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Vendor Selection Problem (VSP) Using Fuzzy Analytical Hierarchy Process (FAHP)

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Abstract: Companies in order to attain the goals of low cost, consistent high quality, flexibility and more customer satisfaction have been increasingly considering better Vendor selection approaches. Vendor selection problem is a Multicriteria Decision Making (MCDM) problems involving high degree of fuzziness. The fuzziness is involved in the multiple criteria used for selecting and ranking the best vendor. The aim of this study is to provide an analytical tool to select the best Vendor. We identify the criteria for selecting vendors and develop a hierarchy through which decision makers can bring about a comparison among the three Vendors by using fuzzy AHP. Using Rembrandt we try to bring about a comparison between the two techniques. We use the same input data for both the techniques.

Key words: Vendor selection, FAHP, fuzzy rembrandt, fuzzy MCDM

INTRODUCTION

Vendor Selection problem essentially deal with the selection of right vendors and their quota allocation. It is complex in nature and possesses high degree of fuzziness due to evaluation based on multiple criteria. The multiple criteria being price charged (including transportation costs), delivery lead-time, material quality, and services etc provided by each vendor are important considerations to the buyer in reaching a decision (Dickson, 1966; Weber, 1991; De Boer *et al*., 2001). Some factors in evaluating potential suppliers are quantitative, and a dollar value can be put on them. Other factors are qualitative and demand some judgment to determine them. The challenge is finding some method by which a buyer can pick the best rated Vendor that can meet the buyer's expected level of performance.

Different methods have been used for vendor selection the most common being Linear weighting method (Gregory, 1986; Monozka and Trecha, 1988; Wind and Robinson, 1968) have been adapted to deal with uncertainty in vendor selection problem where the DM deriving information from incomplete and qualitative data and unstructured purchasing situations. Narsimhan (1983) employed the Analytical Hierarchy Process, Weber *et al.* (2000) presented data envelopment analysis method for selecting vendors and their quota allocation. Mathematical programming approaches have been extensively used for vendor selection problem. They include Linear Programming (LP), Mixed Integer Programming (MIP) and Goal Programming (GP) etc. Moore and Fearon (1972), Pan (1989), Sharma *et al.* (1989) and Ghodsypour and Brein (1998) formulated some mathematical programming approaches. Weber and Current (1993) introduced multi-objective programming technique by decision support system for selecting vendors with their order quantities by multiple conflicting criteria. These are a few methodologies adopted for selection of Vendor.

Very often the choice between a numbers of alternatives is conflicting when decision has to be made on crisp decision criteria. By offering the decision-makers the possibility to express their opinions essentially fuzzy, we would obtain more realistic results. Fuzzy set theory provides a framework for handling the uncertainties involved in data set. Zadeh (1965) initiated the fuzzy set theory followed by Bellman and Zadeh (1970) giving application of fuzzy theories in decision-making processes. Some innovative approaches based on artificial intelligence techniques such as fuzzy logic (Albino *et al*., 1998; Kumar *et al*., 2004; Nassimbeni and Battain, 2000; Taskin and Murat, 2004) match very well with Decision Makers (DM's) situations where suppliers evaluation is also perceptive, DM's express heterogeneous judgments, many decisional rules are implied and unstructured, precise and accurate data are not available.

In this study vendor selection problem and different methods have been used for vendor selection.

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In this study a numerical example explains our methodology and Rembrandt calculations are shown. We conclude by bringing a comparison between the 2 systems AHP and Rembrandt (fuzzy and crisp case). Finally we rank the vendors using both methods.

FUZZY AHP

The Analytical hierarchy Process (Saaty, 1978) is one of the extensively used MCDM analysis tools for modeling unstructured problem in different areas. The AHP assumes that the multicriteria problem can be completely expressed in a hierarchical structure. The data acquired from the DM's are pair-wise comparisons concerning the relative importance of each of the criteria, or the degree of preference of one factor to another w.r to each criterion. Since it is difficult to map qualitative preferences to point estimates, a degree of uncertainty will be associated with some or all pair-wise comparison values is an AHP problem. The problem of generating such a priority vector in the uncertain pair-to-pair comparison environment is called the fuzzy AHP problem.

