

## The Study of Modal Contribution to the Horizontal Motion of Mid-Rise Buildings

Sarwidi

Faculty of Civil Engineering and Planning, Universitas Islam Indonesia (UII),  
 55584 Yogyakarta, Indonesia

**Abstract:** As it is widely known that the story number affects to the building period during earthquake shaking. However, how far the affection is and what other influencing structural parameters are needs to be intensively studied. This study explores the characteristic of the horizontal motion of three mid-rise buildings using modal analysis method. The method is used to calculate the modal parameters of the buildings to evaluate the motion characteristics of the buildings. It has been found that when the story number of a building increases: the building will become more flexible as indicated by the lengthening the modal periods, the building will vibrate longer as indicated by the decreasing the modal damping ratios and the domination of lower modes to the building vibration decreases as indicated by increasing the summation of the absolute values of the modal participation factors for high modes.

**Key words:** Building, dynamic, structure, modal analysis, mid-rise, domination

### INTRODUCTION

The world population grows continuously, yet the land area is constant. To fulfill the need for people settlements, buildings tend to be built vertically. High rise buildings are not many, however, mid-rise building grows significantly in this time. The 3-7 story buildings becomes popular for medium cities (QA, 2010). Therefore, 3-5 and 7 story buildings are chosen to be the subject of this study.

As it is widely known that the story number influences to the natural period of buildings during earthquake shaking. However, how far the effect is and what other influenced modal parameters are needs to be intensively studied. Similar, series of issues were studied either using computer simulations (Sun *et al.*, 2015; Habibi *et al.*, 2013; Louzai and Abed, 2015; Poursha and Amini, 2015) or laboratory experiments (Huo *et al.*, 2016; Caetano and Cunha, 2004).

This study is objected to investigate the effect of the increasing of the story number of the mid-rise buildings to the characteristic of the horizontal motion of the buildings using modal analysis method. The method is used to compute the modal parameters that contribute to the motion characteristics of the buildings.

### MATERIALS AND METHODS

This study contains the data of the studied buildings and the procedure that are used in this study.

**Structural model and parameters:** The structural models and parameters of the studied mid-rise buildings are shown in Fig. 1.

The left, center, right of Fig. 1 are the structural models of 3-5 and 7 story buildings, respectively. Structural parameters of the mid-rise building are mass ( $m$ ) that is lumped to a floor, the stiffness of a story ( $k$ ) and the damping value of a story ( $c$ ). To be comparable, the structural parameters are determined as follows:

$$m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m \quad (1)$$

$$c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = c_7 = c \quad (2)$$

$$k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k_7 = k \quad (3)$$

Where:

$$m = 29.01 \text{ kg see}^2/\text{cm}$$

$$c = 1422.72 \text{ kg see/cm}$$

$$k = 284544.01 \text{ kg/cm}$$

**Modal analysis methods:** For the shear building models as shown in Fig. 1, general equation of motion of the mid-rise buildings can be written as:

$$M\ddot{u} + C\dot{u} + Ku = -M\ddot{u}_g \quad (4)$$

Where:

$M$  = Matrix of mass

$C$  = Matrix of damping value

$K$  = Matrix of stiffness

$1$  = Unit vector

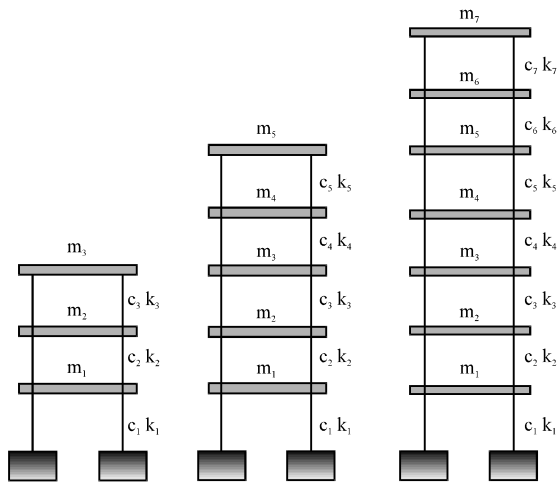


Fig. 1: Structural models and parameters of mid-rise buildings

In Eq. 4,  $\ddot{u}$ ,  $\dot{u}$ ,  $u$ ,  $\ddot{u}_g$  are the vectors of acceleration, velocity, displacement and ground acceleration, correspondingly. Processed by using modal analysis operation (Vamvatsikos and Cornell, 2005), Eq. 1 constitutes the modal equations of motion. The modal equation of motion of a certain mode of vibration can be written as:

$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2u = -\Gamma\ddot{u}_g \quad (5)$$

Where:

$\omega$  = Modal natural angular frequency

$\zeta$  = Modal damping ratio

$\Gamma$  = Modal participation factor

An important modal parameter of buildings is modal natural period, simply called modal period which can be stated as:

$$T = 2\pi / \omega \quad (6)$$

## RESULTS AND DISCUSSION

By applying modal analysis procedures using Eq. 1-6, the modal parameters of the buildings, namely modal periods, modal damping ratios and modal participation factors are calculated and the results are discussed herein.

