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Sliding Mode Controller for Buck DC-DC Converter

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Abstract: This study is a simple and systematic approaches to the design of sliding mode controller for buck DC-DC converters. Various aspects of the design including the practical problems and the proposed solutions are detailed. However, these control strategies can not compensate for large load current and input voltage variations. In this study, a new control strategy by compromising both schemes advantages and avoiding their drawbacks is proposed, analyzed and simulated.

Key words: Buck, DC/DC converters, sliding mode control, drawbacks, voltage, Algeria

INTRODUCTION

The buck type DC-DC converters are used in applications where the required output voltage needed to be smaller than the source voltage. The control of this type DC-DC converters is more difficult than the buck type where the output voltage is smaller than the source voltage. The difficulties in the control of buck converters are due to the non-minimum phase structure, i.e., since the control input appears both in voltage and current equations from the control point of view, the control of buck type converters are more difficult than buck type (Guldemir, 2005). A control technique suitable for DC-DC converters must cope with their intrinsic nonlinearity and wide input voltage and load variations ensuring stability in any operating condition while providing fast transient response. Since, switching converters constitute a case of variable structure systems, the Sliding Mode (SM) control technique can be a possible option to control this kind of circuits. The use of sliding mode control enable to improved and even overcome the deficiency of the control method based on small signal models. In particular, sliding mode control improves the dynamic behaviour of the system, endowing it with characteristics such as robustness against changes in the load, uncertain system parameters and simple implementation.

Study of a fixed-frequency SM controllers has focused on the practical constrains. In view of this, researchers propose in this study a fixed frequency which is based on an indirect sliding mode control technique and is implemented in Pulse Width Modulation (PWM), this controller can offer good large signal control performances with fast dynamical response.

In order to assure that the controlled system operates properly the existence condition and stability must be verified. These are the summarized controller design steps but also the system modeling could be considered as a design step.

MATERIALS AND METHODS

Sliding mode control of buck DC-DC converter: The sliding mode control is based on the variable structure theory (Tan *et al*., 2007; Spiazzi *et al*., 1995) and introduces to the complete system a good dynamic response and also robustness to large load and input voltage variations. The sliding mode control operates in a simplified way as follows: a sliding surface is defined with the equilibrium point and the system is forced to be held into the sliding surface (existence condition) and then the system must reach the equilibrium point (stability).

System modeling: To illustrate the underlying principal, the state space description of the buck converter under sliding mode voltage control where the control parameters are the output voltage, output voltage error dynamic, inductor current and reference voltage (in phase canonical form) (Mattavelli *et al*., 1993) is first discussed. Figure 1 shows the schematic diagram of a sliding mode control buck converter. This study covers the theoretical aspects of the sliding mode control converter. A practical method to determine the sliding coefficients is also introduced:

$$\begin{cases} \dot{V}_r = -\beta V_o + V_{ref} \\ \dot{V}_o = -\dfrac{1}{R_L.C} V_o = \dfrac{1}{C} I_L \\ \dot{I}_L = -\dfrac{1}{L} V_o + \dfrac{1}{L} V_i U \end{cases} \quad (1)$$

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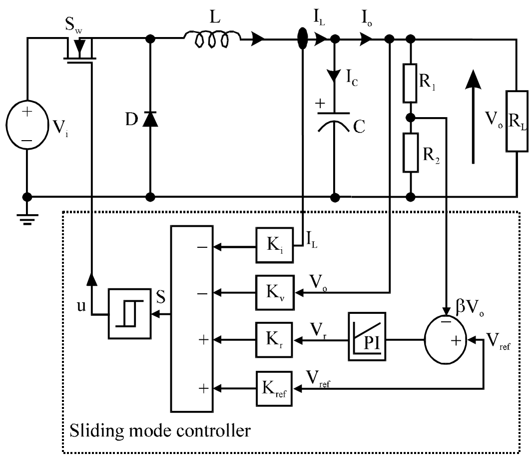

Fig. 1: Proposed sliding mode control circuit for buck DC-DC converter

Where:

