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On $\tilde{\beta}g(\theta)$ Coenvergence and Adherence in Topological Spaces via Grill

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Abstract: A new class of sets called $\tilde{\beta}$-generalized closed sets and $\tilde{\beta}$-generalized open sets in topological spaces and its properties are studied. A subset A of a topological spaces (X, τ) is called $\tilde{\beta}$-generalized closed sets (briefly $\tilde{\beta}$g-closed) if cl(int(cl(A))) contains U whenever A contains U and U is open in X. A new class of $\tilde{\beta}$g-continuous maps and $\tilde{\beta}$g-irresolute maps in topological spaces and study some of its basic properties. In this study, we introduce the notion of $\tilde{\beta}g(\theta)$ convergence and $\tilde{\beta}g(\theta)$-adherence in grill topological spaces and study some of its basic properties and relations among them.

Key words: $\widetilde{\alpha\delta}^{\theta\#}$-convergence, $\widetilde{\alpha\delta}^{\theta\#}$-adherence, grill topological, irresolute maps, properties and relations

INTRODUCTION

Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Indeed a significant the main general topology and real analysis concerns the variously modified forms of continuity, separation axioms etc., by utilizing generalized open sets and the idea of grills on a topological space was first introduced by choquet. The concept of grills has shown to be a powerful supporting and useful tool like nets and filters for getting a deeper insight into further studying some topological notions such as proximity spaces, closure spaces and the theory of compactifications and extension problems of different kinds.

Throughout the present study, spaces X and Y always mean topological spaces. Let X be a topological space and Aa subset of X. For a subset A of a topological space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. In this study, we investigate some more properties of this type of closed spaces. Before entering to our research, we recall the following definitions which are useful in the sequel.

Definition 1.1: A subset A of a topological space (X, τ) is:

- A pre open set (Sheik, 2000) if A⊆ int(cl(A)) and a preclosed set if cl(int(A))⊆A
- A semi open set (Levine, 1963) if A⊆cl(int(A)) and a semi closed set if int(cl(A))⊆A
- A α-open set (Njastad, 1965) if A⊆ int(cl(int(A))) and a α-closed set if cl(int(cl(A)))⊆A
- A semi-preopen set (Andrijevic, 1986) if A⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A)))⊆A
- A regular open set (Stone, 1937) if A = int(cl(A)) and a regular closed set if A= cl(int(A))
- A $\tilde{\beta}$g-closed set (Kannan and Nagaveni, 2012) ($\tilde{\beta}$-generalized closed set) if cl(int(cl(A)))⊆U whenever A⊆U and U is open in X
- Researcher define the $\tilde{\beta}$-closure of A (Kannan and Nagaveni, 2015) as follows:

$$Cl_{\tilde{\beta}g}(A)=\|\{F: F is \tilde{\beta}g-closed in X, A\subset F\}$$

$\tilde{\beta}g$ (θ)-convergence and $\tilde{\beta}g(\theta)$-aderence

Definition 2.1: A grill g on a topological space (X, τ) is defined to be a collection of nonempty subsets of X such that (Kannan and Nagaveni, 2012):

$$A\in G \text{ and } A\subset B\subset X \Rightarrow B\in G$$

And:

$$A, B\subset X \text{ and } A\cup B\in G \Rightarrow A\in G \text{ or } B\in G$$

Definition 2.2: (Kannan and Nagaveni) If g is a grill (or a filter) on a space (X, τ), then the section of g denoted by sec g is given by:

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$$\sec g = \{A \subseteq X : A \cap G \neq \varphi, \text{ for all } G \in g\}$$

Definition 2.3: A grill g on a topological space (X, τ) is said to be a:

- $\hat{\beta}g(\theta)$-adhere (briefly $\hat{\beta}g(\theta)A$) at x∈X if for each $U \in \hat{\beta}go(x)$ and each $G \in g\, Cl_{\hat{\beta}g}(U) G \neq \varphi$
- $\hat{\beta}g(\theta)$-converge (briefly $\hat{\beta}g(\theta)C$) to a point x∈X if for each $U \in \hat{\beta}gO(x)$, there is some G∈g such that $G \subseteq Cl_{\hat{\beta}g}(U)$ (in this case we shall also say that g is $\hat{\beta}g(\theta)$-convergent to x

Remark 2.4: A grill g is $\hat{\beta}g(\theta)C$ to a point x∈X if and only if g contains the collection $\{Cl_{\hat{\beta}g}(U) : U \in \hat{\beta}gO(x)\}$.