**Modal period:** The calculated modal periods of each building for every mode of its vibration are shown in Table 1.

Table 1: Modal periods (sec)

Modes	Story number		
	3	5	7
1	0.4466	0.6984	0.9508
2	0.1594	0.2392	0.3216
3	0.1103	0.1518	0.1988
4		0.1181	0.1485
5		0.1036	0.1228
6			0.1088
7			0.1016

Table 1 demonstrates that the story number influence the modal periods of the buildings in different levels. As the story number increases, the modal periods elongate. It means that, in general the increasing the story number will lead a building to become more flexible. In other word, the building will become less rigid, since in that condition, a building story will support greater masses above it. To detailed exploration of the phenomenon, the numerical results in the table are plotted in Fig. 2-5.

As what has been predicted, Fig. 2 tells that a building structure will be more flexible when its story increases. All modes of vibration have that trend. Try using several fitting curves, the best match are power equations. All curves occupy very good fit that has the coefficients of determination  $R^2$  more than 0.95 or 95 % out of 100%. In addition, the coefficient of the equations measures the increasing degree of modal periods when the number of the mode is greater. The greatest value is for 7 story building while the smallest value is the 3 story building. That means the 7 story building has the longest modal periods while the 3 story building has shortest modal periods.

To measure the degree of decreasing the value of consecutive greater modes for each building, the normalized modal periods are plotted in Fig. 3.

The ordinate of Fig. 3 is normalized modal periods where the modal periods of each building are normalized to one of mode 1. The decreasing level of a consecutive higher mode can be measured by the absolute values of the power of the equations. The greatest absolute value is for 3 story building while the smallest one is for 7 story. To compare the modal period's change for each mode with the story number Fig. 4 and 5 will reveal a clearer explanation.

Figure 4 informs the trend of the modal period change for a certain mode by increasing the story number. When the story number increases, the first modal period will increase linearly. This is indicated by the value of the coefficient of determination  $R_1^2 = 1$ . However, as the number of the mode is greater, the non-linearity trend increases. This phenomenon is detected from the values

