diagram by another graduated, by considering that the flow is constant during an interval of time Δt. In this case, one considers that the bed of the channel is wide and of an almost rectangular form. Let us lay down the condition that erosion occurs in a uniform way according to the width and that the banks are stable.

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The term $\frac{1}{g}\frac{\partial v}{\partial t}$ is defined according to the depth variation, because the flow is constant; However, taking into account the fact that the flow is gradually varied, this term, in more the share of the cases, can be neglected, except for the case where speeds are close to the critical engine failure speeds.

Let us first examine the general case, then the approximate calculation schema will be studied of approximate calculation. The solution of the Eq. 1 and 2 is reduced to the solution of hyperbolic equations by the method of the characteristics worked out by. Khristianovitch (Cunge *et al.*, 1980; Carlier, 1986; Hug, 1975; Rouse, 1938).

$$\frac{\partial z}{\partial t}=\frac{dz}{dt}-\frac{\partial z}{\partial x}\frac{dx}{dt}=\frac{dz}{dt}-\eta\frac{\partial z}{\partial x};\quad \frac{\partial v}{\partial t}=\frac{dv}{dt}-\frac{\partial v}{\partial x}\frac{dx}{dt}=\frac{dv}{dt}-\eta\frac{\partial v}{\partial x} \tag{3}$$

where $\eta=\frac{dx}{dt}$

By substituting these expressions in (1) and (2) and knowing that $\frac{\partial h}{\partial x}=-\frac{Q}{bv^2}\frac{\partial v}{\partial x}$, we obtain:

$\gamma'\frac{\partial z}{\partial t}b=-\frac{\partial Qs}{\partial x}=\gamma f'(v)b\frac{\partial v}{\partial x}$ where: $Qs=\gamma f(v)b$ and $f(v)=\frac{AQv^4}{b}$

$$\gamma'(\frac{dz}{dt}-\eta\frac{\partial z}{\partial x})=-\gamma f'(v)\frac{\partial v}{\partial x}$$

$$\frac{\partial y}{\partial x}=\frac{\partial z}{\partial x}+\frac{\partial h}{\partial x}=-\frac{v}{g}\frac{\partial v}{\partial x}-\frac{1}{g}\frac{\partial v}{\partial t}-I;\quad \frac{\partial z}{\partial x}-\frac{Q}{bv^2}\frac{\partial v}{\partial x}=-\frac{v}{g}\frac{\partial v}{\partial x}-\frac{1}{g}\frac{dv}{dt}+\frac{\eta}{g}\frac{\partial v}{\partial x}-I$$

Thus:

$$\gamma'\eta\frac{\partial z}{\partial x}-\gamma f'(v)\frac{\partial v}{\partial x}=\gamma'\frac{dz}{dt} \tag{4}$$

$$-g\frac{\partial z}{\partial x}+(\eta-v+g\frac{Q}{bv^2})\frac{\partial v}{\partial x}=Ig+\frac{dv}{dt} \tag{5}$$

$$D=\begin{vmatrix}\gamma'\eta & -\gamma f'(v)\\ -g & \eta-v+g\frac{Q}{bv^2}\end{vmatrix}=\gamma'\eta(\eta-v+g\frac{Q}{bv^2})-g\gamma f'=0$$

Thus:

$$\eta^2+v(\frac{gQ}{bv^3}-1)\eta-g\frac{\gamma}{\gamma'}f'(v)=0 \tag{6}$$

By solving the Eq. 6, we find:

$$(\frac{dx}{dt})_1=W=-\left(\frac{gQ}{bv^3}-1\right)\frac{v}{2}\left[\sqrt{1+\frac{4g\frac{\gamma}{\gamma'}f'(v)}{v^2\left(\frac{gQ}{v^3}-1\right)^2}}+1\right] \tag{7}$$

$$(\frac{dx}{dt})_2=\Omega=\left(\frac{gQ}{bv^3}-1\right)\frac{v}{2}\left[\sqrt{1+\frac{4g\frac{\gamma}{\gamma'}f'(v)}{v^2\left(\frac{gQ}{v^3}-1\right)^2}}-1\right] \tag{8}$$

To find the relation z = f (t), it is necessary to calculate the determinant $D_1=0$

$$D_1=\begin{vmatrix}\gamma'\eta & \gamma'\frac{dz}{dt}\\ -g & Ig+\frac{dv}{dt}\end{vmatrix}=\eta\gamma'(Ig+\frac{dv}{dt})+g\gamma'\frac{dz}{dt}-=0;$$

$$dz=-\frac{\eta}{g}(gIdt+dv)$$

Therefore, we will have definitively:

$$\left.\begin{aligned}dx&=Wdt\\ dz&=-\frac{1}{g}W(gIdt+dv)\end{aligned}\right\}C^+ \tag{9}$$

$$\left.\begin{aligned}dx&=\Omega dt\\ dz&=-\frac{1}{g}\Omega(gIdt+dv)\end{aligned}\right\}C^- \tag{10}$$