Definition 2.5: A filter F on a space (X, τ) is $U \in \hat{\beta}gO(x)$ said to $\hat{\beta}g(\theta)A$ x∈X ($\hat{\beta}g(\theta)C$ to x∈X) if for each F∈F and each $U \in \hat{\beta}gO(x)$, $F \cap Cl_{\hat{\beta}g}(U) \neq \varphi$ (resp. to each there corresponds F∈F such that $F \cap Cl_{\hat{\beta}g}(U) \neq \varphi$.

We note at this stage that unlike the case of filters, the notion of $\hat{\beta}g(\theta)A$ of a grill is strictly stronger than that of $\hat{\beta}g(\theta)C$. In fact, we have.

Theorem 2.6: If a grill g on a space (X, τ) $\hat{\beta}g(\theta)A$ at some point x∈X, then g is $\hat{\beta}g(\theta)C$ to x.

Proof: Let a grill g on (X, τ), $\hat{\beta}g(\theta)A$ at x∈X. Then for each $U \in g \circ (x)$ and each G∈g, $Cl_{\hat{\beta}g}(U) \cap G \neq \varphi$ so that $Cl_{\hat{\beta}g}(U) \in \sec g$, for each $U \in g \circ (x)$ and hence $X - Cl_{\hat{\beta}g}(U) \notin g$. Then $Cl_{\hat{\beta}g}(U) \in g$ (as g is a grill and X∈g) for each $U \in \hat{\beta}gO(x)$. Hence, g $\hat{\beta}g(\theta)C$ must to x.

Remark 2.7: Let X be a topological space. Then for any x∈X, we adopt the following notation:

$$g(\hat{\beta}g(\theta), x) = \{A \subseteq X : x \in \hat{\beta}g(\theta)cl(A)\}$$

$$\sec g(\hat{\beta}g(\theta), x) = \left\{A \subseteq X : A \cap G \neq \varphi, \text{ for all } G \in g(\hat{\beta}g(\theta), x)\right\}$$

In the next two theorems, we characterize the $\hat{\beta}g(\theta)A$ and $\hat{\beta}g(\theta)C$ of grills in terms of the above notations.

Theorem 2.8: A grill g on a space (X, τ), $\hat{\beta}g(\theta)A$ to a point x∈X if and only if $g \subseteq g(\hat{\beta}g(\theta), x)$.

Proof: A grill g on a space (X, τ). $\hat{\beta}g(\theta)A$ at x∈X

$$\Rightarrow Cl_{\hat{\beta}g}(U) \cap G \neq \varphi \text{ for all } U \in \hat{\beta}go(\theta), x \text{ and all } G \in g$$

$$\Rightarrow x \in \hat{\beta}g(\theta)cl(g), \text{ for all } G \in g$$

$$\Rightarrow G \in g(\hat{\beta}g(\theta), x), \text{ for all } G \in g$$

$$\Rightarrow g \subseteq g(\hat{\beta}g(\theta), x)$$

Conversely, let $g \subseteq g(\hat{\beta}g(\theta), x)$. Then for all G∈g, $x \in \hat{\beta}g(\theta)cl(g)$, so that for all $U \in \hat{\beta}gO(x)$ and for all G∈g, $Cl_{\hat{\beta}g}(U) \cap G \neq \varphi$. Hence, g is $\hat{\beta}g(\theta)A$ at x.

