

## A Study of the Agglomeration of Sand Particles in Turbulent Flowlines Using the Monte-Carlo Agglomeration Model Based on Smoluchowski Equation

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**Abstract:** In Nigeria, one of the transmission pipeline installation companies was faced with early pipe leakage problem in one of their installations. Several experimental tests were done to determine the root cause and one of the tests suggested strong evidence linking the leakage problem to the interactions between the agglomerated particles and the pipe wall. The limited space in the pipe could cause flow constriction that could lead to pipe burst due to high pressure exertion that could in turn also increase the flow shear stress. Therefore, this research was carried out to find the particle build up pattern and attendant locations. The study uses the monte-carlo agglomeration model that is based on smoluchowski equation to investigate the agglomeration of slurry of particles in fluid transmission pipelines. From the model, particle diameter and aggregate diameter were determined to be 4.8 and 6.0  $\mu\text{m}$ , respectively, with a population of 1.5. These values indicate that the slurry could not form actual aggregates of particles. The cause of leakages could be due to the high flow rate intermixed with high quantity of sand particles in suspension in the pipeline. However, the loss of aggregates may be attributed to proper administration of dispersants in the pipeline and the application of mechanical pigs for routine cleaning.

**Key words:** Aggregates of particles, angularity index, monte-carlo agglomeration model, particle diameter, pipeline, smoluchowski equation

### INTRODUCTION

This research work was carried out to measure the sizes of agglomerates of sand particle slurry aggregates in a transmission fluid pipeline. It has been reported by some researchers that built up inclusions in the pipeline has been found to be responsible for obstructed flow rates (Kiefner and Fischer, 1988) and exerted high pressures on T-joints and welded areas (Edwards *et al.*, 2000). When sand particles in the cause of flow collide and compact together, they form aggregates of particles that are large enough to cause wear of the internal walls of the pipelines, to a point that the thickness of the pipe may not be able to withstand the high pressure from the exerted turbulent flow.

This process of agglomeration is studied here by conducting a dynamic Monte-Carlo Agglomeration Model based on the Smoluchowski equation as was earlier attempted by Rensing (2006).

### MATERIALS AND METHODS

**Description of equations:** The particle aggregates formed are fractal in nature, so fractal dimension is used to characterize the following equations.

The fractal dimension,  $D_f$  relates the scaling of the characteristic length,  $L$ , to the mass of the object,  $M$ , as suggested by Mandelbrot (1982) in Eq. 1:

$$M \propto L^{D_f} \quad (1)$$

In this study, the Sierpinski Triangle with a fractal dimension,  $D_f$  of 1.585 (Sierpinski, 1915) was used.

The proportionality constant is assumed to be the mass of one particle, which gives Eq. 2, relates the number of particles,  $N$ , to the diameter of the particles,  $D_p$  and the diameter of the agglomerate,  $D_A$  (Lattuada *et al.*, 2004).

$$N = \left[ \frac{D_A}{D_p} \right]^{D_f} \quad (2)$$

However, the method to model this agglomeration is the Smoluchowski population balance equation (Steward and Spearing, 1993), which is expressed in Eq. 3:

$$\frac{dN_i(t)}{dt} = \frac{1}{2} \sum_{j=1}^{i-1} B_{i-j,j} N_{i-j}(t) N_j(t) - \sum_{j=1}^{i_{max}} B_{i,j} N_i(t) N_j(t) \quad (3)$$

In which,  $N_i(t)$  is the number of agglomerates of size  $i$  as a function of time  $(t)$ ,  $B_{i,j}$  is the aggregation kernel of particles of size  $i$  and  $j$ . Where, the aggregation kernel is a function that accounts for mobility of particles, fractal dimension and other properties.

Flesch *et al.* (1999) suggested a probability calculation based on the Smoluchowski aggregation kernel, which successfully combined particles in turbulent flow as expressed in Eq. 4:

$$B_{i,j} = 0.31 G v_p \left( x_i^{\frac{1}{D_f}} + x_j^{\frac{1}{D_f}} \right)^3 \quad (4)$$

In which, the aggregation kernel is a function of the volume average shear rate,  $G$ , the particle volume,  $v_p$  and the size of the aggregates,  $x_i$ . The probability was taken to be the ratio of  $B_{i,j}$  against the maximum possible  $B_{i,j}$ . Here, the diameter or size of the aggregate is taken to be the largest distance between two particles.

The probability of accepting the result follows in Eq. 5 as:

$$P_{accept} = \frac{\left( x_i^{\frac{1}{D_f}} + x_j^{\frac{1}{D_f}} \right)^2}{N^2} \quad (5)$$

For Monte-Carlo step a random probability was compared to the  $P_{accept}$ . If the random probability was greater than  $P_{accept}$  then no action was taken, but if this random probability was less than or equal to  $P_{accept}$  then the agglomerates would be combined into one new agglomerate.

**Particle size determination:** The sizes of 500 g of the particles in the pipeline were determined by conducting the particle size distribution analysis with a mechanical sieve. The diameters of the particle  $D_p$  and aggregates  $D_a$  are determined by Eq. 6:

$$D_p, D_a = 0.42 \left[ \sum (x_{a,p})^{1.5} + \left( \frac{\ln(AI)}{0.375} \right) 10^{-3} \right] \quad (6)$$

The angularity of a particle, called the Angularity Index (AI) as expressed in Eq. 7 for a specific particle, is

used to measure the abrasivity of the particle through the wearing of a surface (Steward and Ruther, 1993):

$$AI = \sum_{i=1}^n \left( \frac{\alpha}{n} \right) \quad \text{for } \alpha > 180^\circ \quad (7)$$

Where,

$\alpha$  = The internal angle of protuberances in the particle  
 $n$  = The number of angles measured

It is observed that changes in the outline of the particle projections that extended into the body of the particle result in surface characteristics with angles that are  $< 180^\circ$ . These surface characteristics give rise to no damage to the pipeline material on impact and are therefore, not included in the calculation of the AI (Bahr *et al.*, 2000).

## RESULTS AND DISCUSSION

For different sizes of samples of sand particles  $x_p$  and the corresponding  $x_a$  were determined by the mechanical sieve and are shown in Table 1.

Table 2 shows some of the parameters used for the Smoluchowski Equation calculation.

From the values in Table 1 and 2. When the random probability,

$$\frac{B_{i,j}}{\sum B_{i,j}}$$

was calculated.

It was found that the random probability was greater than  $P_{accept}$  in all the 5 trials. Therefore, it is assumed that the particle slurry under study did not have any actual agglomerate of particle aggregates. However, the particle diameter was  $4.8 \mu\text{m}$ , particle aggregate diameter was approximately  $6.0 \mu\text{m}$  and the number of particles was approximately, 1.5.

Table 1: Sample sizes of  $x_p$  and  $x_a$

Sample (500 g)	$\mu\text{m}$	
	$x_p$	$x_a$
1	3.0	5.2
2	2.9	4.8
3	3.1	5.4
4	3.3	5.8
5	3.5	6.2

Table 2: Parameters used for smoluchowski equation calculation

Variables	Values
Proportionality constant	0.85
Average shear rate (g)	36.5 (Pa sec <sup>-1</sup> )
Mass of particle	45 (g)
Particle volume	16 ( $\mu\text{m}$ ) <sup>3</sup>
Particle angularity index	35

## CONCLUSION

The Monte-Carlo Agglomeration model has been successfully applied. Although, in this study the particle slurry were not able to compact together to form large aggregates of particles. This shows that the particle could have been coarse and non-sticky. This model should be further improved to study the aggregation of particles in pipelines transporting other fluids.

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