APPROXIMATE SOLUTION OF THE EQUATION OF DEFORMATION OF THE BED

As indicated previously, the term $\frac{1}{g}\frac{\partial v}{\partial t}$ can be neglected and the Equ. simplified (1). We have:

$$\gamma'\frac{\partial z}{\partial t}=-f'(v)\frac{\partial v}{\partial x}=f'(h)\frac{q}{h^2}\frac{\partial h}{\partial x}=f'(h)\frac{q}{h^2}(\frac{\partial y}{\partial x}-\frac{\partial z}{\partial x}) \tag{11}$$

$$\frac{\partial y}{\partial x}=-\frac{v}{g}\frac{\partial v}{\partial x}-I=\frac{v}{g}\frac{q}{h^2}(\frac{\partial y}{\partial x}-\frac{\partial z}{\partial x})-I=\frac{q^2}{gh^3}(\frac{\partial y}{\partial x}-\frac{\partial z}{\partial x})-I=\frac{h_{cr}^3}{h^3}(\frac{\partial y}{\partial x}-\frac{\partial z}{\partial x})-\frac{D_1}{h^m}$$

Where:

$$\frac{D_1}{h^m} = I = Dv^{3,5} = \frac{Q^2}{K^2 v^{3,5}} v^{3,5} = \frac{n^2 b^{1,5}}{Q^{1,5}} v^{3,5}$$

$$\frac{n^2 b^{1,5}}{Q^{1,5}} \cdot \frac{Q^{3,5}}{(bh)^{3,5}} = \frac{n^2 q^2}{h^{3,5}} = \frac{n^2 q^2}{h^m}; m = 3,5 \qquad (12)$$

$$\frac{\partial y}{\partial x}\left(1 - \frac{h_{cr}^3}{h^3}\right) = -\frac{D_1}{h^m} - \frac{h_{cr}^3}{h^3}\frac{\partial z}{\partial x}$$

By substituting $\frac{\partial y}{\partial x}$ of the Eq. 12 in the Eq. 11,

we obtain the equation of deformation of the bed in the form:

$$-\gamma' \frac{\partial z}{\partial t} = \frac{q}{h^2} f'(h) \left[\frac{\frac{\partial z}{\partial x} + \frac{D_1}{h^m}}{1 - \frac{h_{cr}^3}{h^3}} \right]$$

$$(h_{cr}^3 - h^3)\gamma' \frac{\partial z}{\partial t} = qhf'(h)\left[\frac{\partial z}{\partial x} + \frac{D_1}{h^m}\right] \qquad (13)$$

where: h = y-z; however y f(x)

Here, it is supposed that $\frac{\partial y}{\partial t} = 0$; this in addition to the assumption neglecting the term $\frac{1}{g}\frac{\partial v}{\partial t}$ means that the deformation of the bed occurs more intensely than the variation of the level of the surface water. Then, the equation takes the form:

$$q\frac{\partial z}{\partial x} + F_1 \frac{\partial z}{\partial t} = -F_2 \qquad (14)$$

Where : $F_1 = \frac{h^3 - h_{cr}^3}{hf'(h)}\gamma'; \quad F_2 = \frac{qD_1}{h^m}; \quad h = y - z$

The value of y can be calculated from the initial conditions or taken equal to the average value during a given interval of time.

The Eq. 14 can be solved by analyzing the auxiliary system:

$\frac{dx}{q} = \frac{dt}{F_1} = -\frac{dz}{F_2}$; from where:

$$dt = \frac{F_1}{q}dx \Rightarrow \int \frac{F_1}{q}dx = t + C_1 = \phi_1 \qquad (15)$$

$$dt = -\frac{F_1}{F_2}dz \Rightarrow -\int \frac{F_1}{F_2}dz = t + C_2 = -\phi_2 \qquad (16)$$

where C_1 and C_2 are functions of the independent variables of x and t. The relation between C_1 and C_2 can be found from the initial conditions.

Since h= f(x) is a very complex relation, it is more convenient, for the solution of the problem to use the grapho-analytical method. For t = 0, $C_1^0 = \phi_1$ and while calculating $\phi_1$and ϕ_2 we can plot the graph $C_2^0 = f(C_1^0)$ (Fig. 1); after that, it will be more practical to express ϕ_1 in this form x ϕ_1. In addition, to time t the relations (15) and (16) give:

$$\phi_1 - C_1 = -\phi_2 - C_2;$$

Thus:

$$C_2 = -(\phi_1 + \phi_2) + C_1 = \phi + C_1 \qquad (17)$$

with: $\phi = -(\phi_1 + \phi_2)$

These equations give a linear relation between C_1 and C_2 providedϕ_1 that ϕ_2 and are given. To calculate ϕ_1 and ϕ_2 it is necessary to give x and the value of deformation of bed z_i ; then one carries the Eq. (17) on the graph (Fig. 2) and the point of intersection of this line with the curve $C_2^0 = f(C_1^0)$ will be the sought after solution.

