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Design of FIR Digital Filter Based on LMS Modeling Algorithm

Firas Mahdi Muhsin Al-Salbi
Department of Electrical and Electronics, Engineering College, Kerballa University, Kerballa, Iraq

Abstract: FIR (Finite Impulse Response) digital filters are designed using traditional methods such as analog to digital transformation techniques with analog filter topologies like butterworth, chebyshive and elliptic, etc. In addition, it can also be designed by using adaptive algorithms. In this study, FIR arbitrary shape filter can be designed using LMS algorithm. Convergence of the adaptive filter to the optimum weight values depends on the statistical characteristics of the input signal (the correlation matrix R_{xx}) which is highly depending on the input signal. Maximum step size of the LMS algorithm has been found and the optimum step value is derived based on wiener-hop formula. The proposed algorithm has been theoretically derived, simulated using Matlab and tested for several filter types. The results shown LMS adaptive technique, more flexibility can be achieved in design response for the required filter.

Key words: Finite impulse response, LMS, digital filters, butterworth, elliptic, Mtalab, Iraq

INTRODUCTION

Filter design is always an important issue in signal processing. Because Finite Impulse Response (FIR) digital filter has good characteristic of linear phase which can avoid phase distortion when FIR filters are used to transmit signals, it attracts many researchers' attentions and is used generally in many fields (Peiqing, 2001; Kuc, 1988). In fact, FIR digital filter design is essentially a multi-criterion optimization problem with multiple local optimums in most cases. There are many contrary factors among the multiple criterions such as the maximal ripple in passband, the width of transition band and the minimal attenuation in stopband. It is very necessary to explore the relations to improve the qualities of FIR digital filters designed (Cheng and Yu, 2000).

Many methods have been proposed for designing FIR digital filters with linear phases (Xiaoping, 1999; Goodwin and Sin, 1984; Cheng and Yu, 2000). FIR digital filter is mostly based on some approximate process on frequency characteristic of ideal filter such as windows function method, equal-ripple approximation method and frequency sampling method. To further improve the accuracy of FIR digital filters, Xiaoping puts forward the Random Sampling Recursive Least Square (RS-RLS) algorithm (Xiaoping, 1999). The RS-RLS algorithm does not deal with the operation about an inverse matrix but an experiential error weight function must be provided. Furthermore, the algorithm does not remarkably improve the precision of the filters. Both window method and frequency sampling method belong to the usual designing approaches of FIR filters. Since both approaches can not accurately control cutoff frequencies of the pass-band and stop-band in the practice applications as a result many scholars have presented some optimal design approaches. The famous method is Remez permutation algorithm based on the rule about maximal error minimum and linear programming algorithm in all the methods. The PLS algorithm has recently been advanced for solving positive definite Quadratic Programming (QP) problems (Lai, 2005). The PLS algorithm can be applied to constrained LS design of FIR filters directly and to constrained MM design of FIR filters in an iterative fashion. The Weight Least Square (WLS) algorithm easily come true and can acquire analytical solution (6-8) but it must calculate an inverse matrix. The matrix's rank is the number of the independent coefficients of the filter. When the filter's degree is very high, it is difficult to compute the inverse matrix (Goodwin and Sin, 1984).

In this study, designing the FIR digital filter by using LSM algorithm was proposed. To design a digital filter, first it needs to establish some performance specifications and find a discrete time liner system according to the application. Second, actualize the system by means of simulations. Finally, examples of FIR digital lowpass and bandpass filters design are given to demonstrate that the method is effective, general and flexible.