The earliest study in fuzzy AHP appeared by Laarhoven (1983), which compared fuzzy ratios, described by triangular membership functions. Buckley (1985) determined fuzzy priorities of comparison ratios with trapezoidal membership functions. Chang (1996) introduced an approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pairwise comparison scale of fuzzy AHP, with the use of triangular fuzzy numbers for pairwise comparison scale of fuzzy AHP and the use of the extent analysis in method for the synthetic extent values of the pairwise comparisons. Cheng (1997) proposed another algorithm for evaluating naval tactical missile system by the fuzzy AHP based on grade value of membership function. Cheng *et al.* (1999) proposed a new method for evaluating weapon systems by an AHP based on linguistic variable weight. Zhu and Change (1999) discussed some extent analysis methods and applications of fuzzy AHP. Leung *et al.* (2000) proposed a fuzzy consistency definition with consideration of a tolerance deviation for alternatives in fuzzy AHP. The work related to this subject would be the research by Wang and Lin (2003) on a fuzzy multicriteria group decision-making approach to select configuration for software developments. A multicriteria decision approach for software development strategy by Buyukozkan use fuzzy AHP method. The most recent work in Fuzzy AHP of Kapoor and Shyam (2006) was application in robot selection.

Extent analysis method on fuzzy ahp: In order to deal with the uncertainty and vagueness from subjective perceptions and experience of human in the decision process a methodology based on Chang (1992) extent fuzzy AHP modeling to assess the tangible and intangible balance is proposed.

Let $X=\{x_1, x_2, \ldots, x_n\}$ be an object set, and $U=\{u_1,u_2,\ldots,u_n\}$ be a goal set. According to the method of extent analysis, each object is taken and extent analysis for each goal, g_i is performed. Therefore m extent analysis values for each object can be obtained with the following signs:

$M_{gi}^1, M_{gi}^2, \ldots, M_{gi}^n$, i = 1, 2,..., n where all the M_{gi}^{ij} (j = 1, 2,..., m) are triangular fuzzy numbers.

The steps of changes extent analysis can be given as follows:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \quad (1)$$

To obtain

$$\sum_{j=1}^{m} M_{g_i}^{j}$$

perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (2)$$

And to obtain

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j}\right]^{-1}$$

perform the fuzzy addition operation of M_{gi}^{j} (j=1,2,…, m) values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \quad (3)$$

And then compute the inverse of the vector in Eq. 3 such that

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \quad (4)$$

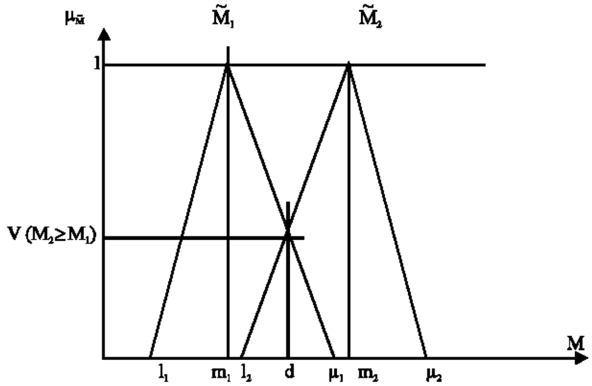

Fig. 1: The intersection between M_1 and M_2

Step 2: The degree of possibility of $M_2=(l_2,m_2,u_2) \geq M_1=(l_1,m_1,u_1)$ is defined as

$$V(M_2 \geq M_1 = \sup_{y \geq x}[\min(\mu_{M_1}(x),\mu_{M_2}(y))]$$

And can equivalently expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d)$$
$$= \begin{cases} 1, & \text{if} \quad m_2 \geq m_1 \\ 0, & \text{if} \quad l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases} \tag{5}$$

Where d is the ordinate of the highest intersection point D between μ_{M1} and μ_{M2} (Fig. 1).

To compare M_1 and M_2 , we need both the values of V ($M_1 \geq M_2$) and V($M_2 \geq M_1$).