Knowing C_1 , it is easy to determine the moment t_i of deformation of the bed at level z_i in section x_i:

$$t = \phi_1 - C_1 \qquad (18)$$

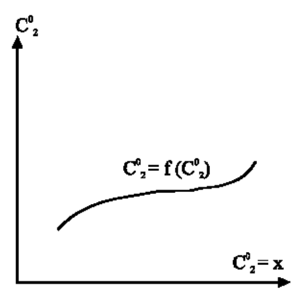

Fig. 1: $C_2^0 = f(C_1^0)$

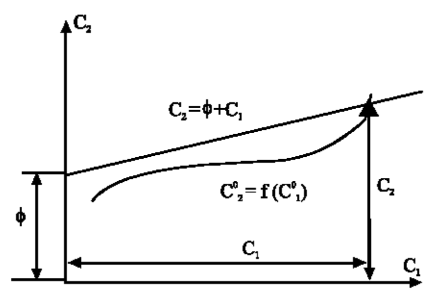

Fig. 2: $C_2 = f(C_1)$

To explain the method of calculation, a numerical example is given afterwards.

In addition to the method described above, it is possible to propose an analytical method; it consists of the search for an analytical relation between C_1^0 et C_2^0 . In certain particular cases (example, for alluviation of a bed), this relation can be obtained easily. It often has the following form:

$$C_2 = MC_1^n \tag{19}$$

where: n - whole or split number.

By tracing an anamorphosis logarithmic curve: lg C_2 = lg M + n lgC_1, we can check the reliability of the relation (19) for the studied channel and to determine, in the affirmative case, the constants M and n. It may be that this relation is valid only for part of the studied section.

To obtain the final relation, it is necessary to substitute C_1 and C_2 in (19) according to x, t and z, by using the Eq. 15 and 16:

$$-\phi_2 - t = M(\phi_1 - at)^n = M(\phi_0 x - at)^n \tag{20}$$

The Equation obtained determines implicitly the value z = f(x, t). To simplify the calculations, we determine x = x (z, t):

$$x = \frac{at + \left[-\frac{1}{M}(\phi_2 + t)\right]^{\frac{1}{n}}}{\phi_0} \tag{21}$$

By giving t and z_1, we determine the functions ϕ_0 et ϕ_2 and we calculate the sought value x. In this manner, we can plot the deformed profile of the bed for any interval of time Δt. It is necessary to notice that with time the level of the free face, because of the continues deformation continues of the bed, increases or decreases; consequently, it is necessary to check from one moment to another the height of the water level.

Thus, by applying the iterative method we can determine not only the deformation allowed of the bed, but also the rise and the lowering of the free surface water level.

FLOW AND DEFORMATION OF THE BEDS

The non stationary flow in the channels over gravel beds is a very complex phenomenon; its analysis runs up against many difficulties. The increase of the speed due to the increase of the flow and the variation of the slope, inevitably involve the erosion of the bed which can reach great values in the sections where speed is maximum. The quantity of the eroded solid matters will be deposited on the sections down streams immediately behind the face of wave, then erosion weakens, simultaneously and proportionately. The character of erosion and the alluvial deposit depends on the type of the flow and above all the speed of the flow. The change of the coast of the bed influences, on the rise or the lowering of the level of the free face. Thus for the examination of the phenomenon in question, it is necessary to take into account the interdependence between the non stationary flow and the deformation of the bed.

To solve the problem arising, it is necessary to examine the system of three equations with three unknown y, z and Q (or y, z and v).

Dynamic equation:

$$-\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\left(\frac{v^2}{2g}\right) + \frac{1}{g}\frac{\partial v}{\partial t} + \frac{v^2}{C^2 R} \tag{22}$$

Equation of continuity:

$$b\left(\frac{\partial y}{\partial t} - \frac{\partial z}{\partial t}\right) + \frac{\partial Q}{\partial x} = 0 \tag{23}$$

Equation of the bed deformation:

$$b\frac{\partial z}{\partial t} = -\frac{\partial Qs_1}{\partial x}; Qs_1 = \frac{Qs}{\gamma'} \tag{24}$$

Using (23) and (24), we can obtain the equation of continuity in form:

$$b\frac{\partial y}{\partial t} = -\frac{\partial (Q + Qs_1)}{\partial x} \tag{25}$$

The left part of this equation represents the speed of the variation of the water level; this variation depends on the liquid flow and the solid discharge.

