

## On the Optimal Pid Regulator for the Control of a Blower System

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**Abstract:** This study is devoted to the determination of an optimal PID regulator which minimizes the settling time of a blower system. Giving the high order of the mathematical model of such a system, the optimization of the regulator parameters is done by genetic algorithms. A discussion on the possibility of the existence of an overshoot is presented. Simulation results confirm the good performances of the implemented control system.

**Key words:** Blower, PID regulator, genetic algorithms, parameter optimization

### INTRODUCTION

The aim of this study considers the determination of an optimal PID regulator of a blower thermal process. It is well known that the dynamical behaviour of such system can be described by a complex mathematical model containing partial derivative terms of the temperature with respect to space variables. To overcome such difficulty, some assumptions will be considered. This yields to model the blower system by a high order large scale system. Thus, the determination of a PID controller becomes difficult. The objective is to minimize the settling time. However, it is practically impossible to express analytically the considered criterion with respect to the parameters of the regulator. For these reasons, the use of Genetic Algorithms (GA) to optimize the regulator parameter is then required.

Because genetic algorithms need a very large computational time, the optimization problem is divided into two steps. The first one localizes the optimal solution by considering a reduced iteration number of the genetic algorithm procedure. The second one considers a unidirectional optimization approach. This study will be done successively. This study is organized as follows. Next section presents a brief description on genetic algorithms.

**Genetic algorithms:** The basic concepts of genetic algorithms were developed by Holland<sup>[1]</sup> and subsequently in several research studies. Goldberg<sup>[2]</sup> provides recent comprehensive overviews and introductions to genetic algorithms. Genetic algorithms are exploratory search and optimization procedures. They

were derived from the principles of natural evolution<sup>[3]</sup> and genetic population. Genetic algorithms are iterative procedures in which a constant size of population of solution candidate is maintained. On the basis of the nature evolution, a new population of candidate solution is formed. In the basic genetic algorithms, the solution structures are determined by genes represented by code terms. These genes are consisted of a number of chromosomes. Each chromosome represents an individual. Genetic algorithms ensure the performance gradual increasing of the good solutions. This is done through checking the new solution generated from old populations by the use of many aspects. There are three most commonly used operators: reproduction, crossover and mutation.

Firstly, stochastic search processes play an important role in genetic algorithms (not deterministic processes as in many other optimization methods).

Secondly, genetic algorithms consider many point in a search space simultaneously, but only one point is considered. Therefore, genetic algorithms have a low chance to converge to a local optimum. Thirdly, genetic algorithms do not require the structures, parameters or other information about the problem.

Finally, genetic algorithms do not work with the search space, but they work with the chromosome space which becomes the search space. In the basic genetic algorithms, the solution structure is represented as binary strings.

**Problem formulation:** Let's consider a fluid flow (air flow) in a blower process whose axis is parallel to (ox) (Fig. 1). We will consider the following assumptions.

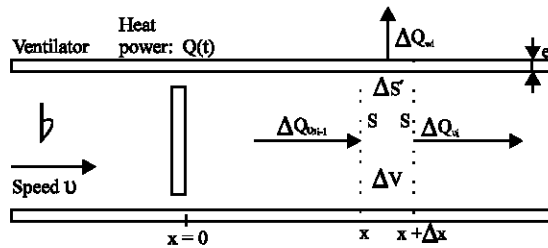


Fig. 1: Schematic representation of the blower system

**Assumption 1:** In all points, the air speed direction is parallel with the (ox) axis. The magnitude of the mean speed  $v$  is practically constant. It is controlled by a ventilator placed in the input of the blower. Such ventilator ventilates an electrical resistance ensuring the propagation of the energy to the output of the blower. Let's consider the power consumed by the resistance. is the control input of the blower. The object of the process consists to regulate the temperature at the output of the blower system.

**Assumption 2:** The pressure gradient is so small in such way that it can be considered that the energy exchange is done at a constant pressure<sup>[4]</sup>.