MATERIALS AND METHODS

Synthesis of digital filters can be accomplished by making use of adaptive modeling techniques. The basic

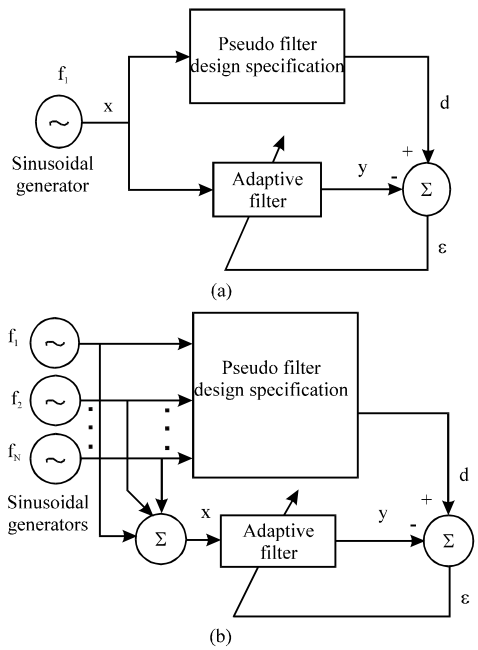

Fig. 1: Scheme for adaptive synthesis of a specified filter; a) excitation of a single frequency, f_1 and b) excitation at N different frequencies

idea for the synthesis of FIR digital filters is represented in the block diagram of Fig. 1. The adaptive filter upon convergence of the adaptive process will assume an impulse response that best satisfies a set of design specifications. These specifications are described by the box labeled psedofilter. The psedofilterdoes not exist because the filter to meet exact specifications will greatly not be physically realizable. This is the purpose is to tie the filter synthesis problem to the plant modeling problem (Golomb, 1964). Assuming that the filter specifications are given in the form of frequency response that is a set of requirements that the filter haveprescribed gain magnitude and phase characteristics at the discrete frequencies f_1, f_2, …., f_N, measured in Hz. Generally, the number of weights to be used in the digital filter will be specified thus defining L, the size (order) of the adaptive filter. Adaptive process finds a design solution that is a best fit (in the minimum mean-square-error sense) to the specifications (Dixon, 1976).

From Fig. 1, the adaptive filter models the psedo filter derived from the design specifications. These specifications cannot in the most cases be met perfectly in their entirety. However, one can imagine the existence of the psedo filter having a frequency response, magnitude and phase perfectly meeting the design specifications. In Fig. 1a, the input sinusoid will be in the form:

$$x(t) = \sin(2\pi f_1 t) \tag{1}$$

Sinusoidal generator is applied to both the psedo filter and adaptive filter. The frequency f_1 is one of the specification frequencies. The output of the psedo filter assuming linear operation is:

$$d(t) = a_1 \sin(2\pi f_1 t + \theta_1) \tag{2}$$

This is the desired response at the output of the adaptive filter. The coefficient a_1 is the design response magnitude at frequency f_1 and the angle θ_1 is the design phase shift at frequency f_1. In order for the specifications to be met (or at least closely approximated) at many frequencies simultaneously, an input comprising a sum of sinusoids, one for each of N specification frequencies is applied to both the psedofilter and the adaptive filter in Fig. 1b. This input is (Etter and Stearns, 1981):

$$x(t) = \sum_{i=1}^{N} \sin(2\pi f_i t) \tag{3}$$

The output of the psedo filter which is the desired response of the adaptive filter is:

$$d(t) = \sum_{i=1}^{N} a_i \sin(2\pi f_i t + \theta_i) \tag{4}$$

When specifications cannot be perfectly met at all frequencies, it is sometimes desirable to have specifications more tightly met at certain frequencies then at others. Certain parts of the design frequency response may be more critical then others. This can be accomplished easily by having input sinusoids with various individual amplitudes rather than all unit amplitudes and scaling the components of d(t) accordingly. The larger the amplitude of the input sinusoid, the more tightly will the specification be held at its frequency. When practicing this technique, the lth individual input sinusoid is scaled be c_l and is given by:

$$c_l \sin 2\pi f_l t \tag{5}$$

with c_l begin a positive constant cost function for all l. The input signal again is a sum of sinusoids as shown:

$$x(t) = \sum_{i=1}^{N} c_i \sin 2p f_i t \tag{6}$$

This desired response, the output of the pseudo filter is:

$$d(t) = \sum_{i=1}^{N} a_i c_i \sin(2\pi f_i t + \theta_i) \quad (7)$$

once again, the desgin specification magnitude is a_i and the phase is θ_i at frequency f_i. The adaptive filter converges to least-squares solution which provides a best least-squares fit to the design specification. The form of this solution is of interest. The least-squares solution has been discussed in many references (Etter and Stearns, 1981). The algebraic form of the least-squares solution in the case of the adaptive linear combiner is well known (Widrow *et al.*, 1981; Soldan, 1988) using the correlation notation the solution is:

$$W^* = R^{-1}P = \begin{bmatrix} \phi_{XX}(0)\ldots\phi_{XX}(L) \\ \vdots \\ \phi_{XX}(L)\ldots\phi_{XX}(0) \end{bmatrix}^{-1} \begin{bmatrix} \phi_{dX}(0) \\ \vdots \\ \phi_{dX}(L) \end{bmatrix} \quad (8)$$

as before, L+1 is the number of weights in the adaptive linear combiner. Since, researchers know the signals d and x, they can compute the correlation functions. Let us define, T_s is the time step between samples then it can be shown that (Sarwate and Parsley, 1980):

$$\begin{aligned} \phi_{XX}(n) &= E\left[x(t-nT)x(t)\right] \\ &= E\left[\sum_{i=1}^{N} c_i \sin 2\pi f_i (t-nT) \sum_{m=1}^{N} c_m \sin 2\pi f_m t\right] \end{aligned} \quad (9)$$

Since, the expected value of the product of two sinusoidal time functions of different frequencies is zero, Eq. 9 becomes:

$$\phi_{XX}(n) = E\left[\sum_{i=1}^{N} c_i^2 \sin 2\pi f_i (t-nT) \sin 2\pi f_i t\right] \quad (10)$$

using the trigonometric identity:

$$\sin 2\pi f_1 (t-nT) = \sin 2\pi f_1 t \cos 2\pi f_1 nT - \cos 2\pi f_1 t \sin 2\pi f_1 nT$$

Equation 10 become:

$$\begin{aligned} \phi_{XX}(n) &= E\left[\sum_{i=1}^{N} \begin{matrix} c_i \sin^2 2\pi f_i t \cos 2\pi f_i nT - \\ c_i^2 \sin 2\pi f_i t \cos 2\pi f_i t \sin 2\pi f_i nT \end{matrix}\right] \\ &= E\left[\sum_{i=1}^{N} c_i^2 \sin 2\pi f_i t \cos 2\pi f_i nT\right] \\ &= \sum_{i=1}^{N} \frac{1}{2} c_i^2 \cos 2\pi f_i nT \end{aligned} \quad (11)$$

In obtaining Eq. 11, recall that sine and cosine waves are uncorrelated and that the mean square of a sine wave is half the square of its amplitude (Proakis, 1983). Thus, Eq. 11 have all of the elements of the R-matrix. The elements of the cross-correlation vector P may be found in like manner.

$$\begin{aligned} \phi_{dX}(-n) &= E\left[x(t-nT)d(t)\right] \\ &= E\left[\sum_{i=1}^{N} c_i \sin 2\pi f_i (t-nT) \sum_{M=1}^{N} a_m c_m \sin(2\pi f_m t + \theta_m)\right] \\ &= E\left[\sum_{i=1}^{N} a_i c_i^2 \sin 2\pi f_i (t-nT) \sin(2\pi f_i t + \theta_i)\right] \\ &= E\left[\sum_{i=1}^{N} a_i c_i^2 \sin(2\pi f_i - 2\pi f_i nT - \theta_i) \sin(2\pi f_i t)\right] \\ &= E\left[\sum_{i=1}^{N} \frac{1}{2} a_i c_i^2 \cos(2\pi f_i nT + \theta_i)\right] \end{aligned} \quad (12)$$