This type of flow was examined by De Vrio and Levy by proposing a solution by the method of the characteristics.

Let us examine, initially, the problem arising by using the solution given by Khristianovitch. Only the case where the flow is gradually varied will be studied, in other words the case of the long waves. The equation of continuity in integral form is written as:

$$\int_{x_1}^{x_2}\int_{t_1}^{t_2} b\frac{\partial y}{\partial t}dtdx = -\int_{t_1}^{t_2}\int_{x_1}^{x_2}\frac{\partial (Qs_1 + Q)}{\partial x}dtdx$$

$$\int_{x_1}^{x_2} by_{t_2}dx - \int_{x_1}^{x_2} by_{t_1}dx = \int_{t_1}^{t_2}(Q_{x_1} + Qs_{x_1})dt - \int_{t_1}^{t_2}(Q_{x_2} - Qs_{x_2})dt \tag{26}$$

where: y_{t_1} and y_{t_2} - water level at times t_1 and t_2.
Q_{x_i} et Qs_{x_i} -liquid flow and solid discharge in section i.

The left part of (26) represents the volume of water which accumulates on the section x_1 - x_2 during the interval t_2 - t_1 (Fig. 3). This difference is determined by hatched surface.
The right part gives respectively:

- The difference of liquid volumes going in and coming out of the section (Fig. 4).
- The difference of the solid particles carried (or settled) in the limit of the section during period t_2-t_1 (Fig. 5).

By adding the two graphs (4) and (5), the rise of the level can be calculated; this rise depends on the deformation (erosion or deposit) of the bed.
The solid discharge can be expressed as follows:

$$Qs_1 = \frac{Qs}{\gamma'} = \frac{\mu Q}{\gamma'} = -\frac{\mu vhb}{\gamma'} = Qf(v)$$

where: μ- average turbidity or concentration of the solid particles. Consequently:

$$\frac{\partial Qs_1}{\partial x} = \frac{\partial(\mu Q)}{\partial x} = \frac{Q}{\gamma'}\frac{\partial \mu}{\partial x} + \frac{\mu}{\gamma'}\frac{\partial Q}{\partial x} \quad (27)$$

replacing this Eq. 25, gives:

$$b\frac{\partial y}{\partial t} = -(1+\frac{\mu}{\gamma'})\frac{\partial Q}{\partial x} - \frac{Q}{\gamma'}\frac{\partial \mu}{\partial x} \quad (28)$$

If μ<<1 this equation takes the form:

$$b\frac{\partial y}{\partial t} = -\frac{\partial Q}{\partial x} - \frac{Q}{\gamma'}\frac{\partial \mu}{\partial x} \quad (28)$$

The additional term $\frac{Q}{\gamma'}\frac{\partial \mu}{\partial x}$ shows that the rate/rhythm of the increase in the water level on a section with variable bed in the case of the non stationary flow depends, to a certain degree on the variation of the turbidity of the water current in length. It can be affirmed a priori that usually (except for the water current with a slope close to the slope critical), the propagation velocity of the face of wave in the case of the non stationary flow in the channels with movable bed, differs very little from the velocity of flow in the case of that with stable bed (Levy, 1967).

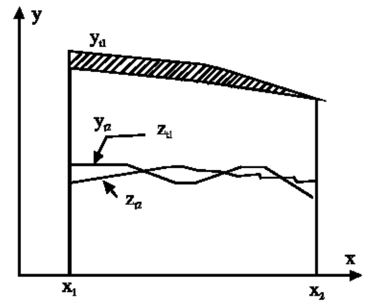

Fig. 3: Variation of free surface

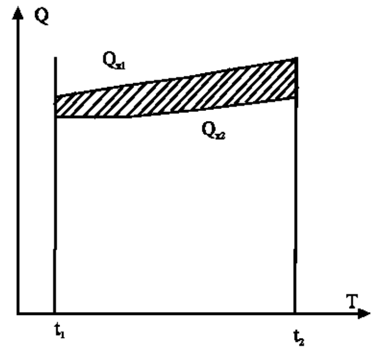

Fig. 4: Q = f(x, t)

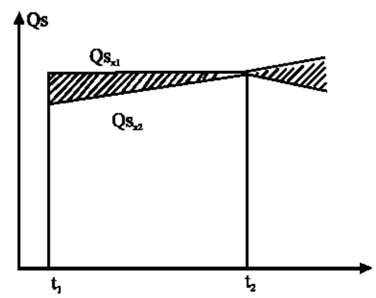

Fig. 5: Qs = f(x, t)