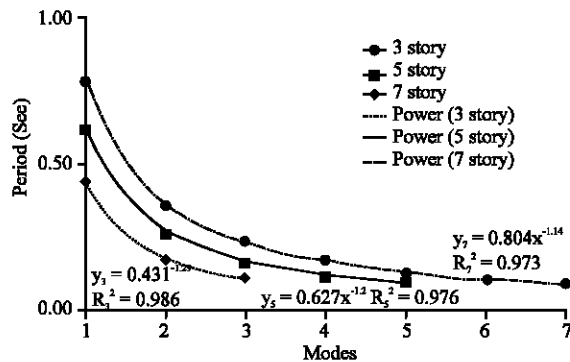


Fig. 2: Number of mode vs. modal periods

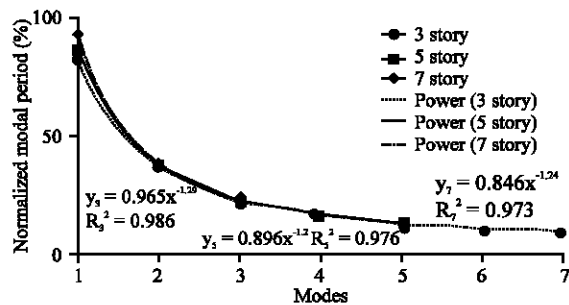


Fig. 3: Number of mode vs. normalized modal periods

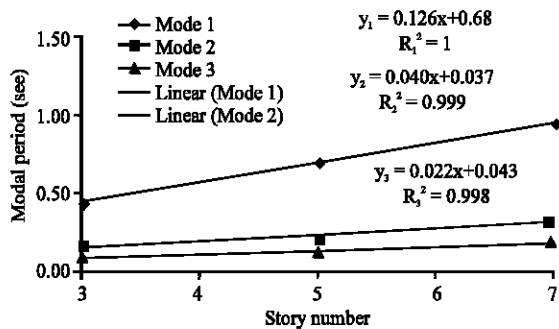


Fig. 4: Story number vs. modal periods

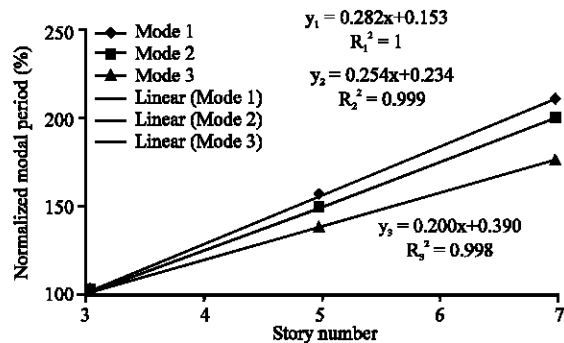


Fig. 5: Story number vs. normalized modal periods

of the coefficient of determination that become smaller when the story number is extended. How big the change of modal periods is can be observed in Fig. 5.

Figure 5 is the relationship between the story number of the buildings and the modal periods that are normalized to one of the 3 story building. The equations of the fitted curves inform that increasing the story number will increase the modal periods in the value of 28.22, 25.44 and 20.05% for mode 1-3, correspondingly. This is indicated by the coefficient of  $x$  in the equations. It should be noted that the nonlinearity trend becomes higher for larger modes of building vibration as it is indicated by the decreasing values of the determination coefficients  $R^2$ .

**Modal damping ratio:** The calculated modal damping ratios of each building for every mode of building vibration are exposed in Table 2.

Table 2 indicates that the story number influences the modal damping ratios of the buildings in different levels. It tells that when the story number increases, the modal damping ratios are getting smaller. It means that, in general, the increasing the story number will lead the vibration of the buildings to become longer.

In the other word, the building vibration will become shorter when the story number decreases. In this condition, adding the story number will make the interaction more complex. To have more a detailed exploration to the phenomenon, the numerical results in the table are plotted in Fig. 6-9.

As what has been predicted, Fig. 6 tells that a structure will vibrate longer as the story number increases. All modes of the vibration have that trend. Try using several fitted curves, the best match is logarithmic equations. All curves engage very good fit that has the coefficient of determination values  $R^2 < 0.95$  or 95% out of 100%.

Furthermore, the coefficients of the equations measure the increasing degree of modal damping ratios as the modes are higher. The smallest value is for 7 story building while the largest one is the 3 story building. That means that 7 story building has smallest modal damping ratios while the 3 story building has the largest modal damping ratio. To measure the degree of increasing the damping ratios of consecutive greater modes for each building, normalized modal periods are plotted in Fig. 7.

The ordinate of Fig. 7 is the normalized modal damping ratios. The modal damping ratios of each building are normalized to one of mode 1. The increasing level of a consecutive higher mode can be measured by the coefficient of the logarithmic term in the equations. The smallest coefficient is for 3 story building while the

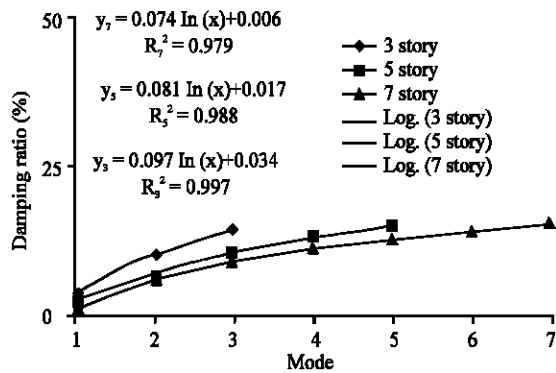


Fig. 6: Number of mode vs. modal damping ratios

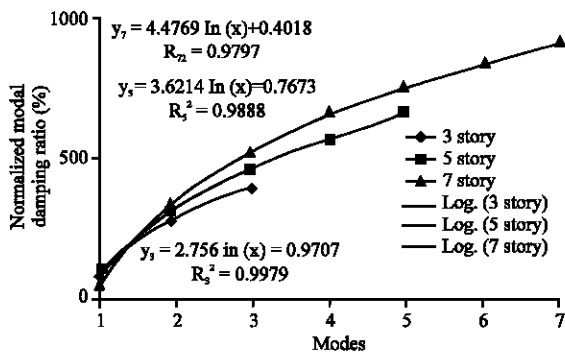


Fig. 7: Number of mode vs. normalized modal damping ratios

Table 2: Modal damping ratio (%)

Modes	Story number		
	3	5	7
1	3.5184	2.2502	1.6527
2	9.8582	6.5683	4.8860
3	14.2456	10.3543	7.9057
4		13.3014	10.5799
5		15.1709	12.7917
6			14.4444
7			15.4659

greatest one is the 7 story building. This means that as the story number increases, the contribution of mode 1 to the lengthening of the building vibration will become smaller. In the other words as the story number increases, high modes will greater contribute inlengthening the building motion. To compare the modal damping ratio's change for each mode to the story number, Fig. 8 and 9 will reveal a clearer explanation.

