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Control Law Based on Backstepping Design for Controlling Lorenz Chaotic System

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Abstract: A simple control law based on the theory of backstepping is proposed to control and to track a Lorenz chaotic system to any desired trajectory. The backstepping design is a step-by-step approach and consists of a recursive procedure, interlacing the choice of a Lyapunov function with the design of a virtual control at each step, at the last step, the true control is obtained. Strong properties of global and asymptotic stability can be achieved. A major advantage of this method is that, it has the flexibility to build the control law by avoiding cancellations of useful nonlinearities.

Key words: Law, backstepping, controlling lorenze

INTRODUCTION

Since the early 1990's, the problems of control of chaos attract attention of the researchers and engineers. Numerous publications have appeared over the recent decade. It seems that T. Li and J.A.Yorke were the first authors, who in 1975 introduced the term chaos or, more precisely, deterministic chaos[1], wich is used widely since then. Various mathematical definitions of chaos are known, but all of them express close characteristics of the dynamic systems that are concerned with supersensitivity to the initial conditions[1]. Recently, great attention has been given to chaos and control, Many researchers had proposed methods to control chaos, (Ott),(Ott and All), used the backstepping as a new framework for nonlinear control design, which is a systematic design approach for constructing both feedback control laws and associated an adequate choice of the Lyapunov 's functions, permitting to guarantee the stability of the system[1].

In[2-6] we found, several nonlinear controllers based on the theory of backstepping were designed and applied to different systems such as[2-4] the 3rd order phase-locked loops, collpits oscilator for controlling the undesirable unstable behavior and pulling the PLL back to the in-lock state, in[5] a backstepping design is proposed as a technique for controlling Lorenz chaos but in it 's thermal convection model (which is obtained from the Lorenz system by substituting for x_3-r, under the assumption that r = const, $\beta = 1$, for u = 0 and 0〈r〈1. Based on recursive application of Lyapunov's direct method, the design enables to drive the chaotic motion towards any desired trajectory. Jinhu[6] described applied the linear feedback techniques (backstepping), to control chaos in Lu system, the effective observers are provided to identify the unknown parameters of Lu system and then the simple feedback functions are designed for controlling Lu system. Also, the proposed method can enable the controlled Lu system to approach any desired points or periodic orbits.

Control law based on backstepping design: Backstepping is a method based on linearization by feedback, eliminating all the non linearity of the system, it gives more flexibility to the designer and robustness is obtained[1,7-9].

Let us consider a Lorenz chaotic system under its parametric form:

$$y = f(x),\ x \in R^n \tag{1}$$

With the control parameter values:

$$\sigma = 10,\ r = 28,\ \beta = 3 \tag{3}$$

$$\begin{aligned} \dot{x}_1 &= -\sigma * (x_1 - x_2) \\ \dot{x}_2 &= -x_1 * x_3 + r * x_1 - x_2 \\ \dot{x}_3 &= x_1 * x_2 - \beta * x_3 + \mu \\ y &= x_1 \end{aligned} \tag{2}$$

The system presents a chaotic behavior and has a three unstable equilibirium points.

$$\begin{aligned} x_{eq0} &= (0, 0, k_1) \\ x_{eqn} &= (-k_2, -k_2, k_1) \\ x_{eqp} &= (k_2, k_2, k_1) \\ k_1 &= r - 1, k_2 = \sqrt{\beta * (r-1)} \end{aligned} \tag{4}$$

For obtaining the 'strict feedback' form, we translate the origin of the system in Eq. 2 to the set point x_{eqp}, we obtain the following system:

$$\begin{aligned} \dot{x}_1 &= 10*(x_1 - x_2) \\ \dot{x}_2 &= (x_1 - x_2) - x_3*(9 - x_3) \\ \dot{x}_3 &= 9*(x_1 - x_2) - 3*x_3 + x_2 + u \end{aligned} \quad (5)$$

Where u = $-\sqrt{b*(r-1)} + \mu$ (6)

The objective of the control law is to have the out put y(t)→ 0 when t→ ∞ (7).

We treats the system as three cascaded subsystems, each one of them with only one input and only one output and during the process a change of variable is done:

$$z_i = \phi(x_i) \quad (8)$$

First step, Let us choice:

$$z_1 = x_1 \quad (9)$$

$$\dot{z}_1 = \dot{x}_1 = -10x_1 + 10x_2 \quad (10)$$

The virtual control is:

$$x_2 = z_2 + \alpha_1(x_1) \quad (11)$$

α_1 (x_1) is the stabilizing function for the first subsystem, z_2 is the new variable.

