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Hierarchical Minimization of Total Completion Time and Number of Tardy Jobs Criteria

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Abstract: In this study, the single machine bicriteria scheduling problem of hierarchically minimizing the total completion time of jobs (C_{tot}) and number of tardy jobs (NT) with release time was explored. Two types of hierarchical minimization models (the case of the total completion time criterion being more important than the number of tardy jobs criterion and the case of the number of tardy jobs criterion being more important than the total completion time criterion) were discussed. Three heuristics (HR4, HR5 and HR6) selected from the literature were used to test the models.

Key words: Hierarchical minimization, heuristics, relative weight, composite objective function, single machine

INTRODUCTION

Since, the realization of the fact that the total cost of a schedule is usually a complex combination of processing costs, inventory costs, machine idle-time costs and lateness penalty costs, amongst others, researchers have been exploring bicriteria scheduling problems (Oyetunji, 2006; Oyetunji and Oluleye, 2008a; 2008b; Ehrgott and Grandibleux, 2000; Hoogeveen, 1992, 2005; Nagar *et al*., 1995; Sayin and Karabati, 1999; Uthaisombut, 2000). In trying to minimize 2 criteria at a time (bicriteria scheduling), unless we are extremely lucky, there may be no schedule that achieves the minimum value for both criteria simultaneously (Hoogeveen, 2005). This implies that we have to give in on the quality of at least 1 of the 2 criteria. An approach to this is to rank the criteria in the other of their relative importance to the organization or firm. Then the less important criterion is minimized subject to the fact that the more important criterion is optimal. This approach is called hierarchical or lexicographical minimization. In many real life situations, criteria often carry unequal weights.

A number of researchers have adopted hierarchical minimization approach to bicriteria scheduling problem. Smith (1956) studied the minimization of the average completion time on 1 machine subject to minimal maximum lateness. Shmoys and Tardos (1993) studied the minimization of the average completion time on unrelated machines subject to the constraint that the makespan must be at most twice the optimal. Hurkens and Coster (1996) showed that there exist instances for the problem of scheduling jobs on unrelated machines such that all optimal average completion time schedules have a makespan of (log n) times optimal.

Ganesan *et al*. (2006) explored the problem of minimizing makespan subject to the minimum completion time variance (CTV) for a given set of n jobs to be processed on each of m machines in a static jobshop.

In many real life situations, minimizing 2 criteria simultaneously may be extremely difficult especially when the criteria are conflicting. Also, in many practical situations, scheduling criteria does not always carry equal weight. This means that some firm attach more importance to some criteria than others. Situations like these are best tackled through the use of hierarchical minimization approach. In this study, the single machine bicriteria scheduling problem of hierarchically minimizing the total completion time of jobs (C_{tot}) and number of tardy jobs (NT) with release time was explored. Two types of hierarchical minimization models (the case of the total completion time criterion being more important than the number of tardy jobs criterion and the case of the number of tardy jobs criterion being more important than the total completion time criterion) were explored.

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Hierarchical minimization models: Oyetunji and Oluleye (2008a) gave equation of the linear composite objective function of 2 scheduling criteria X and Y as:

$$F(X,Y) = \text{"}X + \text{\$}Y \quad (1)$$

where, " and \$ are the relative weights of criteria X and Y, respectively. Using partial differential methods, Oyetunji and Oluleye (2008a) gave (1, 0) and (0, 1) as the ranges of values of " and \$. They also gave the equation of the normalized linear composite objective function as

$$F^1(X,Y) = (\alpha)\left\{\frac{(Y_{max} - Y_{min})(X - X_{min})}{(X_{max} - X_{min})} + Y_{min}\right\} + (\beta)Y \quad (2)$$

In another study, Oyetunji and Oluleye (2008b) explored the bicriteria problem of simultaneously minimizing the total completion time and number of tardy jobs. That is, setting " = \$ = 0.5. In this study, we explore 2 hierarchical minimization models (the case of the total completion time criterion being more important than the number of tardy jobs criterion (model 1: " = 0.6 and \$ = 0.4) and the case of the number of tardy jobs criterion being more important than the total completion time criterion (model 2: " = 0.4 and \$ = 0.6). Using the equation of the normalized linear composite objective function and assuming that criterion X stands for total completion time and criterion Y stands for number of tardy jobs, the 2 hierarchical minimization models can be written as:

$$\text{Model 1} = 0.6*\left\{\frac{(NT_{max} - NT_{min})(Ctot - Ctot_{min})}{(Ctot_{max} - Ctot_{min})} + NT_{min}\right\} + 0.4*NT \quad (3)$$

$$\text{Model 2} = 0.4*\left\{\frac{(NT_{max} - NT_{min})(Ctot - Ctot_{min})}{(Ctot_{max} - Ctot_{min})} + NT_{min}\right\} + 0.6*NT \quad (4)$$

Where,

Ctot = Total completion time $(\sum_{i=1}^{n} C_i)$

NT = Number of tardy jobs $(\sum_{i=1}^{n} U_i)$

$Ctot_{max}$ = Maximum value of total completion time.