Step 3: The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i (i =1,2,..,k) can be defined by

$$\begin{aligned} &V(M \geq M_1, M_2, \ldots, M_k) \\ &= V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and} (M \geq M_k)] \\ &= \min V(M \geq M_i),\ i = 1,2,3,\ldots,k. \end{aligned} \tag{6}$$

Assume that $d'(A_i) = \min V(Si \geq S_k)$

For k = 1, 2,.., n; k≠i. Then the weight vector is given by

$W'=(d'(A_1), d'(A_2),\ldots, d'(A_n))^T$ where

A_i (i=1,2,…,n) are n elements (7)

Step 4: Via normalization, the normalized weight vectors are

$W=(d(A_1), d(A_2), \ldots, d(A_n))^T$,
where W is a nonfuzzy number (8)

REMBRANDT

Refers to a system, which uses Ratio Estimation in Magnitudes or deci-Bells to Rate Alternatives, which are non-dominated, was developed in the Netherlands, led by Lootsma (1992, 1997). This system was designed to address three criticized features of AHP. The first issue was the numerical scale for verbal comparative judgment, that is direct rating is on a logarithmic scale, which replaces the fundamental 1-9 scale presented by Saaty. Lootsma presents a geometric scale where the scale reflects the gradients of decision makers' judgment as follows:

$\frac{1}{16}$: Strict preference for object 2 over base object.
$\frac{1}{4}$: Weak preference for object 2 over base object.
1 : Indifference
4: Weak preference for the base object over object 2.
16: Strict preference for the base object over object 2.

First improvement was that in the ratio of value r_{jk} on the geometric scale is expressed as an exponential function of the difference between the echelons of value on the geometric scale δ_{jk} as well as a scale of parameter y. Lootsma considers two alternative scales y to express preferences. For calculating the weight of criteria $y = \ln \sqrt{2} = 0.347$ is used. In Rembrandt only one hierarchical level (no matter how many criteria) is used, superior to the level of alternatives. For calculating the weight of alternatives on each criterion $y= \ln 2 = 0.693$ are used. The difference is echelons of value δ_{jk} is graded as in Table 1, which compares Saaty's ratio scale with Rembrandt scale.

The second suggested improvement is the calculation of impact scores. The arithmetic mean is subject to rank reversal of alternatives so we use geometric mean for calculation of relative value.

The third improvement proposed by Lootsma is aggregation scores. Rembrandt uses one hierarchical level with the alternative level subordinate to it. The lowest level is normalized multiplicatively so that the product of component equal to one for each of the k factors over

Table 1: AHP scale and corresponding Rembrandt scale

Verbal description	Saaty ratio	Rembrandt
Very strong preference for object k	1/9	-8
Strong preference for object k.	1/7	-6
Definite preference for object k	1/5	-4
Weak preference for object k.	1/3	-2
Indifference	1	0
Weak preference for object j.	3	+2
Definite preference for object j.	5	+4
Strong preference for object j.	7	+6
Very strong preference for object j.	9	+8

which the alternatives are compared. Therefore each alternative has an estimated relative performance w_k for each of the k factors. The components of the hierarchical level immediately superior to the lowest level are normalized additively, so that they add to one yielding weights O (j). The aggregation rule for each alternative j is

$$w_j = \sum_{i=1}^{k} w_i^{O(i)} \qquad (9)$$

ILLUSTRATIVE EXAMPLE

From illustrative point of view, an example has been cited for selecting Vendor via AHP and Rembrandt would take decision amongst the three possible vendors (Chaudhary *et al.*, 1991). The data considered in this example is a modification of the problem (Saaty, 1978) presented here in fuzzy form.

Constructing a vendor selection model: Consider a company that purchases blended gasoline from a network of 3 vendors. This brand of gasoline is composed of one or more of 3 blending constituents, each with a different octane rating, and there is a different vendor for each blending constituent. The blended gasoline is characterized by its overall octane blend.

We propose that the best Vendor can be evaluated by 6 criteria. They are C_1: Price, C_2: Service, C_3: Quality, C_4: Delivery, C_5: Technical Capability, C_6: Others, Fig. 2. represents the AHP hierarchy for the vendor selection problem. The hierarchy represents the various levels of the problem in terms of the overall goal, criteria, subcriteria and the decision alternatives. Once the hierarchy is constructed for the problem perform the pair-wise comparison of elements in one level relative to a single element in a level immediately above it to derive local priorities of these elements that reflect their relative contribution to the subject of comparison. Table 2-8 depict the various pair-wise comparison matrix between the various criteria's and alternatives with respect to various criteria. Table 2-8 weights (local priorities) derived from pairwise comparison are shown. Finally Table 9 gives the combination of priority weights of attributes, alternatives to determine the priority weights for the best Vendor.