The Eq. 25 can be written as follows:

$$Qs_1 = \frac{Qs}{\gamma'} = AQv^4 = \frac{\mu vbh}{\gamma'}; Qf'(v) = 4AQv^3 = \frac{4\mu bh}{\gamma'}$$

thus:

$$h + \frac{Q}{b}f'(v) = h(1 + 4\frac{\mu}{\gamma'}) \quad (29)$$

Levy concluded that the calculation of the non stationary flow in channels with movable bed is characterized by the fact that the propagation velocity of wave is defined by the expression:

$$dx = (v \pm \sqrt{gh_1})dt \quad (30)$$

with:

$$h_1 = h(1+\frac{4\mu}{\gamma'})$$

The dynamic equations, of continuity and deformation of the bed can be written in the form (Levy, 1967; Bolchakov *et al*., 1984):

$$\left.\begin{aligned} &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = -Ig \\ &\frac{\partial h}{\partial t} + h\frac{\partial v}{\partial x} + v\frac{\partial h}{\partial x} = 0 \\ &\frac{\partial z}{\partial t} + \frac{4Aqv^3}{\gamma'}\frac{\partial v}{\partial x} = 0 \end{aligned}\right\} \qquad (31)$$

or in matrix form:

$$\frac{\partial}{\partial t}\begin{pmatrix} v \\ h \\ z \end{pmatrix} + \begin{pmatrix} v & g & g \\ h & v & 0 \\ \frac{4Aqv^3}{\gamma'} & 0 & 0 \end{pmatrix}\frac{\partial}{\partial x}\begin{pmatrix} v \\ h \\ z \end{pmatrix} = \begin{pmatrix} -gI \\ 0 \\ 0 \end{pmatrix} \qquad (32)$$

Knowing that:

$$\frac{\partial v}{\partial t} = \frac{dv}{dt} - \eta\frac{\partial v}{\partial x}$$
$$\frac{\partial h}{\partial t} = \frac{dh}{dt} - \eta\frac{\partial h}{\partial x}$$
$$\frac{\partial z}{\partial t} = \frac{dz}{dt} - \eta\frac{\partial z}{\partial x}$$

with η_= dx / dt the system (11) will take the form:

$$\left.\begin{aligned} &(v-\eta)\frac{\partial v}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = gI - \frac{dv}{dt} \\ &h\frac{\partial v}{\partial x} + (v-\eta)\frac{\partial h}{\partial x} = -\frac{dh}{dt} \\ &\frac{4Aqv^3}{\gamma'}\frac{\partial v}{\partial x} - \eta\frac{\partial z}{\partial x} = -\frac{dz}{dt} \end{aligned}\right\} \qquad (33)$$

Characteristics ηi can be calculated using the determinant:

$$D = \begin{vmatrix} v-\eta & g & g \\ h & v-\eta & 0 \\ \frac{4Aqv^3}{\gamma'} & 0 & -\eta \end{vmatrix} = 0$$

$$-\eta^3 + 2v\eta^2 + (gh - v^2 + 4gAqv^3)\eta - 4gAqv^4 = 0$$

The three roots of this equation are:

$$\eta_1 = (\frac{dx}{dt})_1 = v + \sqrt{gh_1}; \quad h_1 = h(1+\frac{4\mu}{\gamma'}) \qquad (34)$$

$$\eta_2 = (\frac{dx}{dt})_2 = v - \sqrt{gh_1} \qquad (35)$$

$$\eta_3 = f'(v)\frac{q}{\gamma h^3(1-\frac{h_{cr}^3}{h})} \qquad (36)$$

To determine z = f (t), the second determinant should be calculated:

$$D_1 = \begin{vmatrix} -gI - \frac{dv}{dt} & g & g \\ -\frac{dh}{dt} & v-\eta & 0 \\ -\frac{dz}{dt} & 0 & -\eta \end{vmatrix} = 0$$

$$\eta(v-\eta)(gI + \frac{dv}{dt}) - g\eta\frac{dh}{dt} + g(v-\eta)\frac{dz}{dt} = 0$$

$$\frac{\eta}{g}(gI + \frac{dv}{dt}) - \frac{\eta}{v-\eta}\frac{dh}{dt} + \frac{dz}{dt} = 0$$

$$dz - \frac{\eta}{v-\eta}dh + \frac{\eta}{g}dv + \eta I dt = 0 \qquad (37)$$

Since η_has three values, an equation can obtained for each characteristic. Consequently, by solving this system of equations by the method of the characteristics or the finite differences method, the unknowns z, h and v can be determinated.

NUMERICAL APPLICATION

To explain the method described in the paragraph (5), the following example is studied:

The study concerned is given in the form of a profile longitudinally. The principal characteristics of this profile are indicated in the following table (Table 1).