α_1 (x_1) is chosen such as the Lyapunov function V_1 for the first subsystem is

$$V_1 = 1/2\ z_1^2 \quad (12)$$

and its derivative in- time:

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 = z_1(-10x_1 + 10x_2) \\ &= -10x_1^2 + 10x_1x_2 \end{aligned} \quad (13)$$

becomes definite negative.

$$\text{So } \alpha_1(x_1) = 0, z_2 = x_2 \quad (14)$$

and

$$\dot{x}_1 = -10x_1 + 10z_2 \quad (15)$$

Second step,
We are begining with:

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 = (x_1 - x_2) - (9 + x_1)x_3 \\ &= (x_1 - z_2) - (9 + x_1)x_3 (16) \end{aligned} \quad (16)$$

The virtual control is choosen as:

$$x_3 = z_3 + \alpha_2(x_1, x_2) \quad (17)$$

α_2 (x_1, x_2) is the stabilizing function for the second subsystem, z_3 is the new variable.
α_2 (x_1, x_2) is chosen such as the Lyapunov function V_2 for the subsystem 2 is,

$$V_2 = V_1 + 1/2z_2^2 \quad (18)$$

And its derivative in time is:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 \\ &= -10x_1^2 + z_2[10x_1 + x_1 - z_2 - (9 + x_1)x_3] \end{aligned} \quad (19)$$

Becomes definite negative.

$$\text{So: } \alpha_2\ (x_1, x_2) = 11x_1/9+x_1) \quad (20)$$

And

$$\dot{V}_2 = -10x_1^2 - z_2^2 \quad (21)$$

Third step,

$$\begin{aligned} z_3 &= x_3 - \alpha_2(x_1, x_2) \\ \dot{z}_3 &= \dot{x}_3 - \dot{\alpha}_2(x_1, x_2) \\ &= [9(x_1 + x_2) - 3x_3 + x_1x_2 + u]\frac{99(-10x_1 + 10x_2)}{(9 + x_1)^2} \end{aligned} \quad (22)$$

The virtual controlu is choosen such as the Lyapunov function V_3 for the subsystem 3 is,

$$V_3 = V_2 + 1/2z_3^2 \quad (23)$$

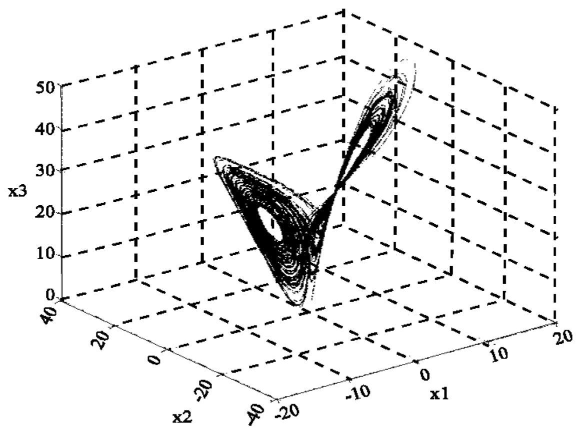
So its derivative in time is:

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + z_3 \dot{z}_3 \\ &= -10x_1^2 - z_2^2 + z_3\begin{bmatrix} 9x_1 - 3z_3 - 3\dfrac{11x_1}{9 + x_1} - u \\ -99\dfrac{(-10x_1 + 10x_2)}{(9 + x_1)^2} + z(9 + x_1) \end{bmatrix} \end{aligned} \quad (24)$$

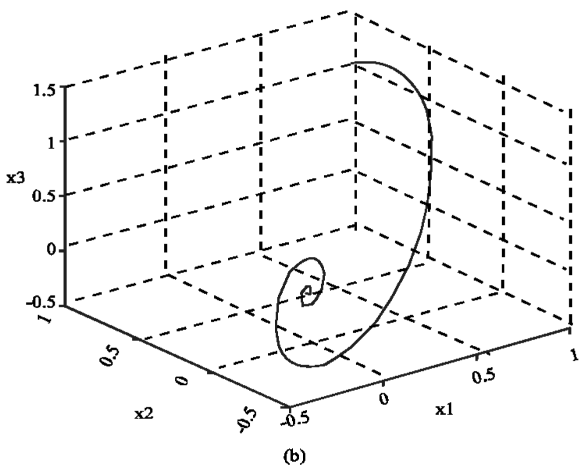
becomes definite negative, if u is chosen as:

$$u = -9x_1 + \frac{33x_1}{(9+x_1)} + 99\frac{(-10x_1 + 10x_2)}{(9+x_1)^2} \quad (25)$$

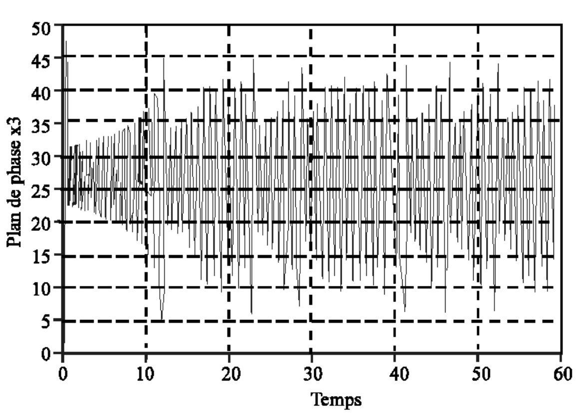
u represents the control law for stabilizing the system Eq. 5in the origin.