Theorem 2.9: A grill g on topological space (X, τ) is $\hat{\beta}g(\theta)C$ to a point x of X if and only if $g \subseteq \sec g(\hat{\beta}g(\theta), x)$.

Proof: Let g be a grill on X, $\hat{\beta}g(\theta)C$ to x∈X. Then for each $U \in \hat{\beta}gO(x)$, there exists G∈g such that $G \subseteq Cl_{\hat{\beta}g}(U)$ and hence $Cl_{\hat{\beta}g}(U) \in g$ for each $U \in \hat{\beta}gO(x)$. Now, $B \in \sec g(\hat{\beta}g(\theta), x)$.

$$\Rightarrow X - B \notin g(\hat{\beta}g(\theta), x)$$

$$\Rightarrow x \notin \hat{\beta}g(\theta)cl(X - B)$$

$$\Rightarrow \text{there exists } U \in \hat{\beta}gO(x) \text{ such that } Cl_{\hat{\beta}g}(U) \cap (X - B) = \varphi$$

$$\Rightarrow Cl_{\hat{\beta}g}(U) \subseteq B, \text{ where } U \in \hat{\beta}gO(x)$$

$$\Rightarrow B \in G$$

Conversely, let if possible, g not a $\hat{\beta}g(\theta)C$ to x. Then for some $U \in gO(x)$, $Cl_{\hat{\beta}g}(U) \in g$ and hence $Cl_{\hat{\beta}g}(U) \notin \sec g(\hat{\beta}g(\theta), x)$. Thus for some $A \in g(\hat{\beta}g(\theta), x)$, $A \cap Cl_{\hat{\beta}g}(U) = \varphi$. But $A \in g(\hat{\beta}g(\theta), x)$

$$\Rightarrow x \in \hat{\beta}g(\theta)cl(A)$$

$$\Rightarrow Cl_{\hat{\beta}g}(U) \cap A \neq \varphi$$

which is a contradiction.

Theorem 2.10: A grill g on a topological space (X, τ), $\hat{\beta}g(\theta)C$ to a point x of (X, τ), if and only if $\sec g(\hat{\beta}g(\theta), x) \subseteq g$.

Proof: Let g be a grill on a topological space (X, τ), $\hat{\beta}g(\theta)C$ to a point x∈X. Then for each $U \in \hat{\beta}gO(x)$ there exists G∈g such that $G \subseteq Cl_{\hat{\beta}g}(U)$ and hence $Cl_{\hat{\beta}g}(U) \in G$ for each Now, $B \in \sec g(\hat{\beta}g(\theta), x) \Rightarrow X/B \notin g(\hat{\beta}g(\theta), x) \Rightarrow x \notin Cl_{\hat{\beta}g}(U) \Rightarrow$ there exists $U \in \hat{\beta}gO(x) \rightarrow B \in G$ such that $Cl_{\hat{\beta}g}(U) \cap (X/B) = \varphi \Rightarrow Cl_{\hat{\beta}g}(U) \subseteq B$, where $U \in (\hat{\beta}gO(x) \Rightarrow B \in G$. Conversely, let if possible, g not to $\hat{\beta}g(\theta)A$ to x. Then for some $U \in (\hat{\beta}gO(x), Cl_{\hat{\beta}g}(U) \in g$

and hence $Cl_{\hat{\beta}g}(U) \notin secg(\hat{\beta}g(\theta),x)$. Thus for some $A \in g(\hat{\beta}g(\theta),x), A \cap Cl_{\hat{\beta}g}(U) = \varphi$. But $A \in g(\hat{\beta}g(\theta),x) \Rightarrow x \notin Cl_{\hat{\beta}g}(A) \Rightarrow Cl_{\hat{\beta}g}(U) \cap U \neq \varphi$.

Definition 2.11: A nonempty subset A of a topological space X is called $\hat{\beta}g$ closed relative to X if for every cover u of A by $\hat{\beta}g$-open sets of X there exists a finite subset u_0 of u such that $A \subseteq U\{Cl_{\hat{\beta}g}(U) : U \subset u_0\}$ If in addition, A = X then X is called a $\hat{\beta}g$-Closed space.