The average width of the river b_{moy} = 400 m; the flow Q = 4000 m^3 s^{-1}; q = Q/b = 10m s^{-1}; v_0 = 0.67 m s^{-1} $h_0 = \frac{q}{v_0}$;

= 15 m; the fall of the water level of section x_0=0 with

Table 1:The principal characteristics

x(m)	z_0 (m)	y (m)	$(y-z_0)$, (m)
-3.000	-0.2	7.8	8.0
0	0.0	7.7	7.7
2000	0.3	7.5	7.2
5000	-0.2	7.2	7.4
7000	-0.1	7.0	7.1
9000	0.05	6.8	6.75

Table 2: The computation results for each section

$C_1^0 = x$	z_0 (m)	y (m)	y-z_0 (m)	$\xi_0 = \frac{y - z_0}{h_0}$	$(\xi_0)^9 10^{-4}$	$C_0^2 = -\phi_2\,(\xi_0) = 3.10^9.\ \xi^9$
-3000	-0.2	7.8	8.0	0.533	34.72	104.10^5
0	0	7.7	7.7	0.513	24.60	74.10^5
2000	0.3	7.5	7.2	0.480	13;53	$40{,}6.10^5$
5000	-0.2	7.2	7.4	0.493	17.20	$51{,}6.10^5$
7000	-0.1	7.0	7.1	0.473	11.85	$35{,}6.10^5$
9000	0.05	6.8	6.75	0.450	7.56	$22{,}7.10^5$

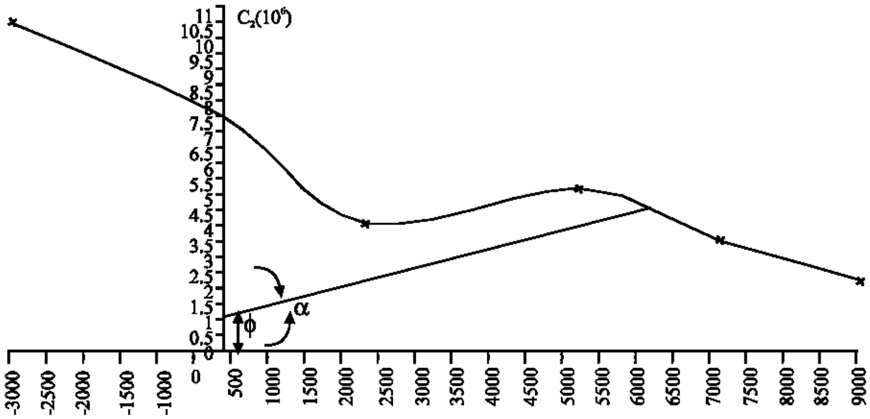

Fig. 6: $C_2^0 = \varphi(C_1^0)$

section x_L=9000m is of 7,7-6,8=0,9m;

$$\frac{\Delta y}{Q^2} = \frac{L}{k^2} = \frac{0{,}9}{(4000)^2} = 562{,}5.10^{-10}$$; A = 0,0002 ; γ` = 1,1T/m^3.

It is asked to calculate the transfer of the bed and to trace its configuration at the end of each interval of time.

Solution: By using the expressions (15) and (16), one will have:

$$\phi_1 = \frac{F_1 x}{q} = \frac{h^2}{q f'(v)} \gamma' x,$$ (while neglecting h^3_{cr} in front h^3)

$$f(v) = A\gamma q v^4 \Rightarrow f'(v) = 4A\gamma q v^3 = \frac{4\gamma A q^4}{h^3}\ \phi_1 = \frac{\gamma'.h^5}{4A\gamma q^5} x \quad (38)$$

$$\phi_2 = \int \frac{F_1}{F_2} dz = \int \frac{\gamma' h^2 h^m}{f'(h).qD_1} dz = \int \frac{\gamma' h^{5+m}}{4\gamma A D_1 q^5}$$

$$dz = \int \frac{\gamma'}{4\gamma A D_1} \frac{(y-z)^{5+m}}{q^5} dz$$

$$\phi_2 = -\frac{\gamma'(y-z)^{m+6}}{4\gamma A D_1 q^5 (m+6)} \quad (39)$$

Let us pose: y - z = $h_0 \xi$, where h_0-depth corresponding to the stop of the phenomenon of erosion.

$$\phi_1 = \frac{\gamma' h_0^5}{4A\gamma q} \xi^5 x; \phi_2 = -\frac{\gamma' h_0^{m+6}}{4A\gamma q^5 D_1 (m+6)} \xi^{m+6}$$

A- Coefficient in the formula has; $q_T = A\gamma v^4 q$; A = 0,0002
Let us take: D_1 = 0.02; γ' =1,1T/m^3; m = 3.0, one obtains:

$$\phi_1(\xi) = \frac{1{,}1.15^5.10^4}{4.2.1.10^5} \xi^5 x \approx 10^4 x \xi^5$$

$$\phi_2(\xi) = -\frac{1{,}1.15^9.10^4.10^2}{4.2.1.2.10^5.9} \xi^9 \approx -3.10^9.\xi^9$$

The computation results for each section are indicated in the following (Table 2).