- C = The capacitance
- L = The inductance
- R_L = The load resistance
- V_{ref} = The reference
- V_i = The input voltage
- $\beta.V_o$ = Sensed output voltage
- I_L = Inductance current
- U = 1 or 0 is the switching state of power switch SW

Then, the state space model of the system (1) can be derived as:

$$\dot{x} = Ax + BU + D \tag{2}$$

Where:

$$A = \begin{bmatrix} 0 & -\beta & 0 \\ 0 & -\dfrac{1}{R_L C} & \dfrac{1}{C} \\ 0 & -\dfrac{1}{L} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{V_i}{L} \end{bmatrix}, D = \begin{bmatrix} V_{ref} \\ 0 \\ 0 \end{bmatrix} \text{and } x = \begin{bmatrix} V_r \\ V_o \\ I_L \end{bmatrix}$$

Controller design: The basic idea of SM control is to design a certain sliding surface in its control law that will direct the trajectory of the state variables towards a desired origin when coincided (Tan *et al*., 2005a). The sliding mode controller has a switching function:

$$U = \begin{cases} 1 & \text{when } S > 0 \\ 0 & \text{when } S < 0 \end{cases} \tag{3}$$

Where S is the instantaneous state variables trajectory and is near to zero, it is described as:

$$S = -k_i I_L - k_v V_o + k_r V_r + k_{ref} V_{ref} \tag{4}$$

With $J^T = [k_i\ k_v\ k_r\ k_{ref}]$ and $k_i\ k_v\ k_r\ k_{ref}$ representing the control parameters termed as sliding coefficients. As in all other SM control schemes, the determination of the ranges of employable sliding coefficients for the SMC converter must go through the process of analyzing the existence condition of the controller converter system using the placement robust pole.

Derivation of existence conditions: To ensure that SM control is realizable in this system, an existence condition must be obeyed:

$$\lim_{S \to 0} S.\dot{S} < 0 \tag{5}$$

Thus by substitution the time derivative of Eq. 4, the condition for SM control to exist is:

$$\begin{cases} \dot{S}_{S \to 0+} = J^T Ax + J^T BU_{S \to 0+} + J^T D < 0 \\ \dot{S}_{S \to 0-} = J^T Ax + J^T BU_{S \to 0-} + J^T D > 0 \end{cases} \tag{6}$$

An illustration is provided for the buck converter.

Case 1 $(S \to 0^+, \dot{S} < 0)$ **:** Substitution of $U_{S \to 0+} = \bar{U} = 0$ and the matrices in Eq. 6 gives:

$$K_r(V_{ref} - \beta V_o) - k_v\left(\frac{1}{C} I_L - \frac{1}{R_L C} V_o\right) + k_i \frac{1}{L} V_o < 0 \tag{7}$$

Case 2 $(S \to 0^-, \dot{S} > 0)$ **:** Substitution of $U_{S \to 0-} = \bar{U} = 1$ and the matrices in Eq. 6 gives:

$$K_r(V_{ref} - \beta V_o) - k_v\left(\frac{1}{C} I_L - \frac{1}{R_L C} V_o\right) + k_i\left(\frac{1}{L} V_o - \frac{V_i}{L}\right) > 0 \tag{8}$$

Equivalent control: The stability analysis of the controller is made with the equivalent control; the equivalent control is substituted into the system model and is verified under that condition.

The equivalent control is the control law when the system is into the sliding surface and it is obtained from $\dot{S} = 0$ but changing U to the equivalent control U_{eq}. To get equivalent control, assume $\dot{S} = 0$:

$$U_{eq} = \frac{L}{V_i}\left[\left(\frac{1}{R_L C}\frac{k_v}{k_i} - \beta\frac{k_r}{k_i} + \frac{1}{L}\right)V_o - \frac{1}{C}\frac{k_v}{k_i} I_L + \frac{k_r}{k_i} V_{ref}\right] \tag{9}$$