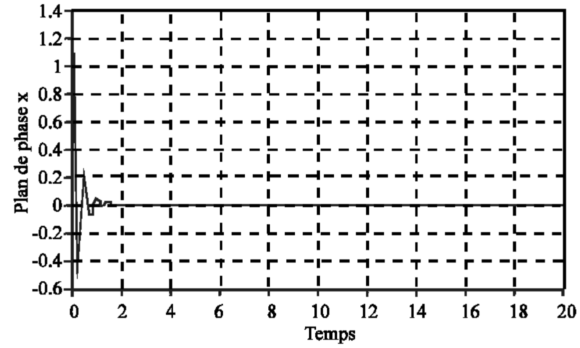


(a) Chaotic behavior origin

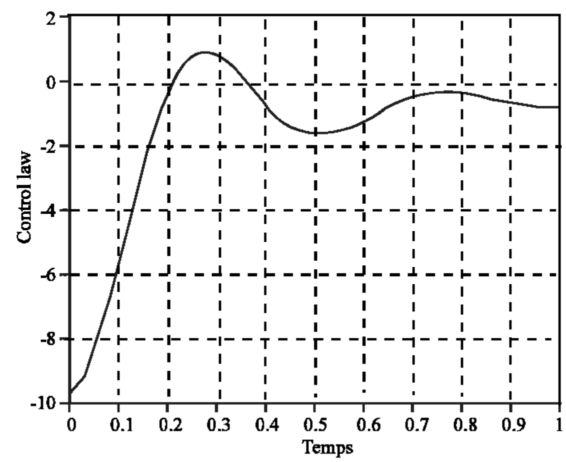


(b) Stabilizing system to the in space wave





Chaotic behavior and stabilizing system in time wave



Variation of control law in time

Tracking of system based on backstepping design: In the tracking problem of non linear system, the objective of the control is not only to stabilize the system globally, but also to force its output to track any desired trajectory.

We have following the same steps on before, excepted, one obtains the error between the output of the second subsystem and the reference signal:

$$x_2 = z_2 - y_r \quad (27)$$

with the Lyapunov function:

$$V_2 = 1/2z_2^2 \quad (28)$$

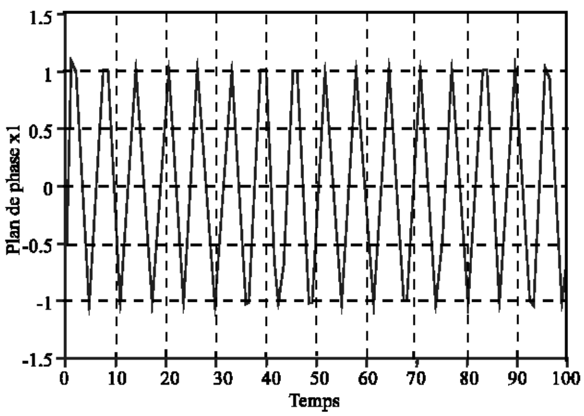
The choice of α_2 for V_2 Becomes definite negative is:

$$x_3 = \alpha_2 = \frac{\left(x_1 - y_r - \dot{y}_r\right)}{(9+x_1)} \quad (29)$$

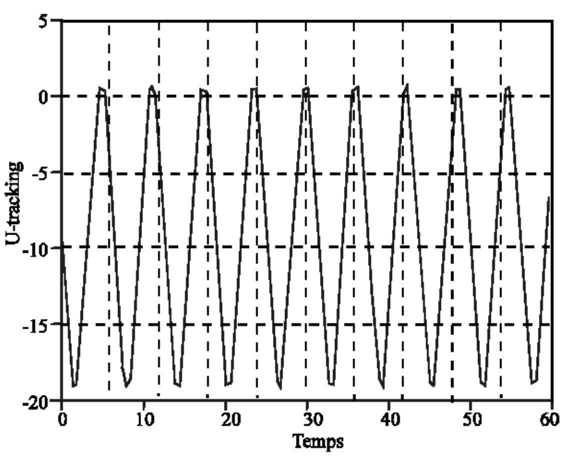
And the tracking law is given as:

$$u = -\frac{(-10x_1 + 10x_2)\left(9 - y_r - \dot{y}_r\right)}{(9 + x_1)^2} - \frac{\left(3x_1 - 3y_r - 2y_r - \ddot{y}_r\right)}{(9 + x_1)}$$
$$-y_r(9 + x_1) - 9x_1 \tag{30}$$

And then 1 x tracks any desired reference signal y_r.



Time wave of tracking $x_1(t)$ for sin (t)



Variation of control law of tracking system in time

CONCLUSION

In this study, we showed that Lorenz's chaotic system can be transformed into a class of nonlinear systems in the so called non-autonomous "strict-feedback" form, then a backstepping process has been used to drive the output to asymptotically equilibrium point and to track it to any arbitrarily given reference signal, strong properties of global stability and asymptotic tracking have been achieved in a finite number of steps, however it has certain drawback one of them is that for high order systems, the nonlinear expression of the controller becomes increasingly complex.

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