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A Universal Decoding Algorithm for t-EC/AUED Codes

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Abstract: The All Unidirectional Error Detection (AUED) codes are among the ways of guarding against erroneous correction and a t-Error Correction (EC)/All Unidirectional Error Detection (AUED) code can be constructed by appending a single check symbol (P) to a linear t-EC code to achieve the AUED property. Whenever, a t-EC/AUED code is used to encode information in a transmission side such information need to be decoded at the receiving side. Methods and techniques proposed in literature for constructing t-EC/AUED codes are either come with no Decoding algorithm or if a decoding a algorithm is proposed, it lacks the mathematical proof. This study adopts a decoding algorithm from the literature and derives a mathematical proof for it. Simulation software is also developed and results for the encoding and decoding processes of four different encoding methods are presented. Based on the mathematical proof and the simulation results obtained, this study claims that the adopted algorithm is a Universal Decoding algorithm.

Key words: Unidirectional error coding, t-EC/AUED codes, decoding algorithms, error correction codes, proof

INTRODUCTION

Computers and digital systems are exposed to faults which are many and varied. The faults lead to errors in data which are usually categorized as follows:

- Symmetric (random) errors: when both (1→0) and (0→1) errors are likely to occur
- Asymmetric errors: when either (1→0) or (0→1) errors occur with different probabilities
- Unidirectional errors: when both types (1→0) and (0→1) errors can occur but in a particular word all errors are of one type

The statistics of errors are strongly dependent on the way the data is organized. If the organization of a RAM memory, for example, consists of several bits per chip, a defect in a memory chip can affect several bits in the same word. Similarly, a defect on a tape due to handling or the presence of foreign particles usually effects one track, however, it will cause possibly a large number of errors on this track.

Early error correcting codes which were designed under the assumption that errors are random may not be entirely appropriate for errors which may occur in most memory systems (Al-Ani and Al-Shayea, 2010; Naydenova and Klove, 2009). This fact has led to the design of a class of codes which are capable of detecting unidirectional errors. This category of errors is the most probable to occur in digital systems and devices. Among the all-unidirectional error detection codes are the m-out-of-n codes (Pradhan and Stiffler, 1980) and the Berger codes (Pradhan, 1986). Unidirectional errors can occur in a large number compared to the limited number of random errors likely to occur.

The theory of unidirectional error correcting/ detecting codes was presented by Bose and Rao (1982) and since then binary t-EC/AUED codes have been extensively studied and many construction methods are pro posed in literature. In general, a t-EC/AUED code can be constructed by appending a check symbol to each word of linear binary t-EC code. An evaluation study of the construction methods found by Nikolos *et al.* (1986) and Andrew (1988) is presented (Mohammed and Al-Jobouri, 2015).

In line with the several number of proposed construction techniques, there are few decoding algorithms proposed. Most and probably all decoding algorithms are proposed with no mathematical proofs (Bose and Rao, 1982; Bruck and Blaum, 1992; Al-Bassam, 2000). On the other hand, other studies did not mention about the decoding algorithm for their proposed construction methods.

In this study, the Decoding algorithm proposed by Bose and Rao (1982) is adopted and a mathematical proof for this algorithm is presented. This adopted algorithm is

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then applied to four construction methods. Two methods are taken from Nikolas *et al*. (1986) and one is from Andrew (1988) while the fourth is from Al-Bassam (2000). In addition to the mathematical proof, a software program is developed whose results supports to the mathematical proof and demonstrates the performance of the decoding algorithm. By this proof and its supportive software, this study claims that the adopted decoding algorithm worked as a Universal Decoding algorithm.

SOME DEFINITIONS AND NOTATIONS

The theory of control coding is concerned with encoding and decoding of data and the mean of implementing them in hardware and software. Coding theory is also concerned with n-bits words that are of the form:

$$\underline{X} = (a_1, a_2, ..., a_n) \quad (1)$$

where, a_1, a_2, ..., a_n are symbol of a set S. let q be the number of symbol that are contained in S. There are q^n possible words of length n of total of q symbols. A code C is defined to be a subset of all possible codewords. If the set of symbol are elements of a finite field with q = 2, then binary codes will be constructed and these codes are the most important, hence, they are considered in this research. There are two main types of codes in common use today block codes in which n is fixed and conventional codes in which n is not fixed. Following are some definitions which are related to codes:

Definition 1: The Hamming distance between two n bit words $\underline{v}$ and $\underline{w}$ denoted $d(\underline{v}, \underline{w})$ is defined as the number of places where they differ.