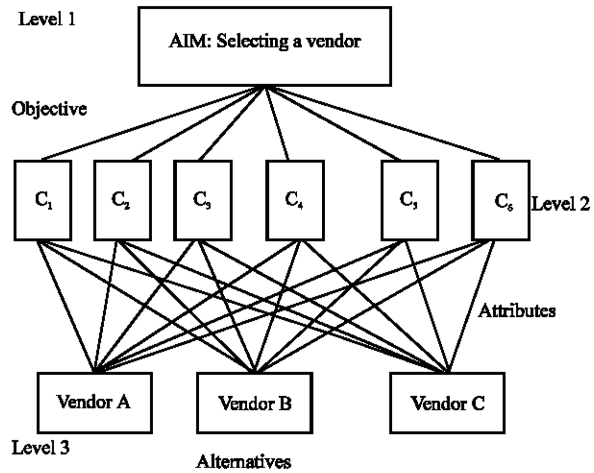

Fig. 2: Hierachy for vendor selection problem

Table 2: The fuzzy evaluation matrix representing pairwise comparison matrix w.r to the goal

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
C_1	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$	$\tilde{4}$	$\tilde{1}$	$\frac{1}{\tilde{2}}$
C_2	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$	$\tilde{4}$	$\tilde{1}$	$\frac{1}{\tilde{2}}$
C_3	$\tilde{1}$	$\frac{1}{\tilde{2}}$	$\tilde{1}$	$\tilde{5}$	$\tilde{3}$	$\frac{1}{\tilde{2}}$
C_4	$\frac{1}{\tilde{4}}$	$\frac{1}{\tilde{4}}$	$\frac{1}{\tilde{5}}$	$\tilde{1}$	$\frac{1}{\tilde{3}}$	$\frac{1}{\tilde{3}}$
C_5	$\tilde{1}$	$\tilde{1}$	$\frac{1}{\tilde{3}}$	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$
C_6	$\tilde{2}$	$\tilde{2}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$

The weights we get are W_G = (.20, .18, .24, 0, .15, .24)

Table 3: Evalution of the alternative A, B, C w. r to criteria C_1

C_1	Vendor A	Vendor B	Vendor C
Vendor A	$\tilde{1}$	$\frac{1}{\tilde{4}}$	$\frac{1}{\tilde{2}}$
Vendor B	$\tilde{4}$	$\tilde{1}$	$\tilde{3}$
Vendor C	$\tilde{2}$	$\frac{1}{\tilde{3}}$	$\tilde{1}$

W_{C1} = (.048, .67, .282)

Table 4: Evalution of the alternative A, B, C w. r to criteria C_2

C_2	Vendor A	Vendor B	Vendor C
Vendor A	$\tilde{1}$	$\frac{1}{\tilde{4}}$	$\frac{1}{\tilde{5}}$
Vendor B	$\tilde{4}$	$\tilde{1}$	$\frac{1}{\tilde{2}}$
Vendor C	$\tilde{5}$	$\tilde{2}$	$\tilde{1}$

W_{C2} = (.052, .196, .752)

Table 5: Evalution of the alternative A, B, C w. r to criteria C_3

C_3	Vendor A	Vendor B	Vendor C
Vendor A	$\tilde{1}$	$\tilde{3}$	$\frac{1}{\tilde{3}}$
Vendor B	$\frac{1}{\tilde{3}}$	$\tilde{1}$	$\tilde{1}$
Vendor C	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$

W_{C3} = (.449, .046, .504)

Table 6: Evalution of the alternative A, B, C w. r to criteria C_4

C_4	Vendor A	Vendor B	Vendor C
Vendor A	$\tilde{1}$	$\frac{1}{\tilde{3}}$	$\tilde{5}$
Vendor B	$\tilde{3}$	$\tilde{1}$	$\tilde{7}$
Vendor C	$\frac{1}{\tilde{5}}$	$\frac{1}{\tilde{7}}$	$\tilde{1}$

W_{C4} = (.299, .7,0)

Table 7: Evalution of the alternative A, B, C w. r to criteria C_5

C_5	Vendor A	Vendor B	Vendor C
Vendor A	$\tilde{1}$	$\tilde{1}$	$\tilde{7}$
Vendor B	$\tilde{1}$	$\tilde{1}$	$\tilde{7}$
Vendor C	$\frac{1}{\tilde{7}}$	$\frac{1}{\tilde{7}}$	$\tilde{1}$

W_{C5} = (.5, .5, 0)

