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# On the Existence and Uniqueness of a Problem Arising from the Theory of Combustion

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**Abstract:** We consider the species and energy equation and we impose some initial and boundary conditions on the equations. It is of great important to establish that a new combustion problem has a unique solution.We re-afirm that the solution exist and it has physical implications.The existence and uniqueness theorem stated can be used for other similar problems in fluid dynamics.

**Key words:** Existence, uniqueness, problem arising, theory of combustion

## INTRODUCTION

The interest of scientists after modeling a problem or designing any model, are the questions: Does a solution exist? How many solutions does the problem have? Is the solution unique? The uniqueness of solution is very important in modeling physical problem or designing a model.

Okoya (1989) gave some conditions for existence and uniqueness of some particular problems. He also gave criteria for multi-solution of another problem.

Olanrewaju (2002, 2005) determines some criteria for a design or model to have only one solution and the effect of certain parameters on these solutions.

Njoku (2000) examined an existence and uniqueness result for a system of non-linear volterra equations and established forward modifications to Urysohn integral operators. In this study, we consider a particular problem from the thesis.

Olanrewaju (2002) and impose conditions so that we have only one solution.

## MATHEMATICAL FORMULATION

We consider the species and energy equations,

$$\rho\left(\frac{\partial y}{\partial t}+\frac{u\partial y}{\partial x}\right)=\frac{D\partial^2 y}{\partial x^2}-Aye^{-E/RT} \quad (1)$$

$$\rho\left(\frac{\partial T}{\partial t}+\frac{u\partial T}{\partial x}\right)=\frac{k\partial^2 T}{\partial x^2}+qAye^{-E/RT} \quad (2)$$

Where a =1 (i.e the order of reaction is 1) Let the initial and boundary conditions be y (x,0) = 0.

$y(L,t) = y_0, t > 0$
$y(L,t) = 0$ or $y(L,0)=0$
$T(x,0) = T_0$
$T(0,t) = T_0, t > 0$
$T(L,t) = T_1, t > 0, T_1 > T_0$
Let $\rho u = a_1$ (constant t)

And

Where
y = Pre-mixed reactants
ρ = Density of the reactants
c = Specific heat
T = Temperature
q = Heat released/unit mass
D = Diffusion coefficient
k = Thermal conductivity
R = Universal gas constant
t = Time
E = Activation energy

Steady case of (1) and (2) becomes

$$u\frac{\partial y}{\partial x}=D\frac{\partial y}{\partial x^2}-Aye^{-E/RT} \quad (3)$$

$$\rho cu\frac{\partial T}{\partial x}=\frac{k\partial^2 T}{\partial x^2}+qAye^{-E/RT} \quad (4)$$

We can write (3) and (4) as

$$qa_1\frac{\partial y}{\partial x}=qD\frac{\partial y}{\partial x^2}-qAye^{-E/RT}e^{-E/RT_0}e^{-E/RT_0} \quad (5)$$

$$a_1c\frac{\partial T}{\partial x}=k\frac{\partial^2 T}{\partial x^2}-qAye^{-E/RT}e^{-E/RT_0}e^{-E/RT_0} \quad (6)$$

Let

$$\varepsilon=\frac{RT_0}{E}$$

Then

$$T = \theta\varepsilon\, T_o + T_o \text{ or } T = T_o\,(\theta\varepsilon + 1)$$

Non-dimensionalizing process

Let

$$y' = \frac{y}{y_0}, \quad u' = \frac{u}{\cup}, \quad x' = \frac{x}{L}, \quad t = \frac{t\cup}{L}$$

Where

$y_0$ = References length along y-axis
U = Characteristic velocity component along x-axis
L = References length along x-axis
t = Time

Non-dimensionalizing (5), (6) and after dropping the "we have

$$\frac{\partial\theta}{\partial x} = \frac{\partial^2\theta}{\partial x^2} + \left[\frac{\frac{1}{2}a\left(e^{1/b^x} - 1\right)}{e^{1/b-1}} - \theta\right] e^{\theta/1+\varepsilon\theta} \tag{7}$$

$\theta\,(0) = 0, \theta(1) = 0.5$

Where

d = Modified dimentionless thermal conductivity,
b = Modified dimentionless thermal conductivity,
a = Frank-Kamenetskii parameter

## METHOD OF SOLUTION

We utilize shooting method in solving (7)

$$\text{Let} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x \\ \theta \\ \theta^1 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \frac{1}{d}\left[x_3 - \frac{\frac{1}{2}a\left(e^{1/b^{x1}} - 1\right) - x_2}{e^{1/b-1}} e^{\frac{x_2}{1+\varepsilon x_2}}\right] \end{pmatrix} \tag{9}$$

Satisfying

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix} \tag{10}$$

## THE EXISTENCE AND UNIQUENESS OF THE SOLUTION

For Eq. 8 which satisfies (9)
(HI): S > 0, 0 = $x_1$ = 1, 0 = $x_2$ = L, å >0 and -m = $x_3$ = M, where L, m and M are positive constants.

**Theorem 1:** Let (HI) hold. Then problem (8) has a unique solution which satisfies (9).

**Proof:**
From (9), we have

$$\left|\frac{\partial f_1}{\partial x_1}\right| = \left|\frac{\partial f_1}{\partial x_2}\right| = \left|\frac{\partial f_1}{\partial x_3}\right| = \left|\frac{\partial f_2}{\partial x_1}\right| = \left|\frac{\partial f_2}{\partial x_2}\right| = 0 \left|\frac{\partial f_2}{\partial x_3}\right| = \left|\frac{\partial f_3}{\partial x_3}\right| = 1$$

$$\left|\frac{\partial f_3}{\partial x_1}\right| = \left|\frac{a}{2bd}\right| = \left|e^{1/b} - 1\right|\left|e^{1/b}\right| = \lambda_1 \lhd \infty$$

$$\left|\frac{\partial f_3}{\partial x_2}\right| = \left|\frac{L}{e^{1+\varepsilon L}}\right|\left|1 + \frac{L}{(1+\varepsilon L)^2}\right| = \lambda_2 \lhd \infty$$

$f_1$, $f_2$, $f_3$, are given by (9) clearly

$$\frac{\partial f_i}{\partial x_i}$$

is bounded for i,j = 1,2,3.

Thus $f_i$ i = 1,2,3 are Lipschitz continuous. Hence there exists a solution of (9) which satisfies (10).

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