**Assumption 3:** The air temperature which depends on the three Cartesian components will be considered as a function of  $x$  and  $t$ :

$$T = T(\mathbf{x}, t).$$

**Assumption 4:** Only conductive energy exchanges through the blower wall will be formulated. The axial conduction will be neglected<sup>[5,6]</sup>. The considered blower is a cylindrical volume (with a section S) whose axis is parallel to the (ox) direction. In order to develop a model of the blower describing the evolution of the air interior temperatures, we will quantify the space (only with respect to the x-coordinate) in N cylindrical sections whose thickness is equal to  $\Delta x$ . For the volume element  $\Delta V = S\Delta x$ , variations of the enthalpy can be expressed as:

$$\Delta H_i = \rho_i C_{pi} \Delta V \frac{\partial T_i}{\partial t} \quad (1)$$

with:  $\delta_i = 1$  for  $i = 1$  and  $\delta_i = 0$  otherwise,  $T_0 = T_a$  and:

$$\Delta Q_{vi} = \rho_i C_{pi} v T_i S$$

$$\Delta Q_{wi} = \frac{2}{R} \times \frac{T - T_a}{\frac{1}{h_f} + \frac{1}{h_c} + \frac{e}{\lambda_{ce}}} \quad (2)$$

where  $\rho$  is the fluid density,  $C_p$  is the air specific heat,  $\lambda_w$  is the blower wall thermal conductivity,  $v$  is the air speed inside the blower and  $h_f$  and  $h_e$  are the internal and external fluid convective coefficients, respectively. It is notable that  $h_f$  corresponds to a turbulent forced convection (inside the blower) and  $h_e$  corresponds to a laminar natural convection (outside the blower).

Thus, we can write the following equations describing the dynamical behaviour of the blower:

$$\rho_i C_{p,i} \Delta x \frac{dT_i}{dt} = v(\rho_0 C_{p,0} T_a - \rho_i C_{p,i} T_i) + Q - \frac{2}{R} \times \frac{(T_i - T_a) \Delta x}{\frac{1}{h_{fi}} + \frac{1}{h_e} + \frac{e}{\lambda_{wi}}}$$

and for  $i=2$  to  $N$ :

$$\rho_i C_{pi} \Delta x \frac{dT_i}{dt} = v (\rho_{i-1} C_{p,i-1} T_{i-1} - \rho_i C_{p,i} T_i) - \frac{2}{R} \times \frac{(T_i - T_a) \Delta x}{\frac{1}{h_g} + \frac{1}{h_e} + \frac{e}{\lambda_{wi}}}$$

Because quantities  $\rho$ ,  $C_p$ ,  $h_f$ ,  $\lambda_p$  and  $\lambda_f$  depend on the temperature, we notified:

$$\rho_i = \rho(T_i), C_{pi} = C_p(T_i), h_{fi} = h(T_i),$$

$$\lambda_{wi} = \lambda_w([T_i + T_a]/2)$$

Identification procedures of parameters  $h_b$ ,  $\lambda_{ws}$ ,  $\lambda_b$ ,  $\rho$  and  $C_p$  give the following approximations<sup>[7,8]</sup>:

$$\begin{aligned}\rho(T) C_p(T) &\approx 210496 T^{-0.9097} \\ h_f(v, T) &\approx 143.9577 v^{0.8} T^{-0.5855} \\ \lambda_w(T) &\approx 1.575 \times 10^{-3} T^{1.2076} \\ \lambda_f(T) &\approx 0.2673 \times 10^{-3} T^{0.8051}\end{aligned}$$

The following are the blower parameters:  $R = 0.125$  m is the radius of the base of the blower,  $L = 1.2$  m its length,  $e = 2$  cm is the thickness of lateral sides,  $v = 3$  m/s is the air speed inside the blower,  $v_a = 0.1$  m/s is the air speed outside the blower,  $N = 100$  sections and  $T_a = 27^\circ\text{C}$  the ambient temperature.

**Proposed optimization algorithm:** The optimization will be done in two steps. The first step uses genetic algorithms in order to localise optimal values of the PID regulator

parameters. An updated optimization procedure will be done by the use of an unidirectional optimization procedure. This is done successively by:

- the optimization with respect to  $K_p$  and keeping  $K_d$  and  $K_i$  constants,
- the optimization with respect to  $K_i$  and keeping  $K_p$  and  $K_d$  constants,
- the optimization with respect to  $K_d$  and keeping  $K_p$  and  $K_i$  constants.

This procedure will be repeated until the convergence to the minimum is reached. The updated optimization procedure of a function  $f(x)$ ,  $x \in \mathbb{R}$ , can be described as follows:

- consider  $x_0$  the optimal value given by genetic algorithms and  $f_0 = f(x_0)$ .
- let  $x_1 = x_0 - \Delta x$  and  $x_2 = x_0 + \Delta x$ .
- compute  $f_1 = f(x_1)$  and  $f_2 = f(x_2)$ .
- let  $x_a = x_1 + 1/3 (x_2 - x_1)$  and  $x_b = x_2 + 1/3 (x_2 - x_1)$ .
- compute  $f_a = f(x_a)$  and  $f_b = f(x_b)$ .
- search the minimum of  $\{f_1, f_a, f_b, f_2\}$ .
- localise the minimum of  $f(x)$  in a reduced interval and repeat the procedure until the minimum of  $f(x)$  is reached.

