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Static and Dynamic Response of a Pile Foundation Subjected to a Vertical Load

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Abstract: In this study a discrete semi analytical method based on a combination of the Finite Elements, Thin Layers and Integral Equations Methods is presented to evaluate the vertical static and dynamic linear response of a pile foundation subjected to a vertical load. This study, which takes into account the soil-pile interaction effects, is able to treat the case of an arbitrary shaped pile. However, the presentation will be limited to the case of a massless square pile with linear elastic behaviour. The pile is embedded in a viscoelastic soil-layer overlaying a bedrock and subjected to static or/and harmonic vertical load, applied at the head of the pile. The aim of this study is to characterise the response of a pile through the force-displacement relationship. Numerical results are presented to illustrate the accuracy of this study.

Key words: Soil, pile, vibration, finite elements, boundary elements, displacement

INTRODUCTION

Piles are frequently adopted as foundations for important structures, such as high buildings, nuclear power plants, bridges, offshore structures or machine foundations, etc. where the study of static and/or dynamic response of these foundations, constitutes a very important element in their conception and the security of structures that they support.

During the last decades, static and dynamic response of piles was the topic of several investigations. Although considerable efforts have been conducted to develop adequate analytical methods, conceptual and computational difficulties still remain essentially due to the three-dimensional nature and semi infinity of soil, as well as the embedding of the pile foundation.

Many researchers proposed approximate solutions using the concept of Winkler foundation. In this study the soil around the pile is replaced by a set of springs and dashpots, where spring constants are obtained from analytical considerations or from experimental data. Such analysis have been proposed by Penzien *et al.*[1], Matlock[2], Novak[3,4] and others.

Finite Elements method was also widely used to evaluate pile response (Kuhlmeyer[5,6], Krishnanand al[7], Guoxi Wu *et al.*[8]). The advantage of Finite Elements method resides in its easy adaptation to problems of complex geometry and strong heterogeneities, but to take into account the infinite nature of soil, appropriate boundaries may be used to eliminate waves reflections toward the inside of the model.

More recently a formulation in Boundary Elements has been used by Kynia and Kausel[9], Sen *et al.*[10] and others, who obtained solutions in displacements of a ring of loads inside a multilayered soil. This method is better adapted to problems of dynamic soil-structure interaction because it is based on analytical solutions satisfying the conditions of radiation at the infinity. However it is hardly adaptable to the non-linear, strong heterogeneities and complex geometry problems.

Rajapakse *et al.*[11-13] proposed a different modeling based on the decomposition of the soil-pile system in continuous soil without excavation and a fictional pile for which Young modulus and mass density resulting from the differences between those of the real pile and those of soil.

In order to be able to take profit of the advantages of the Finite Elements Method and the Boundary Elements Method and to eliminate their individual inconveniences, an alternative approach which consists in the combination of the last two methods has been developed (Messast *et al.*[14], Mendoça[15], Matos Filho[16], Sami Benjama[17], Mansouri[18] and Caro[19]).

In this study, a combined formulation Finite Elements-Boundary Elements is presented to analyze the soil-pile interaction under different static or dynamic load

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combinations (harmonic vibrations). In this formulation, the principle of decomposition of Rajapakse[12] is used.
The system can be decomposed into two fictional elements. The first is a fictional pile that will be modeled by Finite Elements Method as one-dimensional element discretized in beam elements, while the second is the soil modeled by the Boundary Elements Method (BEM) combined with Thin Layers Method as an elastic, continuous, linear, isotropic and homogeneous medium. The flexibility matrix of soil is formulated according to the formalism proposed by E. Kausal[20]. The conditions of compatibility are imposed any where in the volume of the soil (beam of soil) relative to the pile.

This method is general. It permits to treat any shape of pile embedded in any type of soil under different vertical static or dynamic load combinations. To make include the two cases (static and dynamic) in the same procedure, the vibratory case is directory considered and the static case corresponds to a zero circular frequency.