Using Eq. 11, 12 in Eq. 8, it can write explicitly. The least-squares solution for the adaptive filter weights. The solution is:

$$W^* = \begin{bmatrix} \sum_{i=1}^{N} c_i^2 \cdots\cdots \sum_{i=1}^{N} c_i^2 \cos 2\pi f_i T \cdots\cdots \sum_{i=1}^{N} c_i^2 \cos 2L\pi f_i T \\ \sum_{i=1}^{N} c_i^2 \cos 2\pi f_i T \cdots\cdots \sum_{i=1}^{N} c_i^2 \\ \vdots \\ \sum_{i=1}^{N} a_i c_i^2 \cos(2L\pi f_i T + \theta_i) \cdots\cdots \sum_{i=1}^{N} c_i^2 \end{bmatrix}^{-1} * \begin{bmatrix} \sum_{i=1}^{N} a_i c_i^2 \cos(\theta_i) \\ \sum_{i=1}^{N} a_i c_i^2 \cos(2\pi f_i T + \theta_i) \\ \vdots \\ \sum_{i=1}^{N} a_i c_i^2 \cos(2L\pi f_i T + \theta_i) \end{bmatrix}^{-1} \quad (13)$$

Equation 13 provides a simplifications in this formulation result when the various specification frequencies are uniformly spaced and when the input sine waves are all of the same amplitude. The least-squares solution in Eq. 13 does not in itself give much insight into the filter design. In some ways, the adaptive process is much more appealing (Haykin, 1996). The least-squares solution is often valuable in the computer implementation of the design process, however because the R-matrix and the P-vector can be computed directly from the design specification. If the adaptive weights are fixed at W*, the least-squares solution and the mean-square error will be:

$$\xi_{min} = \sum_{i=1}^{N} \frac{1}{2} c_i^2 \left| S - H_i^* \right|^2 \tag{14}$$

Where, S is the complex transfer function of the specified pseudo filter at frequency f_i that is:

$$S \overset{\Delta}{=} a_i e^{j\theta_i} \tag{15}$$

and S* is the complex transfer function of the optimal adaptive linear combiner with weights W* at frequency f_i. The transfer function for the adaptive transversal filter can be given by Zhang and Schmer (2000) and Jones *et al.* (2004):

$$H(z) = \sum_{n=0}^{L} w_n^* z^{-n} \tag{16}$$

and it can be seen that $e^{jw} = e^{jw\Omega T}$ is substituted for z to obtain the frequency response. The normalized angular frequency corresponding to f_i is of course:

$$w_1 = 2\pi f_1 T$$

So, H* in Eq. 16 become:

$$H^* = \sum_{n=0}^{L} w_n^* e^{-j2\pi f_i T} \tag{17}$$

RESULTS AND DISCUSSION

To implement this technique, a simulation program has been written using Matlab 10 simulation program with m-files format. The input signal x(t) has been simulated with sinusoidal function with N different frequencies and amplitude (cost function) as illustrated previously in Eq. 1.

The FIR filter design specifications are chosen by implementing the function in Eq. 7 which defines the required FIR filter type.

The LMS algorithm has been tested using Matlab functions with maximum calculations of the step size, μ_{max}. The evaluation of the maximum step size gives boundaries to the convergence rate of the adaptive process.

Optimum value of the step size, μ_{Opt} has been calculated using Weiner-Hopf formula. This research has been tested for different filter types; LPF, BPF, HPF and arbitrary shape filter with order, L = 32. The simulated response has been compared with standard built-in Matlab functions as well as with ideal filters. Matching has been achieved between the simulated and ideal response with fast convergence of the LMS algorithm to the best weight values.

Using 200 samples for the input and desired signals, the minimum mean square error, ζ_{min} has been got with only 30-50 samples depending on the optimum value of the step size, μ_{Opt} more simulation result can found in Fig. 2-19.