Table 8: Evalution of the alternative A, B, C w. r to criteria C_6

C_6	Vendor A	Vendor B	Vendor C
Vendor A	$\tilde{1}$	$\tilde{7}$	$\tilde{9}$
Vendor B	$\frac{1}{\tilde{7}}$	$\tilde{1}$	$\tilde{7}$
Vendor C	$\frac{1}{\tilde{9}}$	$\frac{1}{\tilde{5}}$	$\tilde{1}$

W_{C6} = (.1,0,0)

REMBRANDT CALCULATION

We consider the same hierarchy and same data as used in AHP model. We first convert the respective data from Saaty's scale to Rembrandt scale. Table 10-17 show the pair-wise comparison matrices between the criteria, alternatives in terms of Rembrandt scale. The fuzzy matrix is defuzzified using Eq. 10 (Appendix) and transformed to a matrix of the form $e^{347\,\delta(jk)}$. Table 11-17 show the fuzzy matrices along with the defuzzified form. With the help of geometric mean weights are evaluated. Table 11-17 depict both the multiplicative and additive weights, respectively. Using Eq. 9 we aggregate the weights for various vendors A, B, C.

Table 9: Main Attributes of the goal

	C_1	C_2	C_3	C_4	C_5	C_6	Alternative priority weight
Weight	0.20	0.18	0.24	0	0.15	0.24	
Alternative							
Vendor A	0.048	0.052	0.449	0.299	0.5	1	0.443
Vendor B	0.67	0.196	0.046	0.700	0.5	0	0.2553
Vendor C	0.282	0.752	0.504	0	0	0	0.3127

Vendor A is the best

Table 10: A fuzzy judgment matrix represented in terms of Rembrandt scale $\bar{\delta}$ (jk)

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	0	0	+3	0	-1
C_2	0	0	1	+3	0	-1
C_3	0	-1	0	+4	+2	-1
C_4	-3	-3	-4	0	-2	-2
C_5	0	0	-2	+2	0	0
C_6	1	1	1	+2	0	0

Table 11: Deffuzification of fuzzy judgment matrix and value of $e^{347\,\delta(jk)}$

	C_1	C_2	C_3	C_4	C_5	C_6	Multiplicative	Additive
C_1	1.13	1.13	1.13	2.83	1.13	1.33	$\sqrt[6]{1.13*1.13*1.13*2.83*1.13*1.13}$ = 1.353	.163
C_2	1.13	1.13	1.59	2.83	1.13	1.33	1.432	.172
C_3	1.13	1.33	1.13	4	2	1.33	1.619	.195
C_4	1.15	1.15	1.09	1.13	1.24	1.24	1.165	.140
C_5	1.13	1.13	1.24	2	1.13	1.13	1.262	.152
C_6	1.59	1.59	1.59	2	1.13	1.13	1.474	.178

Table 12: Comparison of alternatives A, B, C w.r to criteria C_1 Defuzzification and value of $e^{693\,\delta(jk)}$ for C_1

C_1	A	B	C	C_1	A	B	C	Multiplicative	Additive
A	0	-3	-1	A	1.26	1.31	1.78	$\sqrt[3]{1.26*1.31*1.78}$ =1.432	.2186
B	+3	0	+2	B	8	1.26	4	3.43	.5233
C	1	-2	0	C	2.51	1.53	1.26	1.69	.646

Table 13: Comparison of alternatives, A, B, C w.r. to criteria C_2 Defuzzification and value of $e^{693\,\delta(jk)}$ for C_2

C_2	A	B	C	C_2	A	B	C	Multiplicative	Additive
A	0	-3	-4	A	1.26	1.31	1.2	1.256	.1658
B	+3	0	-1	B	8	1.26	1.78	2.618	.3457
C	+4	1	0	C	16	2.51	1.26	3.699	.4884

Table 14: Comparison of alternatives A, B, C w.t to criteria C_3 Defuzzification and value of $e^{693\,\delta(jk)}$ for C_3

C_3	A	B	C	C_3	A	B	C	Multiplicative	Additive
A	0	2	-2	A	1.26	4	1.53	1.98	.382
B	-2	0	0	B	1.53	1.26	1.26	1.34	.429
C	2	0	0	C	4	1.26	1.26	1.85	.270

Table 15: Comparison of alternatives A, B, C w.r to criteria C_4, Defuzzification and value of $e^{693\,\delta(jk)}$ for C_4