ANALYSIS METHOD

Figure 1 represents a pile foundation embedded in viscoelastic soil and subjected to a vertical load. The whole system consists of two different mediums: pile and soil. The pile is considered as an one-dimensional element of length l and square section of side B with l >> B. Pile's material is considered linear elastic characterized by Young modulus E_p, density ρ_p and Poisson's ratio v_p. The pile is embedded in a homogeneous viscoelastic soil limited in depth by a deep enough substratum in such a way that the soil will be consider as a semi infinite medium. The soil is modelled as an ideal continuous medium characterized by densityρ_s, Poisson ratio v_s, shear modulus $G_s = E_s/2\,(1 + v_s)$ and damping ratio β. To take into account the effect of material damping, shear modulus of soil is replaced by its complex modulus $G = G_0 e^{i\omega t}$. The harmonic load will be also expressed under complex form ($P = P_0 e^{i\omega t}$) with $i=\sqrt{-1}$ and ω is the vibration frequency. In the following the factor $e^{i\omega t}$ will be omitted for simplicity.

The equilibrium equation of the pile can be expressed in general form as:

$$\{U\} = [F].\{p\} \tag{1}$$

Where U is the displacement vector of the pile, F is the flexibility matrix of the pile which takes into account the contribution of soil in the response of pile and P is the external load vector.

The whole system soil-pile can be decomposed into two fictional mediums, where the total response can be expressed as the superposition of their responses. The

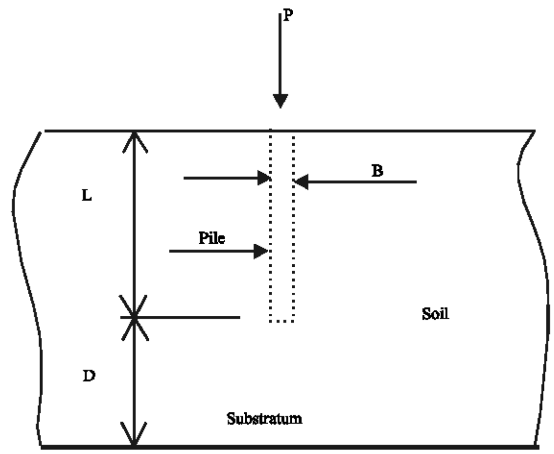

Fig. 1: Pile-soil system

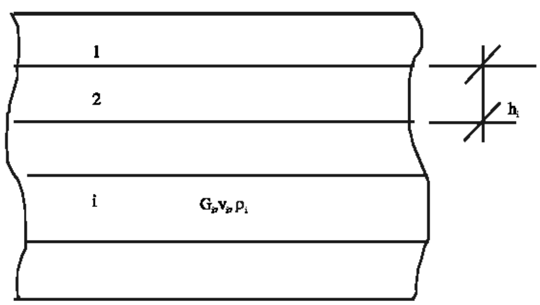

Fig. 2: Vertical discretization of soil

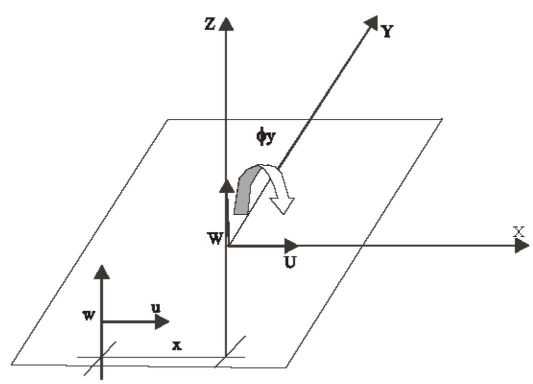

Fig. 3: Rigid body displacements

first modelled as a beam of soil represents the soil equivalent to the pile. The second is a fictional pile for which $E = E_p - E_s$ and $\rho = \rho_p - \rho_s$. Each of the fictional elements equilibrates a fictional load, which represents a part of the applied exterior load.

EQUILIBRIUM EQUATION OF SOIL BEAM

In the following, U_1 and P_1 designate respectively displacements and loads vectors at interfaces nods while

u_t and p_t refer, respectively to displacements and loads vectors at disks nods constituting the soil-beam.

The equilibrium equation of soil-beam can be written as:

$$u_t = F_t \cdot p_t \tag{2}$$

F_t refer to the flexibility matrix of soil which can obtained by the formalism proposed by Kausel and Peek[20]. According to this formalism, the soil will be divided into underlayers with small thicknesses as compared to the wavelengths of the interest (Fig. 2).