The curve $C_2^0 = \varphi(C_1^0)$ is represented on the graph (Fig. 6); this curve has an opposite form compared to that relating to the variation initial depth on the section concerned.

Table 3: The calculation of the deformation of the bed

x (m)	y (m)	z (m)	y-z (m)	$\xi=\frac{y-z}{h_0}$	$\xi^9\ 10^{-4}$	$-\phi_2.10^5$	$\frac{\phi_1(\xi)}{x}=k$	$\phi_1(\xi).10^5=kx$	$\phi.10^5=-\phi_2-\phi_1$	C'₁	$t10^4$(s)
		-0.05	7.75	0.517	26.4	79.2	369	0	79.2	-400	15
		-0.1	7.8	0.520	27.8	83.4	380	0	83.4	-600	23
0	7.7	-0.2	7.9	0.526	30.8	92.4	403	0	92.4	-1300	52
		-0.3	8.0	0.533	34.7	104.1	430	0	104.1	-2000	86
		-0.4	8.1	0.540	39.0	117.0	459	0	117.0	-2800	129
		-0.5	8.2	0.547	43.8	131.4	490	0	131.4	-3000	147
2000	7.5	0.1	7.4	0.493	17.2	51.6	291	5.8	45.8	1300	20
		0.0	7.5	0.500	19.5	58.5	313	6.3	52.2	1000	32
		-0.1	7.6	0.507	22.1	66.3	335	6.7	59.6	700	43
		-0.2	7.7	0.513	24.6	73.8	355	7.1	66.7	400	57
		-0.3	7.8	0.520	27.8	83.4	380	7.6	75.8	-200	83
		-0.4	7.9	0.526	30.8	92.4	403	8.1	84.3	-1000	121
		-0.5	8.0	0.533	34.7	104.1	430	8.6	95.5	-1500	151
		-0.6	8.1	0.540	39.0	117.0	459	9.2	107.8	-2300	198
5000	7.2	-0.1	7.3	0.486	15.1	45.3	271	13.5	31.8	5400	11
		-0.2	7.4	0.493	17.2	51.6	291	14.5	37.1	1800	93
		-0.3	7.5	0.500	19.5	58.5	313	15.6	42.9	900	128
		-0.5	7.7	0.513	24.6	73.8	355	17.8	56.0	800	150
		-0.6	7.8	0.520	27.8	83.4	380	19.0	64.4	500	171
		-0.7	7.9	0.526	30.8	92.4	403	20.2	72.2	100	198
7000	7.0	-0.2	7.2	0.480	13.5	40.5	255	17.9	22.6	6500	13
		-0.4	7.4	0.493	17.2	51.6	291	20.4	31.2	5300	50
		-0.5	7.5	0.500	19.5	58.5	313	21.9	36.6	1700	166
		-0.6	7.6	0.507	22.1	66.3	335	23.5	42.8	1300	191
9000	6.75	-0.1	6.85	0.456	8.5	25.5	197	17.7	7.8	8600	8
		-0.2	6.95	0.463	9.8	29.4	213	19.2	10.2	8500	11
		-0.25	7.0	0.466	10.4	31.2	220	19.8	11.4	8400	13
		-0.45	7.2	0.480	13.5	40.5	255	22.9	17.6	7500	38
		-0.65	7.4	0.493	17.2	51.6	291	26.2	25.4	6100	84
		-0.75	7.5	0.500	19.5	58.5	313	28.2	30.3	5600	107
		-0.85	7.6	0.507	22.1	66.3	335	30.2	36.1	1700	245

The calculation of the deformation of the bed is carried out in table (Table 3). The function ϕ_1 can be written in form $\phi_1 = kx$, where in the example studied $K = 10^4\xi^5$. By writing C_1 in the Eq. 17 pennies forms: $C_1 = kC_1'$, one obtains: $\phi_1 = \phi_2 - C_1 = k(x - C_1') = -(\phi_2 + C_2)$; from where: $C_2 = \phi + kC_1'$; where:

$\phi = -\phi_2 - kx$. By using this formula, one calculates the deformation of the bed (Table 3) and the time of deformation:

The variation of the coast of the bottom of the bed according to time is represented on the graph (Fig. 7). On Fig. (8) the deformation of the bed at the end of the following periods of time is represented: $t_1 = 20.10^4$ s; $t_2 = 50.10^4$ s; $t_3 = 100.10^4$ s and $t_4 = 150.10^4$ s.