Where U_{eq} is continuous, parameter k_v/k_i and k_r/k_i are to be determined which corresponds to the desired sliding mode controller dynamics that will be discussed. A necessary and sufficient condition of local existence of sliding regimes on S is:

$$-1 < U_{eq} < 1 \qquad (10)$$

Equation of state in sliding mode: The equation of state in sliding mode is obtained by replacing in the system (2) the discontinuous control U by the equivalent control (Spiazzi *et al*., 1995). The equation of state becomes:

$$\begin{bmatrix}\dot{V}_r\\ \dot{V}_o\\ \dot{I}_L\end{bmatrix} = \begin{bmatrix}0 & -\beta & 0\\ 0 & -\frac{1}{R_L C} & \frac{1}{C}\\ 0 & \frac{k_v}{k_i}\frac{1}{R_L C}-\beta\frac{k_r}{k_i} & -\frac{1}{C}\frac{k_v}{k_i}\end{bmatrix}\begin{bmatrix}V_r\\ V_o\\ I_L\end{bmatrix}+\begin{bmatrix}1\\ 0\\ \frac{k_r}{k_i}\end{bmatrix}V_{ref} \qquad (11)$$

The dynamics of the system in sliding mode is given by the characteristic equation:

$$p(s) = \det\left(s1 - A^*\right) = s\left[s^2 + \left(\frac{1}{C}\frac{k_v}{k_i} + \frac{1}{R_L C}\right)s + \frac{\beta}{C}\frac{k_r}{k_i}\right] = 0 \qquad (12)$$

Let us notice that a root of the characteristic equation is null because of the linear dependence of the variables of state when the system is in sliding mode S (x, t) = 0. The dynamics of the system is thus influenced by the parameters of the sliding surface k_v/k_i, k_r/k_i and the time constant of the filter R_LC.

However, the dynamics of the system is not affected by the variations of the voltage V_i and the inductance L. This is due to an inherent quality of the sliding modes which is the robustness.

Synthesis of the coefficients of the surface of commutation: When the load R_L varies, the poles of the system moves in the plan complexes, consequently, the performances are modified.

In this case, it is convenient to make a robust placement of the poles and to determine the coefficients of a return of state. Rather than to choose quite distinct poles; researchers impose a field in the complex plan in which the poles of the system must be whatever the disturbances of the load.

Choosing a domain of poles: Researchers proceed to determine the image of the domain poles in the plane K. Researchers adopt the imposition domain of poles within

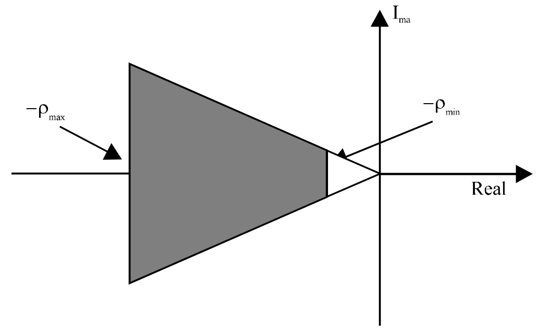

Fig. 2: Domain of poles imposed in complex plan K

Fig. 2. This domain in the left plane half of the plan K is delimited by two vertical lines corresponding, respectively to actual values $-\rho_{min}$ and $-\rho_{max}$ and by two inclined lines with ±45°, corresponding to complex conjugate poles combined with the real part equal to the imaginary part, the choice of these poles corresponds to an optimal relative damping.

When choosing a pair of complex conjugate poles inside the domain imposed $p_{1,2} = -\rho \pm j\omega$, the corresponding characteristic equation is:

$$p(s) = s^2 + 2\rho s + \rho^2 + \omega^2 = 0 \qquad (13)$$

Robust pole placement: The most predominant perturbation in a converter supply voltage V_i and the load R_L. The coefficients of the sliding surface are to identify and fix once and for all so as to minimize the effect of a change in R_L on the dynamics of the system in sliding mode.

To satisfy this condition, it is wise to choose as in the case of a step-down area of poles in which the poles of the closed-loop systems must be whatever the value of R_L. The root locus of the characteristic equation is kept within the imposed domain regardless of the variation in R_L.