Figure 8 informs the trend of the modal damping ratio change for a certain mode by the alteration of the story number. When the story number increases, modal damping ratios of all modes decrease. This is indicated by

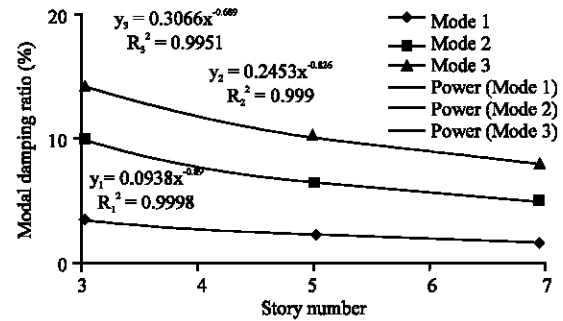


Fig. 8: Story number vs. modal damping ratios

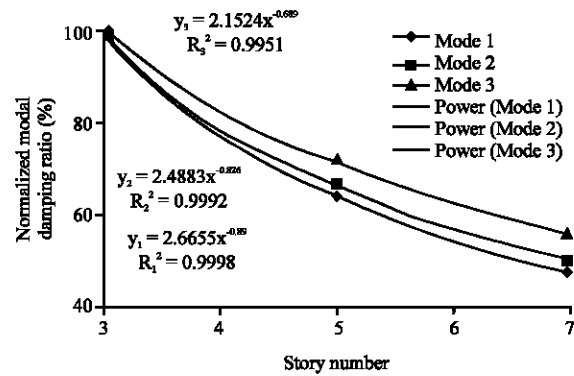


Fig. 9: Story number vs. normalized modal damping ratios

the values of the power of the equations. Moreover, as the mode number is greater, the nonlinearity trend increases.

The occurrence is pointed out by the values of the coefficients of determination that become smaller as the modenummer is greater. How big the change of the modal damping ratios is can be explored from Fig. 9.

Figure 9 is the relationship between the story number of the buildings and the modal damping ratios that are normalized to one of the 3 story building. The equations of fitted curves inform that increasing the story number will decrease the modal damping ratios. This is indicated by power values of the equations.

**Modal participation factor:** The modal participation factors of each building for every mode of the building vibration are shown in Table 3. The table shows that the story number influences the modal participation factors of the buildings in different levels. As the story number increases, the absolute values of modal participation factors are larger.

It should be noted that, the contribution of each mode not always in the same motion direction with other modes. The negative sign points out that the direction of

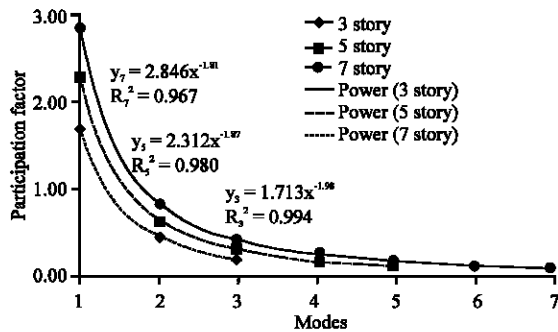


Fig. 10: Number of mode vs. modal participation factor

Table 3: Modal participation factor

Modes	Story number		
	3	5	7
1	1.6560	2.0971	2.4566
2	-0.4740	-0.6602	-0.7947
3	0.1820	0.3480	0.4472
4		-0.1938	-0.2868
5		0.0885	0.1876
6			-0.1150
7			0.0549

contribution opposites with the positive one. To have a more detailed exploration to the phenomenon, the numerical results in the table are plotted in Fig. 10-12.

Figure 10 plots the relationship between the story number and the modal participation factors in absolute values. The figure tells that all modal participation factors are increased as the story number increases. All modes have that trend. Try using several fitted curves, the power equations best match.