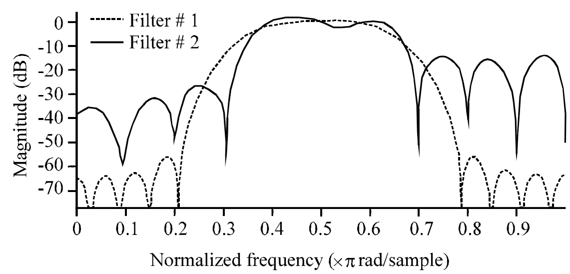

Fig. 2: Magnitude response of 32 order BPF

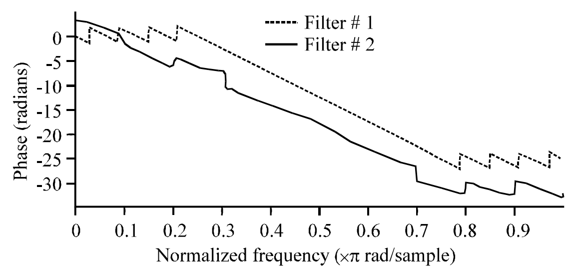

Fig. 3: Phase response of 32 order BPF

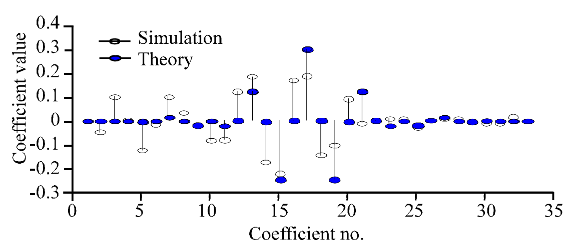

Fig. 4: Weight coefficients of 32 order BPF

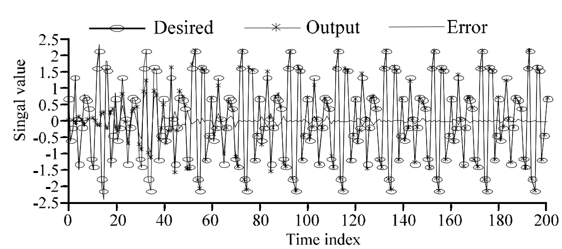

Fig. 5: Error signal of 32 order BPF

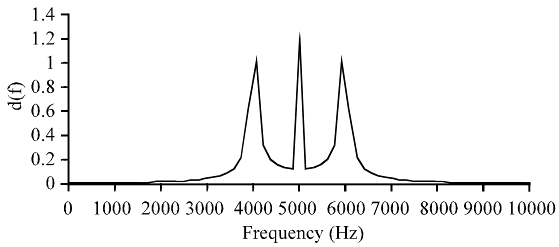

Fig. 6: Frequency response of desired signal (Singal-sided amplitude spectrum of d(t))