Determining the coefficient k_{ref}: The coefficient k_{ref} has an influence on the position of the right shift compared to the original therefore according to the inequality (Eq. 16), the area of sliding mode varies according to k_{ref}. k_{ref} is chosen so that the field of sliding mode contains the desired operating region (Utkin *et al*., 1999).

Constant frequency operation: To control the switching frequency of the converter, the relationship between the hysteresis band Δ and switching frequency f_s must be known. Figure 3 shows the magnified view of the phase trajectory when it is operating in sliding mode.

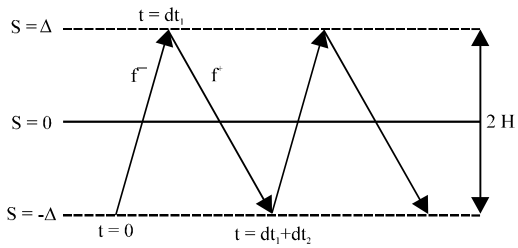

Fig. 3: Magnified view of phase trajectory in sliding mode operation

f^- and f^+ are the vectors of state variable velocity for U = 0 and U = 1, respectively. It was previously derived that:

$$dt_1 = \frac{2H}{\nabla S.f^-},\quad dt_2 = \frac{-2H}{\nabla S.f^+} \tag{14}$$

Where dt_1 is time taken for vector f^- to move from position a-b and dt_2 is the time taken for vector f^+ to move from b-c.

$$\nabla S.f = \sum \frac{\partial S}{\partial x_i}\frac{dx_i}{dt} = \frac{dS}{dt} = \dot{S} \tag{15}$$

Where:

$$f = \begin{cases} f^- & \text{for} \quad u = 0 \\ f^+ & \text{for} \quad u = 1 \end{cases}$$

So, we have:

$$dt_1 = \frac{2H}{\dot{S}_{u=0}.f^-} \qquad dt_2 = \frac{-2H}{\dot{S}_{u=1}.f^+} \tag{16}$$

the time period for one cycle is $T = dt_1+dt_2$. Therefore, the time period for one cycle in which the phase trajectory moves from position a-c is equivalent to (Eq. 17):

$$f_s = \frac{1}{T} = \frac{V_o(V_i - V_o)}{2\Delta V_i L} \tag{17}$$

Researchers can see clearly that the frequency is influenced by any variation on the input voltage, a way to get a fixed frequency sliding regime is to the equivalent relationship between the Pulse Width Modulation (PWM) and the sliding regime (Tan *et al*., 2006a, b).

It is possible to synthesize the duty cycle corresponding to the Pulse Width Modulation (PWM) from the theory of VSS (Fig. 4). This duty cycle derived from the equivalent command, applied to the converter has the advantage of robustness compared to the

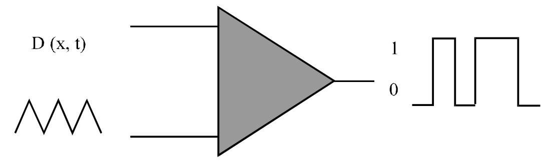

Fig. 4: Duty cycle derived from the equivalent control

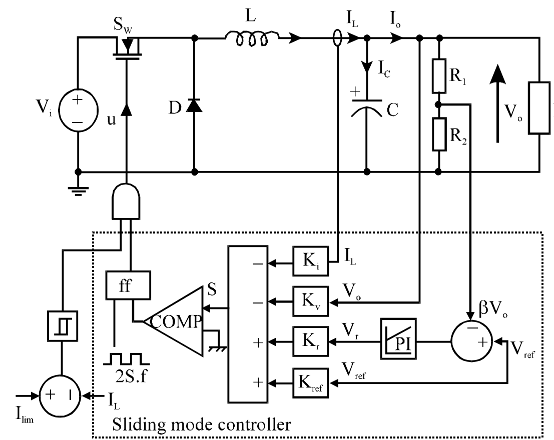

Fig. 5: Proposed sliding mode control with fixed frequency and current limitation