$Ctot_{min}$ = Minimum value of total completion time.

Nt_{max} = Maximum value of number of tardy jobs.

Nt_{min} = Minimum value of number of tardy jobs.

In real life, there are situations where firms place more importance on 1 criterion than others. Equations (3 and 4) models 2 different situations in which a firm sees number of tardy job criterion as more important than the total completion time criterion and vice versa. Note that $Ctot_{max}$, $Ctot_{min}$, NT_{max} and NT_{min} are constant values to be determined as stated in Oyetunji and Oluleye (2008b).

MATERIALS AND METHODS

The 3 heuristics (HR4, HR5 and HR6) proposed by Oyetunji and Oluleye (2008b) are used to test the above 2 models. The HR4 heuristic made use of both HR1 and HR3 heuristics (HR1 and HR2 were proposed by Oyetunji (2006) for the problem of minimizing the total completion of jobs and number of tardy jobs on a single machine with release time, respectively). The HR5 heuristic is a combination of the HR1 and DAU (DAU was proposed by Dauzere (1995), for the problem of minimizing the number of tardy jobs on a single machine with release time). Also, the HR6 heuristic is a combination of the AL1 (AL1 was proposed by Oyetunji (2006) for the problem of minimizing the total completion of jobs on a single machine with release time) and DAU.

Therefore, in order to assess the performance of the heuristics against the proposed models, 50 problems each were randomly generated for 22 different problem sizes ranging from 3-500 jobs. In all, a total of 1100 randomly generated problems were solved. The processing time of jobs were randomly generated with values ranging between 1 and 100 inclusive. The ready time of jobs were also randomly generated with values ranging between 0 and

$$(\sum_{i=1}^{n} P_i)$$

inclusive. The due dates were also randomly generated with values ranges between $(r_i + p_i)$ and $(r_i + 2*p_i)$ inclusive.

A program was written in Microsoft visual basic 6.0 to apply the solution methods (HR4, HR5, HR6 and BB) to the problems generated. The program computes the value of the normalized linear composite objective functions (models 1 and 2) obtained by each solution method for each problem. The data was exported to Statistical Analysis System (SAS version 9.1) for detailed analysis. SAS is a very versatile statistical package and was employed to enable credible conclusions to be drawn from the results. The hardware used for the experiment is a 2.4 GHz Pentium IV with 512 MB of main memory.

The general linear model (GLM) procedure in SAS was used to compute the mean value of the normalized linear composite objective function for each problem size (50 problems instances were solved under each problem size) and by solution methods. Also, the ranking of each solution method under each problem was computed. For example, if a solution method gives the lowest value of the total completion time, it was ranked first. Then, the number of times each solution method was ranked first out of the 50 problems solved under each problem size was computed. By this we were able to know the proportion of time that a solution method gives the best solution. This was done for all the solution methods tested under each of the models.

RESULTS

The results obtained when the mean value of the normalized composite objective functions were computed for each solution method and problem size are shown in Table 1 and 2 for models 1 and 2, respectively.

Based on the minimum mean value of the normalized composite objective function model 1 (the case of the total completion time criterion being more important than the number of tardy jobs criterion), a ranking order BB, HR4, HR6, HR5 was obtained for 3 # n # 10 problems while a ranking order BB, HR6, HR5, HR4 was obtained for 12 # n # 500 problems (Table 1). However, with the normalized composite objective function model 2 (the case of the number of tardy jobs criterion being more important than the total completion time criterion), a ranking order BB, HR4, HR6, HR5 was obtained for 3 # n # 7 problems while a ranking order BB, HR6, HR5, HR4 was obtained for 8 # n # 500 problems (Table 2).

It was observed that the cross over point (the point at which the performance of HR6 exceeds that of HR4) occurred when n = 12 and n = 8 for models 1 and 2, respectively. This means that in a bicriteria hierarchical minimization problem, where the number of tardy jobs criterion is more important than the total completion time criterion (model 2), the HR6 and HR5 heuristics perform better compared with the bicriteria hierarchical minimization problem where the total completion time criterion is more important than the number of tardy jobs

Table 1: Means of the normalized composite objective function (model 1)