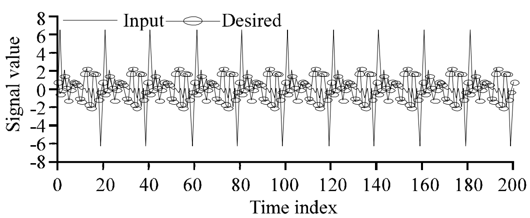

Fig. 7: Comparison between input and desired signals in time domain

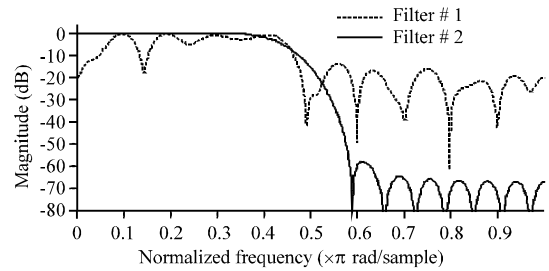

Fig. 8: Magnitude response of 32 order LPF

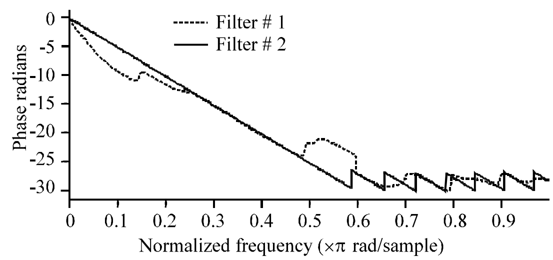

Fig. 9: Phase response of 32 order LPF

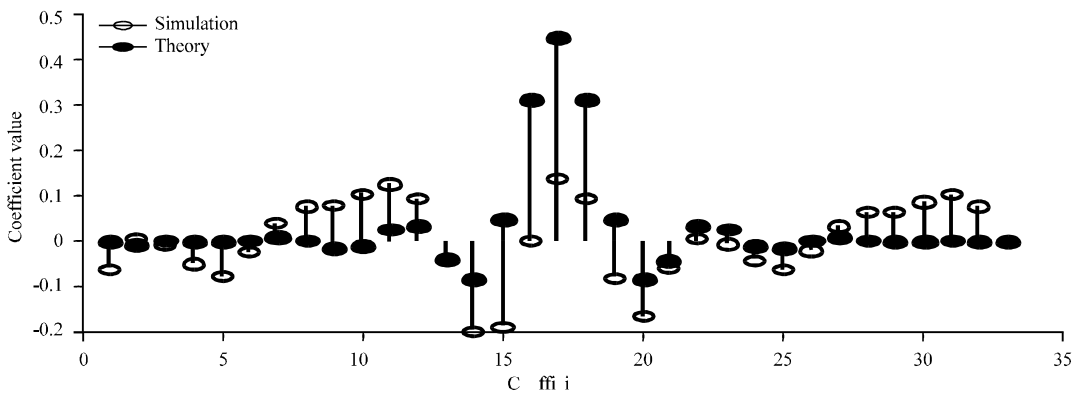

Fig. 10: Weight coefficients of 32 order LPF

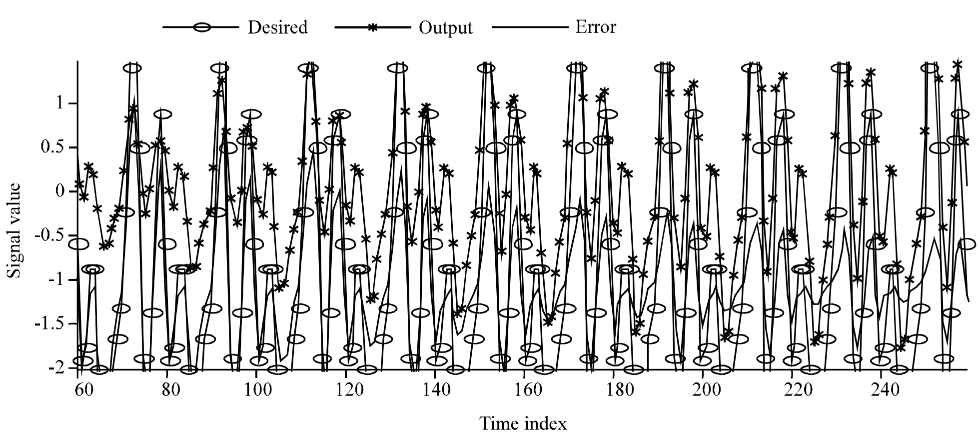

Fig. 11: Error signal of 32 order LPF

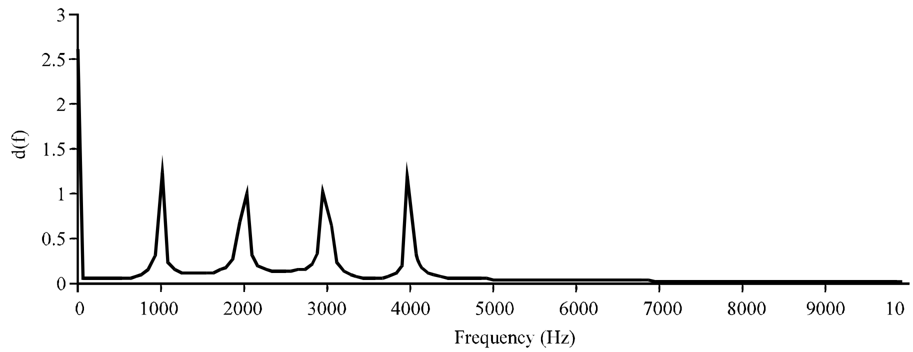

Fig. 12: Frequency response of desired signal (Singal-sided amplitude spectrum of d(t)