Theorem 2.12: For a topological spaceX, the following statements are equivalent:

- X is $\hat{\beta}g$-closed
- Every maximal filterbase $\hat{\beta}g(\theta)C$ to some point of X
- Every filterbase $\hat{\beta}g(\theta)$ -adhere to some point of X
- For every family $\{V_a : a \in I\}$ of $\hat{\beta}g$-closed sets that $\cap\{V_i : i \in I\} - \varphi$,there exists a finite subset I_0 of I such that $\cap\{Int_{\hat{\beta}g}(V_i) : i \in I_0\}$.

Proof (a)⇒(b): Let F be a maximal filterbase on X. Suppose that F does not $\hat{\beta}g$-converge to any point of x. Since, T is maximal, T does not $\hat{\beta}g(\theta)$-accumulate at any point of X. For each x∈X, there exist F_x∈F and $V_x \in \hat{\beta}gO(X,x)$ such that $Cl_{\hat{\beta}g}(V_x) \cap F_x = \varphi$. The family is $\{V_a : x \in X\}$ a cover of X by $\hat{\beta}g$-open sets of X. By (a), there exists a finite number of points $x_1, x_2, x_3, \ldots x_n$ of X such that $X = U\{Cl_{\hat{\beta}g}(V_{xi}) : i = 1, 2, \ldots, n\}$. Since, F is a filter base on X, there exists F_o∈F such that $F_o \subseteq \cap\{F_{xi} : i = 1, 2, \ldots, n\}$ Therefore, we obtain $F_o = \varphi$. This is a contradiction.

(b)→(c): Let F be any filterbase on X. Then, there exists a maximal filterbase F_o such that F⊆F_o. By (b), F_o $\hat{\beta}g(\theta)$-converges to some point x∈X. For every F∈F and every $V \in \hat{\beta}gO(X,x)$, there exists F_o∈F_o such that $F_o \subseteq Cl_{\hat{\beta}g(\theta)}(V)$; hence $\varphi \neq F_o \cap Cl_{\hat{\beta}g}(V) \subseteq F$. This shows that F $\hat{\beta}g(\theta)$-accumulates at x.

(b)→(c): Let $\{V_a : \alpha \in I\}$ be any family of $\hat{\beta}g$-closed subsets of X such that $\cap\{V_a : x \in X\} = \varphi$. Let Γ (I) do note the ideal of all finite $\hat{\beta}g$-closed subsets of A. Assume $\cap\{Int_{\hat{\beta}g}(V_a) : \alpha \in I\} = \varphi$ that for every I∈ΓI). Then, the family $F = \{\bigcap_{a \in I} Int_{\hat{\beta}g}(V_a) : I \in \Gamma(I)\}$ is a filterbase on X By (c), F $\hat{\beta}g(\theta)$-accumulates at some point x∈X. Since, $\{X/V_\alpha : \alpha \in I\}$ is a cover of $X, x \in X/V_{\alpha o}$ for some V_{xo}∈I. Therefore, we obtain $X/V_{\alpha 0} \in \hat{\beta}gO(X,x)$, $Int_{\hat{\beta}g}(V_{\alpha o}) \in F$ and $Cl_{\hat{\beta}g}(V_{\alpha o}) \bigcap Int_{\hat{\beta}g}(V_{\alpha o}) = \varphi$ which is a contradiction.

(d)→(a): Let $\{V_\alpha : \alpha \in I\}$ be a cover of X by $\hat{\beta}g$-open sets. Then, $\{X/V_\alpha : \alpha \in I\}$ is a family of $\hat{\beta}g$-closed subset, so fX such that $\bigcap\{X/V_\alpha : \alpha \in I\} = \varphi$. By (d), there exists a finite subset I_0 of I such that $\bigcap\{Int_{\hat{\beta}g}(X/V_a) : a \in I_0\} = \varphi$ hence $X = \bigcup\{Cl_{\hat{\beta}g}(V_x) : a \in I_0\}$. This shows that X is $\hat{\beta}g$-closed.