Problem Size	Solution methods BB	HR4	HR5	HR6
3×1	0.91	0.98	1.29	1.28
4×1	1.54	1.67	1.83	1.83
5×1	1.82	2.01	2.49	2.48
6×1	2.30	2.61	2.81	2.80
7×1	2.73	3.18	3.41	3.40
8×1	3.15	3.94	4.15	4.15
9×1	3.69	4.33	4.52	4.51
10×1	4.11	4.84	4.96	4.93
12×1	4.87	5.97	5.80	5.75
15×1	6.54	7.98	7.54	7.45
20×1	9.05	11.39	9.92	9.73
25×1	11.14	14.75	12.30	11.99
30×1	13.12	17.75	14.71	14.29
40×1	17.93	24.62	20.05	19.34
50×1	22.90	31.96	25.33	24.26
100×1	45.65	66.81	49.67	47.13
120×1	54.66	81.09	60.53	57.44
140×1	63.93	94.80	69.36	65.63
200×1	90.20	136.97	99.07	93.63
300×1	-	206.86	148.69	140.36
400×1	-	277.00	197.51	186.55
500×1	-	346.80	246.65	232.73

Sample size = 50

Table 2: Means of the normalized composite objective function (model 2)

Problem Size	Solution methods BB	HR4	HR5	HR6
3×1	0.81	0.87	1.17	1.17
4×1	1.55	1.68	1.78	1.78
5×1	1.80	2.03	2.47	2.47
6×1	2.38	2.72	2.85	2.85
7×1	2.83	3.41	3.48	3.47
8×1	3.25	4.36	4.30	4.30
9×1	3.90	4.75	4.74	4.72
10×1	4.34	5.32	5.22	5.20
12×1	5.16	6.66	6.14	6.10
15×1	7.12	9.03	8.19	8.13
20×1	9.97	12.96	10.68	10.55
25×1	12.31	16.89	13.39	13.19
30×1	14.55	20.35	16.05	15.77
40×1	20.02	28.34	22.00	21.52
50×1	25.67	36.68	27.76	27.05
100×1	51.35	76.43	54.35	52.65
120×1	61.79	92.73	66.27	64.21
140×1	71.85	108.61	75.92	73.43
200×1	101.96	156.69	108.27	104.64
300×1	-	236.58	162.51	156.96
400×1	-	316.83	215.82	208.51
500×1	-	396.61	269.49	260.21

Sample size = 50

Table 3: Percentage of times solution methods was ranked first with respect to normalized composite objective function (model 1)

Problem Size	Solution methods BB	HR4	HR5	HR6
3×1	100	62	34	34
4×1	100	20	22	28
5×1	100	18	14	14
6×1	100	16	16	16
7×1	100	8	0	8
8×1	100	2	0	0
9×1	100	0	4	14
10×1	100	0	4	8
12×1	100	0	0	12
15×1	100	0	0	10
20×1	100	0	0	24
25×1	100	0	0	14
30×1	100	0	0	58
40×1	100	0	0	12
50×1	100	0	0	4
100×1	100	0	0	6
120×1	100	0	0	12
140×1	100	0	0	26
200×1	100	0	0	24

Sample size = 50

Table 4: Percentage of times solution methods was ranked first with respect to normalized composite objective function (model 2)

Problem Size	Solution methods BB	HR4	HR5	HR6
3×1	100	62	34	34
4×1	100	20	22	28
5×1	100	18	14	14
6×1	100	16	16	16
7×1	100	8	0	10
8×1	100	2	0	0
9×1	100	0	4	14
10×1	100	0	6	8
12×1	100	0	0	12
15×1	100	0	0	12
20×1	100	0	0	24
25×1	100	0	0	14
30×1	100	0	0	72
40×1	100	0	0	12
50×1	100	0	2	8
100×1	100	0	0	16
120×1	100	0	0	0
140×1	100	0	0	26
200×1	100	0	0	22

Sample size = 50

Table 5: Means of execution time (seconds) with respect to normalized composite objective functions (models 1 and 2)

Problem Size	Solution methods BB	HR4	HR5	HR6
3×1	0.8791	0.0002	0.0003	0.1409
4×1	0.6761	0.0004	0.0001	0.1406
5×1	0.8009	0.0002	0.0003	0.1418
6×1	1.0627	0.0006	0.0001	0.1424
7×1	1.5514	0.0016	0.0010	0.1390
8×1	1.8374	0.0006	0.0013	0.1462
9×1	2.2335	0.0011	0.0018	0.1283
10×1	2.7916	0.0004	0.0002	0.1296
12×1	3.0853	0.0019	0.0016	0.1296
15×1	3.422	0.0016	0.0016	0.1315
20×1	7.4707	0.0024	0.0034	0.1349
25×1	8.9151	0.0033	0.0039	0.1512
30×1	12.4608	0.0044	0.0048	0.1515
40×1	18.6614	0.0081	0.0087	0.1721
50×1	34.6055	0.0108	0.0109	0.1721
100×1	102.5728	0.0421	0.0453	0.3615
120×1	140.3210	0.0581	0.0634	0.3956
140×1	270.2090	0.0787	0.0840	0.4681
200×1	480.5320	0.1533	0.1646	1.7628
300×1	-	3119	0.3234	1.7071
400×1	-	5443	0.5631	2.4587
500×1	-	8434	.8984	4.2134

Sample size = 5

criterion (model 1). This appears to be as a result of the fact that the DAU heuristic, which is one of the heuristics that constitutes the HR6 and HR5 heuristics, performed very well with the single criterion problems of minimizing the number of tardy jobs with release dates on a single machine.