Now let us examine the problem of the variation of the level of the free face after erosion. For that, let us calculate the value of;

$$K^2 = \frac{Q^2}{I};\ \frac{K^2}{K_1^2} = \left(\frac{h}{h_1}\right)^m;$$

K_1 is calculated for the average depth with the state initial $h_1 = 7{,}2$ m.

$$K_1 = \frac{Q^2.\ell}{\Delta y_1} = \frac{(4000)^2.9000}{0{,}90} = 16.10^{10}$$

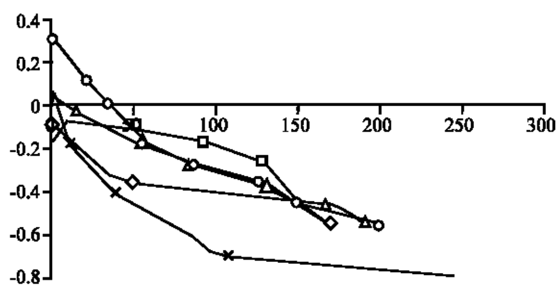

Fig. 7: z = f (t)

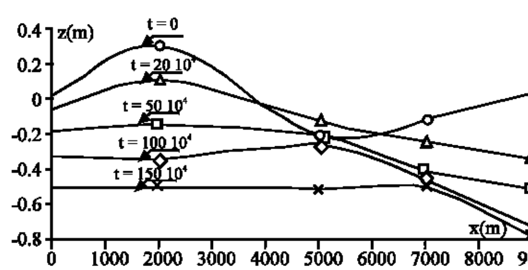

Fig. 8: z = f (x,t)

Table 4:The lowering of the water level is proportional to theti meoferosion

z(m)	y(m)	y-z(m)	$\xi = \frac{y-z}{h_0}$	$\zeta^{\circ}10^4$	$-\phi_2(\xi)$	$\phi(\xi)$	K	C'_1	$t.10^4$(s)
-0.1	7.7	7.8	0.520	27.8	$83.4.10^4$	$83.4.10^5$	380	-600	23
-0.2	7.66	7.86	0.524	29.8	$89.4.10^5$	$89.4.10^5$	395	-1100	43
-0.3	7.63	7.93	0.528	31.9	$95.7.10^5$	$95.7.10^5$	410	-1500	62
-0.4	7.59	7.99	0.532	34.1	$102.3.10^5$	$102.3.10^5$	426	-2000	85
-0.5	7.55	8.05	0.536	36.5	$109.5.10^5$	$109.5.10^5$	442	-2500	111
-0.6	7.52	8.12	0.541	39.7	$119.1.10^5$	$119.1.10^5$	463	-3000	139

After 150.10^4 s, the average depth is 7,8m, therefore:

$$K^2 = 16.10^{10}\left(\frac{7,8}{7,2}\right)^3 = 20,34.10^{10};\ \Delta y = \frac{\ell}{K^2}.Q^2 = \frac{9.10^3.4^2.10^6}{20,34.10^{10}} \approx 0,72\text{m}$$

instead of 0.90 m

Consequently, the lowering of the level of the free face on the whole of the studied section, will decrease by 0,90-0,72 m = 0,18 m under the influence of the deformation of the bed.

At the beginning of the section, this circumstance influences the result of calculation and involves the increase in the deformation of the bed. One remakes calculation for the section x = 0, by considering that the lowering of the water level is proportional to the time of erosion (Table 4).

Calculations show that the surface of the bottom of the bed is flattened progressively and tends to being parallel to the level of the free face, i.e., the mode of flow tends towards the permanent flow.

The examined example explains sufficiently the method of calculation of deformation of the bed of the river in the various practical cases for the gradually varied modes of flow.

CONCLUSION

This study was devoted to the flow gradually varied in the rivers with a variable longitudinal profile and to the deformations of these rivers in length and width. On the basis of simplifying assumptions and by using the method of the characteristics for the resolution of the released quasi linear equations, we could lead to convincing results such as the points of the highest rivers and particularly the tops of the undulations formed by sands move towards the downstream more quickly than the points located low.

It is now proven that the carriage of geochemical, biological and geomorphological natural equilibrium or the dynamic deceleration of the waves of floods passes by the solution of very complex problems related to the non stationary flows which are: knowing: - strong curves of the surface profiles;

- Variations of speeds (maximum attacks at the time of the flood) from a section to another of the channel;
- Depths of flood and duration of submergence which returns brutally or gradually;
- Longitudinal profiles of the channels;
- The hydraulic parameters applicable to the topography of the ground such as roughness or the Manning-Strickler coefficient, etc…

The slowing of the flow consists precisely in using fitting up solutions in order to slowing down the propagation waves of flows and the effects induced by these flows which are erosion (Boukrana, 1983), transfer and deposit of sediments in one or the other part of the channel or the work of storage.

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