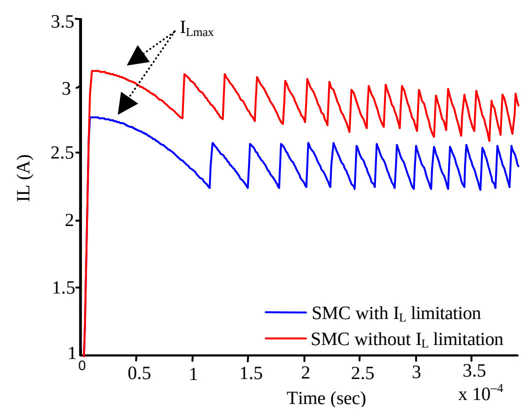

Fig. 6: Magnified inductor current I_L with and without current limitation. The values of coefficients used are k_r = 55, k_v = 0.045 and k_i = 1, k_{ref} = 1

variation of the input voltage V_i. The sequence (0.1) as a result causes an oscillation of the trajectory of state around the integral manifold of the controlled system.

Current limitation: The large inductor current could not be tolerated by the converter devices for two reasons; it can cause the inductor core to saturate with consequent even high peak current value or can be simply greater than the maximum allowed switch current. Thus, it is convenient to introduce into the controller a protection circuit which prevents the inductor current from reaching dangerous values. This feature can be easily incorporated into the sliding mode controller by a suitable modification of the sliding line (Tan *et al.*, 2005b). The scheme of the sliding mode controller associated with a PWM comparator to allow stabilization of the switching frequency and an approach to current limitation method is shown in Fig. 5.

For instance, the current limiter shown in Fig. 5 overrides sliding mode control when the switch current exceeds threshold I_{Lim}. If this happens, the control maintains the switch current at the value I_{Lim}. A simulation of the system with and without the current limitation is shown by Fig. 6.

RESULTS AND DISCUSSION

In this study, simulation results of the proposed sliding mode controller are provided to validate the

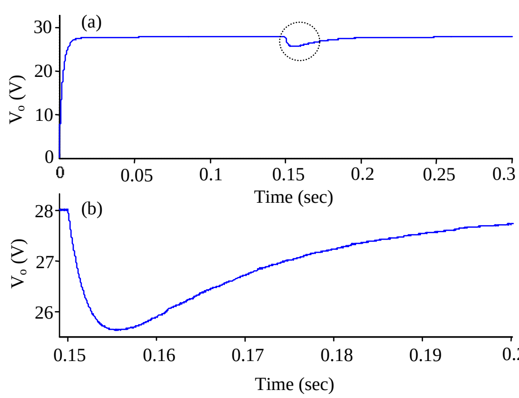

Fig. 7: Time response of output voltage V_o under step load variation (from 15-20 Ω) . The values of coefficients used are k_r = 55, k_v = 0.045 and k_i = 1, k_{ref} = 1

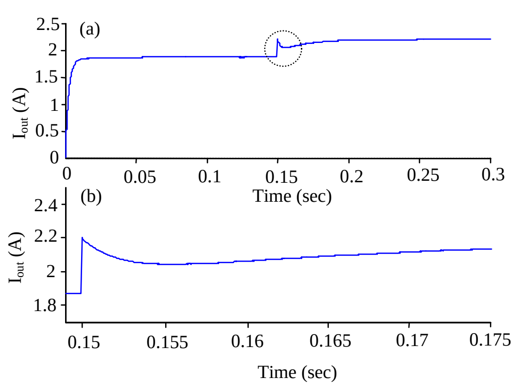

Fig. 8: Time response of output current I_{out} under step load variation (from 15-20 Ω). The values of coefficients used are k_r = 55, k_v = 0.045 and k_i = 1, k_{ref} = 1