C_4	A	B	C	C_4	A	B	C	Multiplicative	Additive
A	0	-2	4	A	1.26	1.53	16	3.136	.280
B	2	0	6	B	4	1.26	64	6.86	.613
C	-4	-6	0	C	1.2	1.13	1.26	1.195	.107

Table 16: Comparison of alternatives A, B, C w.r to criteria C_5, Defuzzification and value of $e^{693\,\delta(jk)}$ for C_5

C_5	A	B	C	C_5	A	B	C	Multiplicative	Additive
A	0	0	6	A	1.26	1.26	64	4.67	.44
B	0	0	6	B	1.26	1.26	64	4.67	.44
C	-6	-6	0	C	1.13	1.13	1.26	1.17	.112

Table 17: Comparison of alternatives A, B,C w.r to criteria C_6, Defuzzification and value of $e^{693\,\delta(jk)}$ for C_6

C_6	A	B	C	C_6	A	B	C	Multiplicative	Additive\
A	0	+6	+8	A	1.26	64	256	27.43	.872
B	-6	0	4	B	1.13	1.26	16	2.83	.090
C	-8	-4	0	C	1.1	1.2	1.26	1.18	.038

Aggregation: Vendor A: $(1.432)^{.163} * (1.26)^{.172} * (1.98)^{.195} * (3.14)^{.140} * (4.67)^{.152} * (27.43)^{.178} = 3.372$, Vendor B: $(3.43)^{.163} * (2.62)^{.172} * (1.34)^{.195} * (6.86)^{.140} * (4.67)^{.152} * (2.83)^{.178} = 3.042$, Vendor C: $(1.69)^{.163} * (3.69)^{.172} * (1.85)^{.195} * (1.19)^{.140} * (1.17)^{.152} * (1.18)^{.178} = 1.66$

Table 18: Comparison between Fuzzy Rembrandt and Fuzzy AHP

Vendors	Fuzzy rembrandt	Fuzzy AHP
A	0.4176(I)	0.443(I)
B	0.3767(II)	0.2553(III)
C	0.2055(III)	0.3127(II)

Table 19: Comparison between Crisp Rembrandt and Crisp AHP using additive mean

Vendors	Crisp rembrandt	Crisp AHP
A	0.289(II)	0.378(I)
B	0.324(I)	0.361(II)
C	0.169(III)	0.289(III)

Table 20: Comparison between Crisp Rembrandt and Crisp AHPUsing Geometric mean

Vendors	Crisp rembrandt	Crisp AHP
A	0.377(II)	0.377(II)
B	0.515(I)	0.383(I)
C	0.106(III)	0.250(III)

This study compares the use of fuzzy Rembrandt and fuzzy AHP (Table 18) using same input data for both the techniques. We observe that by Fuzzy Rembrandt and Fuzzy AHP Vendor A is a common choice between the 2 methods. From Table 19 where a comparison is made between Rembrandt and AHP dealing with a non-fuzzy case and using the additive mean method. There is a total difference in opinion regarding the best-rated vendor since none of the participants indicate any desire to change their personal rating when presented evidences about these differences. For instance Vendor B was ranked as the best by Crisp Rembrandt method and Vendor A was ranked as best by Crisp AHP. Only Vendor C was a common choice between the two methods. Differences of opinion such as this are features of group interaction, which need to be considered when selecting a method of support. Coming to the geometric mean value case (Table 20) where there is no difference of opinion between the two a method indicating that geometric mean aggregation rule avoids rank reversal. But we are not interested in a deterministic problem and the optimal results of these deterministic formulations may not serve the real purpose of modeling the problem. Due to this we have considered the model as a fuzzy model, which deals with real-life situation where many input information related to the various vendors are not known with certainty.

Our aim is to find an analytical tool to select the best vendor and it is immaterial about the second and third choice here. Moreover there is a contradiction between the second and third position of the vendors (Table 18). It is better to avoid the contradiction and proceed with Vendor A only for allocation of order.

APPENDIX

Triangular Fuzzy Numbers (TFN) their corresponding Membership Function (MF)

TFN	MF
$\tilde{0}$	(0,0,1)
$\tilde{1}$	(1,1,2)
$\tilde{2}$	(1,2,3)
$\tilde{3}$	(2,3,4)
$\tilde{4}$	(3,4,5)
$\tilde{5}$	(4,5,6)
$\tilde{6}$	(5,6,7)
$\tilde{7}$	(6,7,8)
$\tilde{8}$	(7,8,9)
$\tilde{9}$	(8,9,9)

Fuzzy terminology: Fuzzy sets are generalization of conventional set theory that was introduced by Zadeh 1965 as a mathematical way to represent vagueness in everyday life.