Theorem 2.13: A topological space X is $\hat{\beta}g$-closed if and only if every grill on X is $\hat{\beta}g(\theta)$-convergent in X.

Proof: Let g be any grill on a $\hat{\beta}g$-closed space X. Then by Theorem 2.6, sec g is a filter on X. Let B∈sec g, then $X/B \notin g$ and hence B∈g (as g is a grill). Thus, secg⊆g. Then by Theorem 2.6 (b), there exists an ultrafilter u on X such that sec g⊆u⊆g. Now as X is $\hat{\beta}g$-closed in view of Theorem 3.2, the ultrafilter u is $\hat{\beta}g(\theta)$-convergent to some point x∈X. Then for each $U \in (\hat{\beta}gO(X,x)$, there exists F∈u such that $F \subseteq Cl_{\hat{\beta}g}(U)$. Consequently, $Cl_{\hat{\beta}g}(U) \in u \subseteq g$, that is $Cl_{\hat{\beta}g}(U) \in g$, for each $U \in \hat{\beta}gO(X,x)$. Hence, g is $\hat{\beta}g(\theta)$-convergent to x. Conversely, let every grill on X be $\hat{\beta}g(\theta)$-convergent to some point of X. By virtue of Theorem 3.2 it is enough to show that every ultrafilter on X is $\hat{\beta}g(\theta)$-convergent in X which is immediate from the fact that an ultrafilter on X is also a grill on X.

Theorem 2.14: A topological space X is $\hat{\beta}g$-closed relative to X if and only if every grill ? on X with A∈g, to a point in A.

Proof: Let A be $\hat{\beta}g$-closed relative to X and g a grill on X satisfying A∈g such thatg does not $\hat{\beta}g(\theta)$-convergent to any a∈A. Then to each a∈A, there corresponds some $U_0 \in \hat{\beta}gO(X,a)$ such that $Cl_{\hat{\beta}g}(U_a) \notin g$. Now $\{U_\alpha : a \in A\}$ is a cover of A by $\hat{\beta}g$-open sets of X. Then, $A \subseteq U_{i=1}^{n} Cl_{\hat{\beta}g}(U_{ai}) = U$ (say) for some positive integer n Since g is a grill, $U \notin g$; hence $A \notin g$ which is a contradiction.

Conversely, let A be not $\hat{\beta}g$-closed relative to X. Then for some cover $u = \{U_a : a \in A\}$ of A by $\hat{\beta}g$-open sets of X, $F = \{A / U_{\alpha \in Io} Cl_{\hat{\beta}g}(U_\alpha) : I_u \text{ is finite subset of } I\}$ is a filter base on X. Then, the family F can be extended to an ultra filter F*on X. Then F* is a grill on X with A∈F* (as each Fof F is a subset of A). Now for each x∈A, there must exists β∈I such that x∈U_β as U is a cover of A. Then for any G∈F*, $G \bigcap (A/Cl_{\hat{\beta}g}(U_\beta) \neq \varphi$, so that $G \supset Cl_{\hat{\beta}g}(U_\beta)$ for all G∈g. Hence, F*, cannot $\hat{\beta}g(\theta)$-convergent to any point of A. The contradiction proves the desired result.

CONCLUSION

By using generalized closed sets and topology, new class of sets in topological spaces namely $\hat{\beta}$-generalized closed sets and $\hat{\beta}$-generalized open sets have been introduced and some of their properties are investigated. Various functions, namely $\hat{\beta}g$-continuous functions, almost contra $\hat{\beta}g$-continuous functions in topological spaces and also introduces a new spaces like $\hat{\beta}g(\theta)$-convergence and $\hat{\beta}g(\theta)$-adherence in grill topological spaces have been defined and their characteristics are investigated.

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