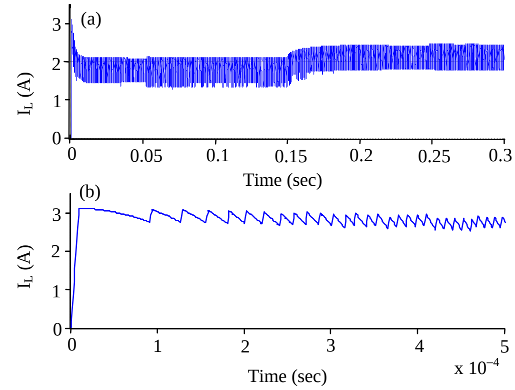

Fig. 9: Time responses of inductor current I_L under step load variation (from 15-20 Ω). The values of coefficients chosen are k_r = 55, k_v = 0.045 and k_i = 1, k_{ref} = 1

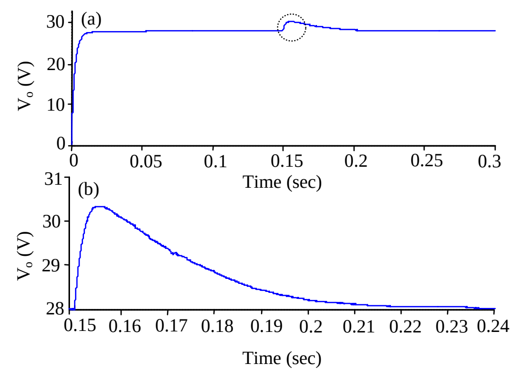

Fig. 10: Time response of output voltage V_o under step load variation (from 15-10 Ω) . The values of coefficients used are k_r = 55, k_v = 0.045 and k_i =1, k_{ref} = 1

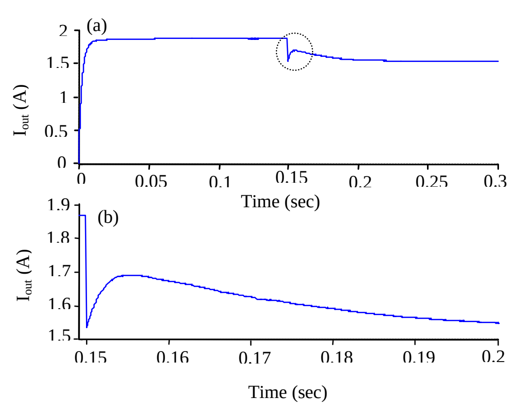

Fig. 11: Time responses of output current I_{out} and magnified output current under step load variation (from 15-10 Ω). The values of coefficients used are k_r = 55, k_v = 0.045 and k_i = 1, k_{ref} = 1

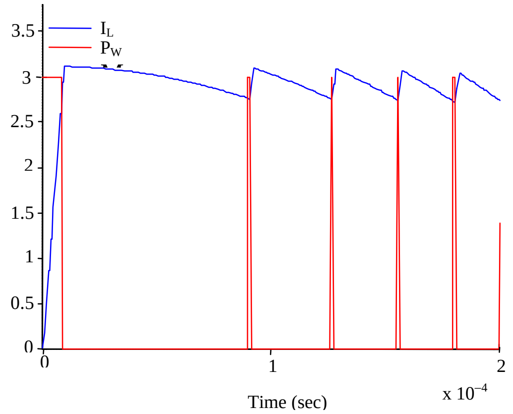

Fig. 12: Inductor current and PWM switching voltage magnified

theoretical design. The simulation program is developed from Eq. 8 for 60 W buck converters with specification theoretical design. The simulation program is developed shown in Table 1. The control parameters adopted are $V_{ref} = 2.8$ V, $\beta = 0.1$, $k_r = 55$, $k_v = 0.045$, $k_i = 1$ and $k_{ref} = 1$. They are chosen to comply the design restrictions in Eq. 9 and 11 and have been fine tuned to respond to a desired regulation and dynamic response. Figure 7-12 show the full schematic diagram of the simulation model.

CONCLUSION

The performance of the sliding mode controller buck DC-DC converter are analyzed under normal and disturbance conditions. Sliding mode control is able to ensure system stability even for large input voltage and load variation, good dynamic response and simple implementation. A fast response that operates at a fixed frequency is proposed for buck converter. The various aspects of the controller which includes the choice of sliding surface, the placement pole method for determination of controller parameters. Also additional function was implemented to provide inductor current limitation.

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