**Simulation results:** Let us consider  $T = T_N$  the output temperature of the blower. In order to determine optimal values of PID parameters, we will consider the following criterion:

$$J = \min \tau_s$$

where  $\tau_s$  is the settling time, that is to say,  $\forall t > \tau_s$ :

$$\left| \frac{T(t) - T(\infty)}{T(0) - T(\infty)} \right| < 0.05$$

We will note the rise-time  $\tau_r$  by the necessary time which corresponds to the evolution of the temperature  $T$  from 10 to 90% of  $|T(0) - T(\infty)|$ . Moreover, we will consider the overshoot  $O_s\%$  which corresponds to the following quantity:

$$O_s = \max \left( 0, \max \frac{T(t) - T(\infty)}{T(\infty) - T(0)} \right) \quad (4)$$

The control output of the PID regulator is then expressed by:

$$u = Q = K_p (T_d - T) - K_d \frac{dT}{dt} + K_i \int_0^t (T_d - T) d\tau$$

for which parameters  $K_p$ ,  $K_d$  and  $K_i$  will be determined by genetic algorithms and updated by the proposed optimization algorithm.  $T_d$  is the desired temperature. It is to be noted that the above iterative procedure reduces the interval of the solution by 1/3 or 2/3. It is also well known that we can use other iterative procedures (particularly, the Fibonacci procedure<sup>[9]</sup>). In order to obtain aperiodic dynamics, we modify the optimized criterion in the above algorithm, as follows:

$$J = \begin{cases} \tau_s & \text{if } O_s = 0 \\ \tau_s + 1000 \times O_s & \text{otherwise} \end{cases} \quad (5)$$

Because genetic algorithms are very slow procedures, we will consider a first optimization phase in which relative small iteration number will be done. Results are relatively near the optimal solution. Thus, the optimal solution is localized. In a second step, an updated optimization procedure will be done by the use of the above detailed algorithm.

The obtained results are presented in Fig. 2, 3, 4 and 5. Figure 2 represents the response of the system (the blower output temperature). In this study, the PID regulator parameters are optimized by genetic algorithms. It is obvious that the output temperature presents an overshoot and the output reach the desired temperature. The settling time is about 15s with more than 10% of the overshoot.

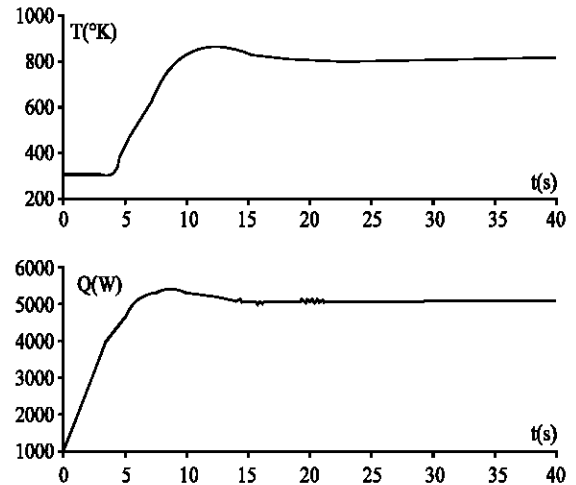


Fig. 2: Variations of the output temperature  $T$  and of the input control power  $Q$ . The parameters of the PID regulator are determined by genetic algorithms

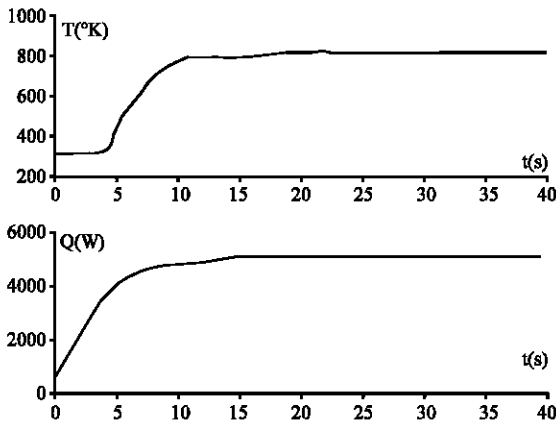


Fig. 3: Variations of the output temperature  $T$  and of the input control power  $Q$ . The parameters of the PID regulator are determined by genetic algorithms and improved by the iterative optimisation procedure