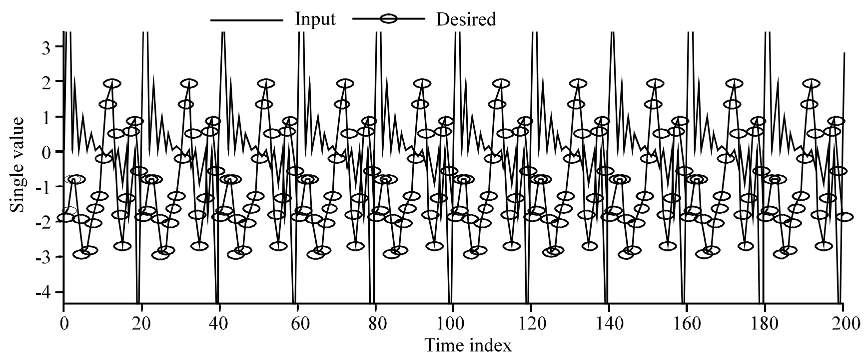

Fig. 13: Comparison between input and desired signals

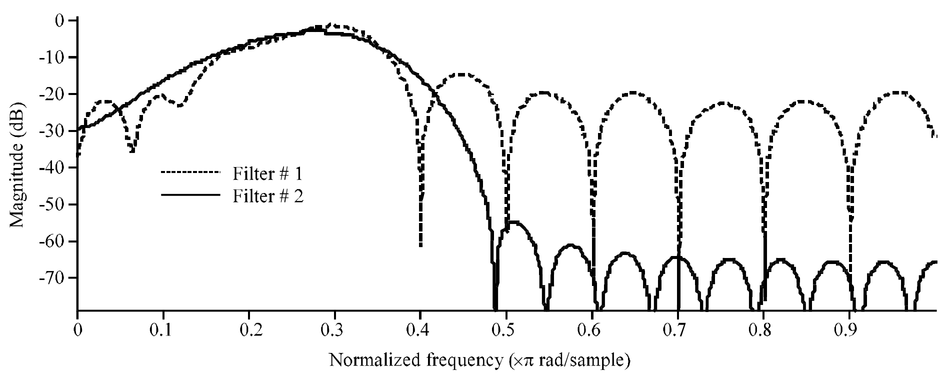

Fig. 14: Magnitude response of 32 order arbitrary shape filter

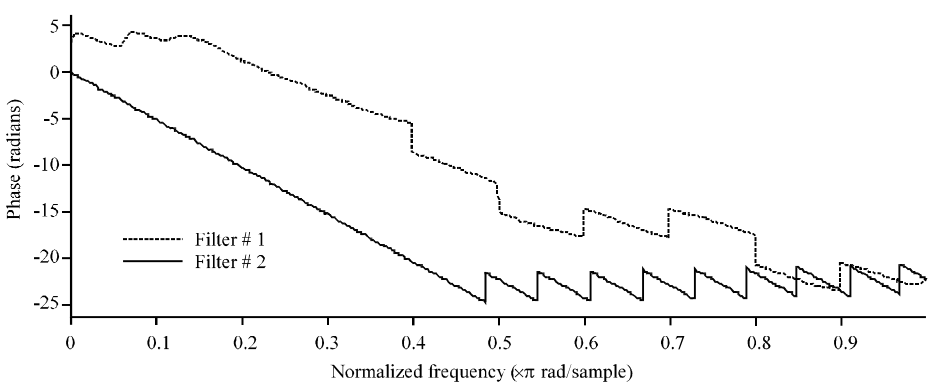

Fig. 15: Phase response of 32 order arbitrary shape filter

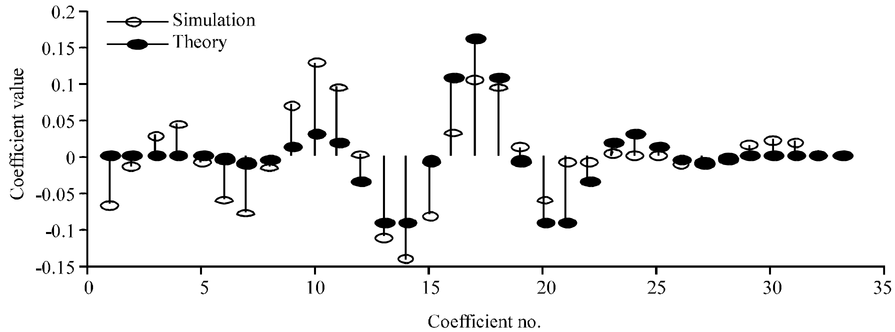

Fig. 16: Weight coefficients of 32 order arbitrary shape filter

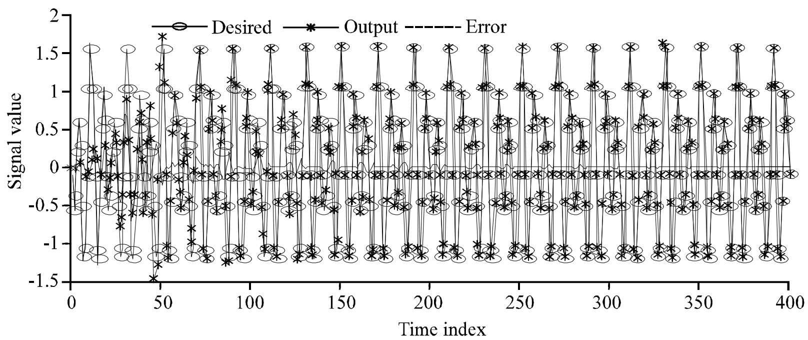

Fig. 17: Error signal of 32 order arbitrary shape filter

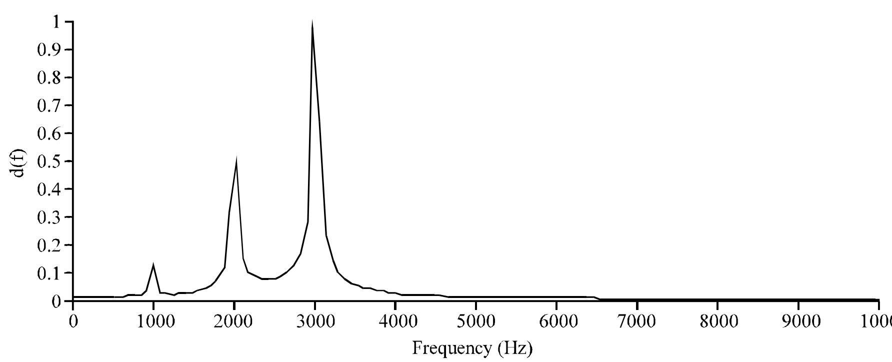

Fig. 18: Frequency response of desired signal (Singal-sided amplitude spectrum of d(t))

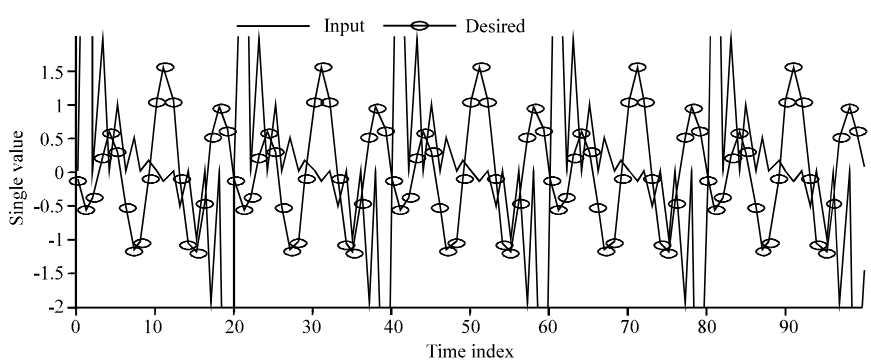

Fig. 19: Comparison between input and desired signals

CONCLUSION

In this research, FIR (Finite Impulse Response) digital filter have been designed and implemented using LMS adaptive algorithm. This technique is better used for FIR digital filter design with large number of weights. More flexibility in this synthesis approach is obtained and unusual filter designs can be realized compared with the classic filter designs. By using Weiner-Hopf formula, optimum step size of the LMS algorithm has been obtainedand fast convergence to the FIR filter weights have achieved.

As a future research need more on analytical solution of cost function (c_i) since, no theoretical evaluation has been found for the cost function till now. Different adaptive techniques can be used rather than LMS method and tested to find faster and better response of the designed FIR filter.

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