The rigid body condition imposed anywhere to the soil beam may be expressed as (Fig. 3):

$$U_d^i = R_d^i U_s^n$$

where:

$$R_d^i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -x_i \end{bmatrix} \tag{3}$$

$U^n_s = \{U\ W\ \Phi_y\}^t$ where U, W and Φ_y are the three degrees of freedom of sections of the soil beam.

$U^i_d = \{u\ w\}^t$ where u and w are the two degrees of freedom of a disk.

After assembling matrices, it can be written that:

$$u_t = RU_1$$

and

$$P_1 = R^t p_t \tag{4}$$

The total equilibrium equation of the soil beam can be written as:

$$u_t = F_t p_t \tag{5}$$

From Eq. (4) and (5) it can be written:

$$R^t F_t^{-1} R U_1 = P_1 \tag{6}$$

If: $$K_1 = F_t^{-1} R^t R$$

equation (6) becomes :

$$K_1 U_1 = P_1 \tag{7}$$

MOVEMENT OF FICTIONAL PILE

The fictional pile is modelled by Finite Elements Method, then divided into beam elements which have the same number of underlayers. its vibrations are considered to be in a vertical plane. The movement equation of the fictional pile can be expressed as:

$$K_2 U_2 - \omega^2 M_2 U_2 = P_2 \tag{8}$$

Where K_2 and M_2 are, respectively the stiffness and masses matrix of the fictional pile obtained using standard procedures of the structural analysis.

P_2 is the part of the external load supported by the fictional pile. U_2 is the displacement of the fictional pile and ω is the circular frequency.

COUPLING OF FEM AND BEM METHOS

The coupling of the FEM and BEM methods can be accomplished through imposing the compatibility condition between the beam of soil and the fictional pile. This condition can be expressed as:

$U = U_1 = U_2$ and

$$P = P_1 + P_2 \tag{9}$$

Putting $M = M_2$ and $K = K_1 = K_2$ where K is the stiffness matrix of the real pile.

From Eq. (7) and (8) it can be written:

$$(K - \omega^2 M)U = P \tag{10}$$

Where P and U are respectively external load vector and displacement vector of pile.

If $K_{tot} = K - \omega^2 M \Rightarrow K_{tot} U = P$, the vector displacement can be given by the following equation:

$$U = K_{tot}^{-1} P \tag{11}$$

From equation (11), displacements of pile can be obtained for different combinations of static or dynamic loads.

NUMERICAL RESULTS

In order to compare this study in which the pile have a square form and others works cited in the literature with circular piles, it is necessary to establish the dimensional

equivalence between these two shapes. The equivalence between the side of a square cross section and the radius of a circle, is given by the following expression[18,21].
For translator modes:

$$B = \left(\sqrt{p}\right) r_0$$

For rocking modes:

$$B = \left(\sqrt[4]{3p}\right) r_0$$

r_0 refer to the radius of the circular pile and B the side of square cross section.

The corresponding non dimensional parameters are :
Non dimensional frequency: $a = \omega r_0 / c_5$

Where ω is the circular frequency of the excitation, c_s is the propagation speed of the shearing waves in soil.
Relative length: $l_r = L/r_0$

The others parameters remain unaltered.

Relative stiffness: $e_r = E_p / E_s$, relative mass: $m_r = \rho_p/\rho_s$, relative depth: $h_r = D/B$.

Let's define respectively the vertical and the horizontal discratization ratio as: $n_s = h_i/B$ and $n_d = 2_r/B$, where h_i refers to the thickness of an underlayer i and r is the radius of equivalent disks.

The optimal discretization ratios can be given by the following expressions[18]:
Horizontal modes: $n_s = 1.5$, $n_d = 0.5$
Vertical modes: $n_s = 1.5$, $n_d = 0.333$

Static case: In order to assess the accuracy of this method, several comparisons with other theoretical methods and experimental measurements are carried out.