Let U be the universe of discourse $U=\{u_1, u_2,\ldots,u_n\}$. A fuzzy set $\tilde{A}$ of U is a set of ordered pairs

$$\{(u_1, f_{\tilde{A}}(u_1)), (u_2, f_{\tilde{A}}(u_2)),\ldots\ldots,(u_n, f_{\tilde{A}}(u_n))\}$$

Where $f_{\tilde{A}}$, $f_{\tilde{A}}$; $U->[0, 1]$, is the membership of $\tilde{A}$ and $f_{\tilde{A}}(u_i)$ indicates the grade of membership of u_i in $\tilde{A}$.

Definition 1: A fuzzy set $\tilde{A}$ of the universe of discourse U is convex if and only if for all μ_1, μ_2 in U, $f_{\tilde{A}}[\lambda u_1 + (1-\lambda) u_2] \geq \min[f_{\tilde{A}}(u_1), f_{\tilde{A}}(u_2)]$, where $\lambda \in [0, 1]$.

Alternatively, a fuzzy set is convex if all α-level sets are convex.

Definition 2: A fuzzy set $\tilde{A}$ of the universe of discourse U is called a normal fuzzy set implying that

$$\exists u_i \in U,\ f_A(u_i) = 1$$

Definition 3: A fuzzy number is a fuzzy subset in the universe of discourse U that is both convex and normal.

Definition 4: According to Kaufmann and Gupta (1991), a fuzzy number $\tilde{A}$ of the universe of discourse U may be characterized by a triangular distribution function parameterized by a triplet (a, b, c) shown in Fig. 3. The membership function of the fuzzy number $\tilde{A}$ is defined as

$$f_{\tilde{A}}(u) = \begin{cases} 0, & u < a \\ \dfrac{u-a}{b-a}, & a \leq u \leq b, \\ \dfrac{c-u}{c-b}, & b \leq u \leq c, \\ 0, & u > c. \end{cases}$$

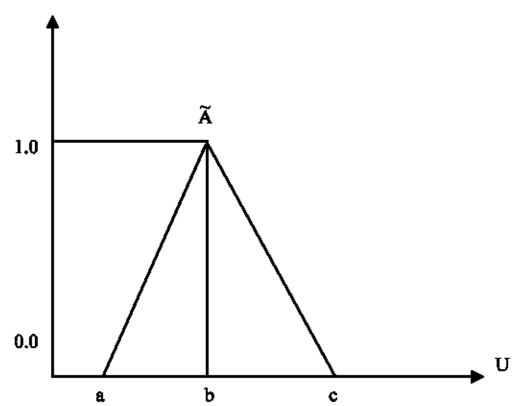

Fig. 3: A triangular fuzzy number

Let $\tilde{A}$ and $\tilde{B}$ be Two Fuzzy Numbers (TFN) parameterized by the triplet say (a_1, a_2, a_3) and (b_1, b_2, b_3), respectively.

Then the operations of fuzzy numbers are expressed as:

$$\tilde{A}\,(+)\,\tilde{B} = (a_1, a_2, a_3)(+)(b_1, b_2, b_3) = (a_1+b_1, a_2+b_2, a_3+b_3),$$
$$\tilde{A}\,(-)\,\tilde{B} = (a_1, a_2, a_3)(-)(b_1, b_2, b_3) = (a_1-b_3, a_2-b_2, a_3-b_1),$$
$$\tilde{A}\,(\times)\,\tilde{B} = (a_1, a_2, a_3)(\times)(b_1, b_2, b_3) = (a_1b_1, a_2b_2, a_3b_3),$$
$$\tilde{A}\,(\div)\,\tilde{B} = (a_1, a_2, a_3)(\div)(b_1, b_2, b_3) = (a_1/b_3, a_2/b_2, a_3/b_1).$$

Defuzzification of a Trapezoidal fuzzy number: Consider a trapezoidal fuzzy number (Kaufmann and Gupta, 1991) parameterized by quadruple (a, b, c, d), where e is defuzzified value of the fuzzy number

$$e = \frac{a+b+c+d}{4} \qquad (10)$$

Defuzzification of a TFN parameterized by a triplet (a, b, c) is equal to

$$e = \frac{a+2\times b+c}{4}$$

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