The percentage of the times each solution method was ranked first with respect to the value of the normalized composite objective functions (models 1 and 2) was computed and results are shown in Table 3 and 4, respectively. The Branch and Bound (BB) procedure, as expected, gave the best result (i.e. ranked first) 100% of the time for all the problem sizes under models 1 and 2. The percentage of the times HR4 was ranked first was similar under both models 1 and K2. There was little improvement in the percentage ranking of HR5 and HR6 heuristics when moving from model 1 to model 2 especially, with increasing number of jobs (Table 3 and 4).

Table 6: Test of means of execution time with respect to normalized composite objective functions (models 1 and 2)

Heuristics	Heuristics BB	HR4	HR5	HR6
BB	-	*	*	*
HR4	*	-	X	*
HR5	*	X	-	*
HR6	*	*	*	-

Sample size = 50; Note: * = Indicate significant result at 5% level; X = Indicate non significant result at 5% level; - = Indicate not necessary

Table 5 shows the mean time taken (seconds) to solve an instance of a bicriteria problem under various problem sizes and by the solution methods evaluated. There was no solution method that was consistently better between HR5 and HR4 across the problem sizes (Table 5). The difference in the time taken by HR5 and HR4 is not significant (p#0.05). However, both HR5 and HR4 were faster than HR6 and BB for 3 = n = 500. Also, the time taken by HR6 heuristic was significantly different from (faster than) that of BB (Table 6).

DISCUSSION

The rationale for HR4, HR5 and HR6 heuristics followed the idea of Stein and Wein (1997). Stein and Wein (1997) showed that given an optimal makespan schedule and an optimal total weighted completion time schedule, a valid schedule can be constructed from both schedules through a process called truncation and composition of schedules. Therefore, having selected good solution methods for the single criterion problems of minimizing total completion time of jobs with release time on a single machine and minimizing number of tardy jobs with release time on a single machine based on their performance, it is expected that the HR4, HR5 and HR6 heuristics would reflect the behaviors of the solution methods that constitute each heuristic.

This perhaps explains the reason for the good performance of the HR6 heuristic over the HR4 and HR5 heuristics. The HR6 combines the AL1 (the best, according to Oyetunji (2006), for the problem of minimizing the total completion time of jobs with release time on a single machine) and DAU (the best, according to Oyetunji (2006), for the problem of minimizing the number of tardy jobs with release time on a single machine) methods.

There is need for closer interaction between the analyst and the decision maker (production manager) in a firm in order to apply the models being put forward. In order to aid the production manager, the analyst is expected to do the followings:

- Study the environment of the firm or company.
- Ask what importance the manager/firm attaches to the 2 criteria.
- Use your ingenuity based on outcome of the above to determine the relative weights for the 2 criteria.
- Accommodate their opinion, but give them many options with their respective implications for the firm/company.
- Allow the manager to freely make his/her selection .

CONCLUSION

The single machine bicriteria scheduling problem of hierarchically minimizing the total completion time of jobs (C_{tot}) and number of tardy jobs (NT) with release time was explored. Two types of hierarchical minimization models (the case of the total completion time criterion being more important than the number of tardy jobs criterion and the case of the number of tardy jobs criterion being more important than the total completion time criterion) were discussed. Combining the 2 criteria ensures that both the manufacturer's and customer's concerns were taken care of in the decision making process. Minimizing the total completion time takes care of the manufacturer's concern while the customer's concern was taken care of by the minimization of the number of tardy jobs.

Experimental results show that in a hierarchical minimization model 1 where the total completion time criterion is more important than the number of tardy jobs criterion, the HR4 heuristic performed well for 3 # n# 10 problems while HR6 heuristics performed well for 12 # n# 500 problems. The cross over point occurred when n = 12. Also, in a hierarchical minimization model 2 where the total completion time criterion is less important than the number of tardy jobs criterion, the HR4 heuristic performed well for 3 # n# 7 problems while the HR6 heuristic performed well for 8 # n # 500 problems. The cross over point occurred when n = 8.

Observed that as we move from model 1 to model 2, the performance of HR6 method becomes better (the cross over point was reduced from 12-8). This appears to be as a result of the fact that the DAU heuristic, which is one of the heuristics that constitutes the HR6 heuristic, performed very well (Oyetunji, 2006) with the single criterion problems of minimizing the number of tardy jobs with release dates on a single machine (which is now the more important criterion in the bicriteria problem).

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