The first example from R. Matos Filho *et al.*[16] consisting of a pile, 12.2m in depth and 0.61m in diameter, subjected to a vertical load of 1100kN. The Young's modulus was $2.067x10^7$kN/m^2 for the pile and 72400kN/m^2 for the soil, whose Poisson ratio was 0.5. This pile was also tested by Whitker and Cooke[22]. The vertical displacement measured at the head of the pile was 0.284cm and the one calculated by the method proposed by R. Matos Filho et al was 0.2867cm. The presented method gives a displacement of 0.285 cm, which is in good agreement with the previous results.

The second example is the one treated by Vallabhan and Sivakumar[22] and by R. Matos Filho *et al.*[16], consisting of a pile, 6.096m in depth and 0.6096m in diameter submitted to a vertical load of 726.40kN, The Young's modulus of the pile was Ep = 21111000kN/m^2 and the one of soil defined by $E_p/E_s = 100$ and the Poisson ration of the soil is 0.2. Figure 4 shows the variation of the

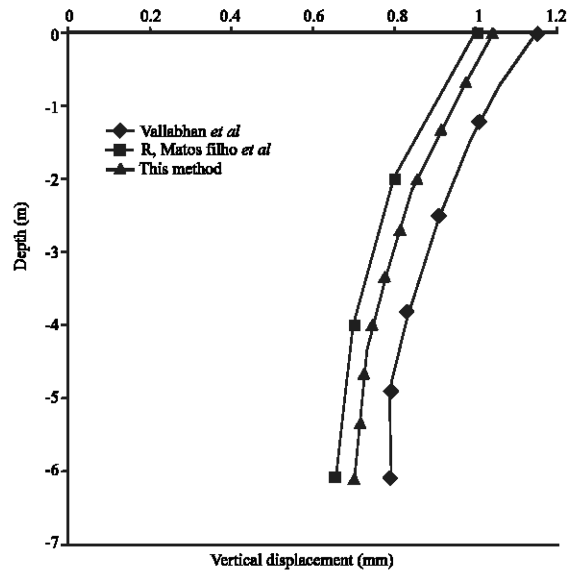

Fig. 4: Vertical displacements along pile

Table 1: Comparison of the vertical impedance normalized by its corresponding static impedance

	Real part			Imaginary part		
a	0.1	0.2	0.3	0.1	0.2	0.3
P. M*	1.11	1.22	1.25	0.41	0.61	0.81
[12]	1.13	1.26	1.36	0.51	0.59	0.8
[6]	1.07	1.19	1.15	0.41	0.58	0.81

* Present method

vertical displacement , there is good agreement between the present model and those of Vallabhan and Sivakumar[22] and by R. Matos Filho *et al.*[16].

Dynamic case: Table 1 presents a comparison of the vertical impedance normalized by its corresponding static impedance given by the present approach with those given by Rajapakse and al[11] and Kuhlmeyer[6], for no dimensional frequencies a = 0.1, 0.2, 0.3,

Other details:

$$\frac{E_p}{E_s} = 100 \quad \frac{\rho_p}{\rho_s} = 1 \quad \frac{L}{B} = 30 \quad \nu_s = 0.25 \quad \beta = 0.0$$

Table 1 shows a good agreement between results of the present method in this study and in [18] and those of Rajapakse *et al*[11] and Kuhlmeyer[6].

CONCLUSION

In this study, a method combined FEM-BEM has been presented for the numerical analysis of pile-soil interaction. This study allows taking into account the infinite aspect of the massif of soil and therefore to avoid the limitation of the geometric model, that would impose to choose adequate boundaries to the limits of the model and would not exclude any parasitic reflections. The pile-soil system is decomposed into two fictional mediums. A beam of soil representing the soil equivalent to the pile and a fictional pile whose mechanical features result from the difference between those of the real pile and those of soil. The flexibility matrix of soil is determined according to the formalism presented in the study of E. Kausel (1982). The pile is modeled by one-dimensional elements in elements beams. The response of the real pile is given after coupling the equilibrium equations of the fictional pile and the beam of soil.

The presented method has a general aspect that permits to treat the different types of shapes of piles subjected to different static or dynamic loads.

This method has been used to obtain the response of single pile subjected to vertical loads. Several cases of the numerical simulations have been included, which confirm, by the agreement of results with those expected, the formulation and treatment proposed in this study are adequate for these types of problems.

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