Definition 2: The weight $w(\underline{v})$ of a word $\underline{v}$ is the number of non-zero digits in it. Linearity forms a common feature for a large number of codes which are referred to as linear codes (Lin and Costello, 2004). An (n, k) binary block code C is called linear if its set of 2^k n-tuples codewords are a subset of all n-tuples over GF(2) and can be expressed as a linear combination of a set of k basis vectors. The following notations are used in this study given X and Y as n-tuples over GF(2) then:

- d(X, Y) = Refers to the Hamming distance between X and Y
- N(X, Y) = Denotes the number of $(1\rightarrow0)$ crossovers from X to Y
- P = Denotes the appended check symbol to achieve the AUED property

The Hamming distance can be expressed in terms of crossovers as follows:

$$d(X, Y) = N(X, Y) + N(Y, X) \quad (2)$$

A characterization of when a code is a t-EC/AUED is known as per the following theorem:

Theorem: A code C is capable of correcting up to (t) random errors and detects all unidirectional errors if:

$$\text{both } N(X, Y) \geq t+1 \text{ and } N(Y, X) \geq t+1 \text{ for all } X, Y \in C \text{ and } X \neq Y \quad (3)$$

A UNIVERSAL DECODING ALGORITHM

The decoding algorithm: The following decoding algorithm is taken from Bose and Rao (1982). The steps of this algorithm which are shown in Fig. 1 are as:

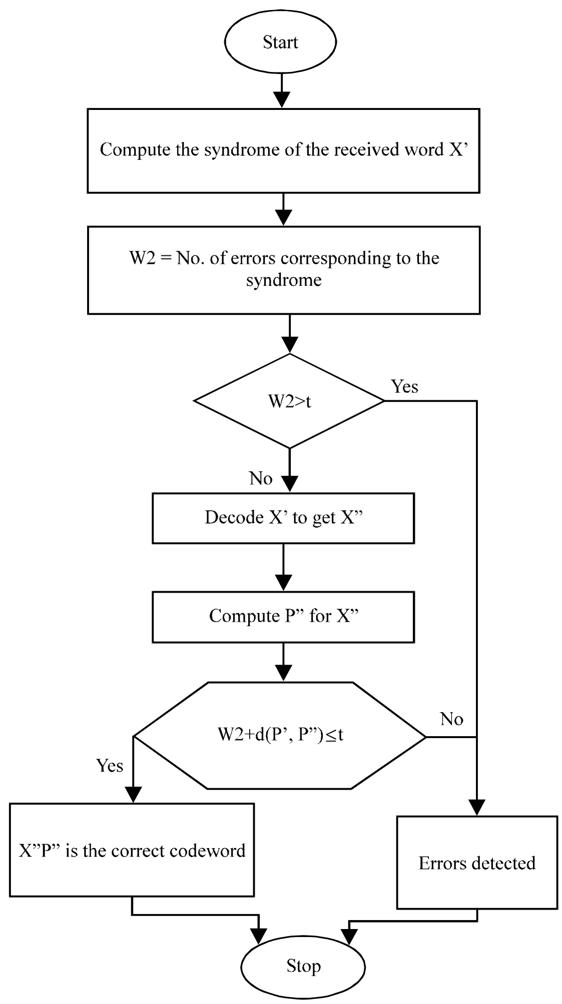

Fig. 1: Flow chart of the Universal Decoding algorithm

Step 1: Compute the syndrome of X', let W2 be the number of errors corresponding to the syndrome.

Step 2: If W2>t, then output errors detected and stop.

Step 3: Correct X' using a Decoding algorithm to get X" and compute P" for X".

Step 4: If W2+d(P', P")≤t then X"P" is the correct codeword, else output errors detected and stop.

The mathematical proof: Let [XP] be received as [X'P'] where d(X, X') = l_1, d(P, P') = l_2. Suppose X' is correctable and is corrected to X'' for which the corresponding AUED check symbol is P''. Then if $l_1+l_2 \le t$ [X"P"] = [XP] and d([X"P"], [X'P'])≤t. If $l_1+l_2>t$ and the errors are unidirectional d([X"P"], [X'P'])≥t+1.

Proof: Since, $l_1 \le t$ then X" = X, it follows that P" = P. Therefore, d([X"P"], [X'P']) = d([XP], [X'P']) = $l_1+l_2 \le t$. There are two cases to consider:

Case 1: $l_1 \le t$. X' is corrected to X. Thus, X" = X and P" = P. Therefore, d([X"P"], [X'P']) = d([XP], [X'P']) = $l_1+l_2 \ge t+1$.

Case 2: $l_1>t$. X' is erroneously corrected to X". There are three cases to consider:

Case 2a: W(X")>W(X), Since W(X")>W(X) then N(X", X)>N(X, X") and since N(X", X)+N(X, X") = d(X, X")≥2t+1, then N(X", X)≥t+1. Correction therefore, requires that it is 0→1 errors which are made in X. Also N(X', X")≤N(X', X")≤t. Since, the code is AUED then N(X, X")+N(P, P")≥t+1. But, since the errors are unidirectional and only 0→1, then N(P', P")≥N(P, P") whence N([X'P']), [X", P"])≥N([XP]), [X", P"])≥t+1 and d([X'P'], [X"P"])≥t+1.

Case 2b: W(X")<W(X). The proof is similar to that of the previous case (case 2a) with 1→0 errors in X occurring.

Case 2c: W(X") = W(X). This case can not occur since: N(X, X") = N(X", X)≥t+1.