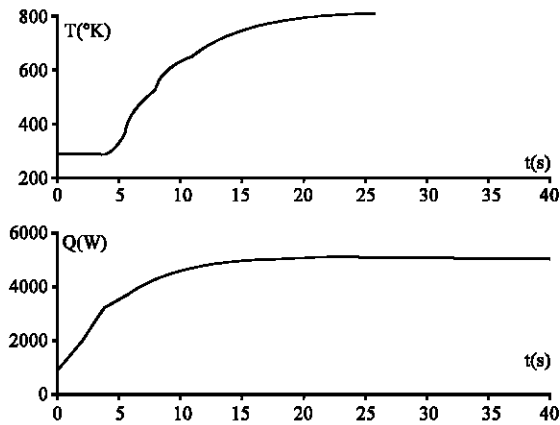


Fig. 4: Variations of the output temperature  $T$  and the input control power  $Q$ . The parameters of the PID regulator are determined by genetic algorithms and improved by the iterative optimisation, for the modified criterion (without overshoot)

Figure 3 presents the obtained results after an updated optimization of the parameters of the PID regulator, by optimizing the modified criterion. It is obvious that the settling time is decreased to almost 10% and the overshoot is also decreased to less than 1%. However the rise time is slightly increased.

Figure 4 presents the evolution of the output temperature and the input control power for the modified criterion. It is obvious that the response do not present any overshoot. Thus, the response is aperiodic. However, the value of the settling time and the rise time are increased. More details can be observed in Table 1.

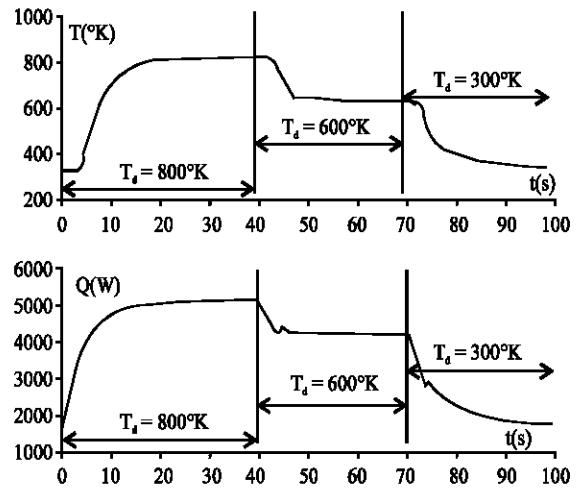


Fig. 5: Variations of the output temperature  $T$  and of the input control power  $Q$ , following variations on the desired trajectory. The parameters of the PID regulator are determined by genetic algorithms and improved by the iterative optimization procedure, for the modified criterion (without overshoot)

Table 1: Evaluation of different performance indexes ( $T_d = 800$  K)

	$\tau_d(s)$	$\tau_s(s)$	$O_s(\%)$
GA	4.274	14.94	10.3
min $\tau_s$	4.78	10.19	0.4
min $\tau_{s_s}$ for $O_s = 0$	9.85	16.75	0

In the following, we consider the PID parameters given by the optimal settling time without overshoot.

Figure 5 presents the evolution of the output temperature and the control power of the blower for different values of the desired output temperature. For the initial condition  $T = 300K$  and for the ambient temperature  $T_a = 300K$ , three steps are considered:

- from  $t = 0$  to  $t = 40s$ , the desired output temperature is equal to 800K,
- from  $t = 40$  to  $t = 70s$ , the desired output temperature is equal to 600K,
- and finally, from  $t = 70$  to  $t = 100s$ , the desired output temperature is equal to 300K.

Figure 5 illustrates the good performances obtained by the optimized PID regulator.

## CONCLUSION

The aim of this study consisted to determine optimal parameters of a PID regulator of a blower. Then a large scale mathematical model which

describes the dynamical behaviour of the system was developed.

Analytical expression of the settling time, which was considered as the objective function to be optimized, cannot be determined. Then, the use of genetic algorithms, as an optimization procedure, was required. In order to accelerate the convergence of the iterative procedure, two steps were considered. The first one was based on a genetic algorithm procedure. The second one used an unidirectional approach for the determination of optimal solutions. Simulation results were presented in two cases: with and without overshoot. A discussion of the obtained results was presented. As an illustration, a variable desired trajectory was considered.

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