All curves occupy very good fit that has the values of determination coefficients  $R^2$  more than 0.95 or 95% out of 100%. To measure the degree of decreasing the values of consecutive greater modes for each building, normalized modal participation factors are plotted in Fig. 11.

The ordinate of Fig. 11 is normalized modal participation factors. The modal participation factors of each building are normalized to one of mode 1. Decreasing level of the participation factor of a consecutive higher mode can be measured by the coefficient values and the absolute values of the power of the equations.

The greatest absolute value of the power is for 3 story building while the smallest one is for 7 story. To compare the modal participation factor change for each mode to story number, Fig. 12 will reveal a clearer explanation.

Figure 12 informs the trend of the modal participation factor change for a certain mode by increasing the story number. When the story number increases, absolute values of all modal participation factors will increase.

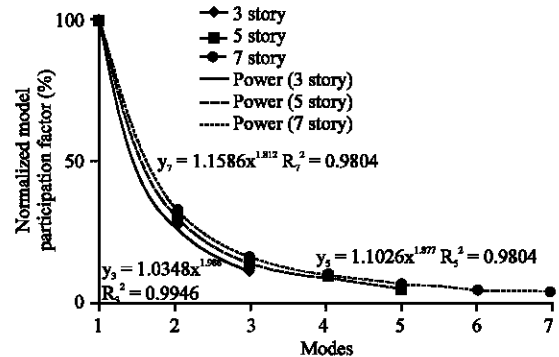


Fig. 11: Number of mode vs. normalized modal participation factor

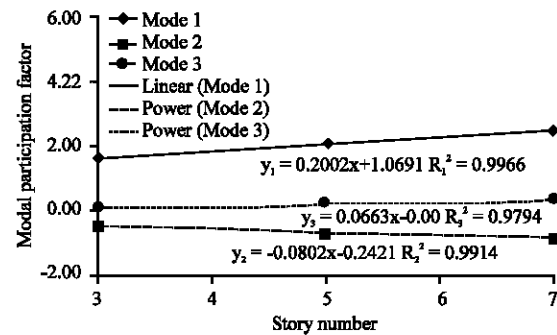


Fig. 12: Story number vs. modal participation factor

However, as the modenumber is greater, the nonlinearity trend increases as it is indicated by decreasing the coefficients of determination  $R^2$ .

## CONCLUSION

Referring the results and discussion, when the story number of a building increases: the building will become more flexible as indicated by the lengthening the modal periods, the building will vibrate longer as indicated by the decreasing the modal damping ratios and the domination of lower modes to the building vibration decreases as indicated by increasing the summation of the absolute values of the modal participation factors for high modes.

## ACKNOWLEDGEMENTS

The researcher appreciates to all persons and institutions that have supported this research. The study program of Civil Engineering and the Post Graduate Program of the Faculty of Civil Engineering and Planning of Universitas Islam Indonesia has sponsored this study.

## REFERENCES

- Caetano, E. and A. Cunha, 2004. Experimental and numerical assessment of the dynamic behaviour of a stress-ribbon footbridge. *Struct. Concr. London Thomas Telford Limited*, 5: 29-38.
- Habibi, A.R., M. Izadpanah and A. Yazdani, 2013. Inelastic damage analysis of RCMRFS using pushover method. *Iran. J. Sci. Technol. Trans. Civil Eng.*, 37: 345-352.
- Huo, L.S., X. Li, Y.B. Yang and H.N. Li, 2016. Damage detection of structures for ambient loading based on cross correlation function amplitude and SVM. *Shock Vib.*, 2016: 1-12.
- Louzaï, A. and A. Abed, 2015. Evaluation of the seismic behavior factor of reinforced concrete frame structures based on comparative analysis between non-linear static pushover and incremental dynamic analyses. *Bull. Earthquake Eng.*, 13: 1773-1793.
- Poursha, M. and M.A. Amini, 2015. A single-run multi-mode pushover analysis to account for the effect of higher modes in estimating the seismic demands of tall buildings. *Bull. Earthquake Eng.*, 13: 2347-2365.
- QA., 2010. Avenues and mid-rise buildings study. *Quadrangle Architects, Toronto, Ontario*.
- Sun, X.H., L.P. Xie and F. Ji, 2015. Dynamic analysis of concrete frame structure with story adding steel structure at the top. *J. Appl. Mech. Mater.*, 744: 65-70.
- Vamvatsikos, D. and C.A. Cornell, 2005. Direct estimation of seismic demand and capacity of multidegree-of-freedom systems through incremental dynamic analysis of single degree of freedom approximation 1. *J. Struct. Eng.*, 131: 589-599.