Case study: Consider case 2a above and assume having a codeword (X = 111010001000000) from the (15.7) 2-EC code and its AUED check symbol (P = 01010101). This codeword represents an example calculated using one of the methods taken from Nikolas *et al.* (1986) and studied by Mohammed and Al-Jobouri (2015). Assume that this codeword is received as (X'P' = 111 111111110000 11011111) after some 0→1 errors in X and P. The word will be erroneously corrected to X" = 111 111110110000 with the assumption that $l_{1=1}$. Its AUED check symbol is P" = 10101010. In this case:

- N(X", X)≥t+1 = 6
- N(X, X")≤N(X', X")≤t = 1
- N(X, X")+N(P, P")≥t+1 = 5

Since, the errors in P are unidirectional only, then N(P', P") = N(P, P") = 4. Whence N([XP], [X"P"]) = N([X'P'], [X"P"])≥t+1 = 5 and d([X'P'], [X"P"])≥t+1 = 6. This corresponds to the detection of the unidirectional errors.

RESULTS

Another case study is presented here to demonstrate how the simulation software is working in order to process the encoding and decoding of different t-EC/AUED codes. A (16.7) code with even weight that is taken from (Andrew, 1988) and studied in (Mohammed and Al-Jobouri, 2015) is considered to explain the details of such processes. Consider a 7 bits message m = (1010010). This message is encoded by being multiplied by the following generator matrix G':

$$G' = \begin{bmatrix} 1110100010000001 \\ 0111010001000001 \\ 0011101000100001 \\ 0001110100010001 \\ 0000111010001001 \\ 0000011101000101 \\ 0000001110100011 \end{bmatrix}$$

The result of (m×G') is a 16 bit codeword X = 1101010111100101. A check symbol P is then calculated and appended to X such that the transmitted codeword XP = 1101010111100101 110001. Now consider that the received word is X'P' = 1101000011100101 110001. The (n-k) bits syndrome of X' is calculated, S = 100101010. By referring to a table of the correctable error patterns (e) and their corresponding syndromes, it is found that the correctable error pattern corresponding to S = 100101010 is e = 0000010100000000. Note that the table which we have to refer to has:

$$\left(1+\binom{16}{1}+\binom{16}{2}\right) = 137 \text{ entries}$$

Table 1: Simulation results of encoding and decoding processes of a t-EC/AUED code based on an even weight (16, 7) 2-EC code

7 bits message	Transmitted codeword XP	Received codeword X'P'	Syndrome S	Corresponding error pattern (e)	W2	Corrected word X" =X'-e	P"	W2+d (P', P")	Decoding Result
1000011	1110110001100111 110001	1110110000000111 110001	1011100	0000000001100000	2	1110110001100111	110001	2	Errors corrected
1001000	1111010110010000 100011	1011010110010000 100000	10000001	0100000000000000	2	1111010110010000	100011	3	Errors detected
1001	000111010110010 100011	1111110101100101 001111	110000001	-	-	-	-	-	Non correctable word
1010111	1101100011001111 110001	1101110011001111 010001	10001001	0000010000000000	1	1101100011001111	110001	2	Errors corrected
1010111	1101100011001111 110001	1101000011001111 110001	100010001	0000100000000000	1	1101100011001111	110001	1	Errors corrected
1010110	1101101101101100 110001	0001101101100111 010101	110001111	-	-	-	-	-	Non-correctable word
1010001	1101000100000011 000111	1011010101001010 101001	101100	0001000000001000	2	1010010101000010	111	6	Errors detected
1011110	1100011001111101 110001	1100001001110101 110011	0100001000	0000010000001000	2	1100011001111101	110001	3	Errors detected

Syndromes are calculated using the equation $S = e H^T$ where H is the parity check matrix. The value of the above error pattern indicates that the two random errors that have occurred in X' are at the sixth and eighth bits. Therefore, set W2 = 2. Because $W2 \leq t$ the errors are correctable and the corrected codeword X" = X'-e = 1101010111100101 and X"P" = 1101010111100101 110001.

More examples from the same code are tabulated and shown in Table 1. In Table 1, different scenarios of possible errors are demonstrated. To suit the space available and for data be to consistent, the table has been tilted to the left.

CONCLUSION

Among the ways of guarding against erroneous correction of possible errors in information being handled in computers and digital systems is by the use of codes which detect all unidirectional errors. It's a common fact that whenever information is encoded they need to be decoded. In literature, a researcher can find several methods and techniques proposed for constructing t-EC/AUED codes. Most of the those methods were proposed without mentioning how to decode the resulting codes while decoding algorithms were proposed with some of the others. In the later, there were no mathematical proofs of the proposed decoding algorithms. This study adopts one the decoding algorithms found in literature and a mathematical proof was derived. Four different construction (encoding) methods are also adopted and simulation results for the encoding and decoding processes are presented. The simulation results are obtained from the run of software that is developed for this purpose.

The Adopted Decoding algorithm is based on the basic theory of unidirectional error detection and correction and after being proved mathematically and by developing the simulation software, this study claims that this algorithm is a Universal Decoding algorithm.

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