Journal of Engineering and Applied Sciences 13 (12): 4551-4558, 2018
ISSN: 1816-949X

Shear Deformation in Voided Beams on Elastic Foundations

Khalida A. Daud, Raid A. Daud and Adel A. Al-Azzawi
College of Engineering, Al-Nahrain University, Baghdad, Iraq

Abstract: The response of voided beams winkler type foundation interaction is investigated numerically in this research. The finite difference method is formulated to model beam with different thin and deep theories resting on elastic springs and programmed in Fortran language. Brick elements and spring elements are used to model the voided beam and the soil, respectively in ABAQUS finite element software. A comparison with previous studies is performed to validate the selected numerical method. The void percentage or void dimensions, void shape, vertical subgrade reaction, beam depth and type of loading are the main study parameters. The maximum deviation in central deflection between the two numerical methods and exact solution is recognized to be 5%.

Key words: Finite difference, finite element, voided beams, winkler foundation, theories, percentage

INTRODUCTION

Beams are one dimensional structural member resist the applied loads through internal forces that developed in beams. The old thin beam equation ($d^4w/dx^4 = q/EI$) which relates the distributed load (q) to the beam deflection (w) does not include the deformation due to transverse shear (Al-Azzawi, 2017).

For beam depth to span ratio <0.1, the thin beam theory can be utilized. While for ratio exceeds 0.2, Timoshenko theory can be used (Henry, 1986; Al-Azzawi, 2011).

Deep or timoshenko beams are one dimensional members with greater depth to effective span proportion or ratio. Hence, the assumption of linear strain distribution is no longer valid and the deformations due to shear must be considered.

Governing differential equations: The following timoshenko beam assumptions are used in deriving the differential equations of beam on elastic springs (Al-Jubory, 1992):

- Linear strain distribution in the study
- The cross section has independent rotation resulting from transverse shear
- Constant shear stress distribution is assumed through using shear correction factor (c^2)

Based on these assumptions, the differential equations for Timoshenko beam on one-parameter elastic foundations (Selvadurai,1979) are given by:

$$Gc^2A\left(\frac{d\psi}{dx}+\frac{d^2w}{dx^2}\right)+q-K_z w = 0 \quad (1)$$

$$EI\frac{d^2\psi}{dx^2}-Gc^2A\left(\psi+\frac{dw}{dx}\right) = 0 \quad (2)$$

Where:

- $w = w(x)$ = The beam deflection
- $\psi = \psi(x)$ = The beam rotation
- K_z = The soil resistance modulus
- G = The material modulus for shear
- A = The beam area
- EI = The member flexural rigidity
- q = The distributed load

The shear correction factor for voided section is equal to Amany and Pasini (2009):

$$c^2 = \frac{20(1-c)(1-c*d)^{-1}(1-c*d^3)^2}{3(7c*d^5+8c^2*d^5-30*c*d^3+15*c*d-8*c+8)} \quad (3)$$

where, $c = b/B$ and $d = h/H$ as shown in Fig. 1.

Finite difference technique: In this method, the differential equations are converted into differences at any point across the member. The resulting nodes will form later the finite difference grid or mesh. The member mesh will contain nodes with intervals of Δx (Fig. 2). The mesh have total number of nodes (n) across the beam. The node with (i) indicator represent the node under consideration. The differential equations in finite differences at node i are:

Corresponding Author: Khalida A. Daud, College of Engineering, Al-Nahrain University, Baghdad, Iraq

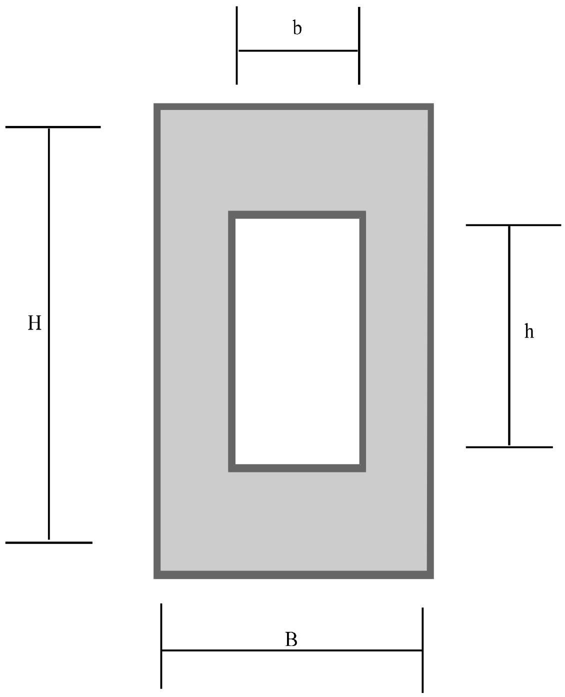

Fig. 1: Voided section

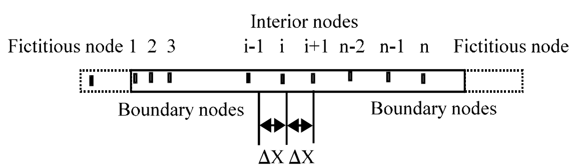

Fig. 2: Finate difference discretization

$$Gc^2A\left[\frac{\psi_{i+1}-\psi_{i-1}}{2\Delta x}\right]+\frac{Gc^2A}{\Delta x^2}[w_{i+1}-2w_i-\frac{K_z w_i\Delta x^2}{Gc^2A}+w_{i-1}]+[q_i+\frac{EA}{2}(\frac{w_{i+1}-2w_i+w_{i-1}}{(\Delta x)^2})(\frac{w_{i-1}-w_{i-1}}{2\Delta x})^2]=0 \qquad (4)$$

$$\frac{EI}{\Delta x^2}[\psi_{i+1}-2\psi_i-\frac{Gc^2A\psi_i\Delta x^2}{EI}+\psi_{i-1}]-[w_{i+1}\frac{Gc^2A}{2\Delta x}\frac{w_{i-1}Gc^2A}{2\Delta x} \qquad (5)$$

MATERIALS AND METHODS

Finite element method: The linear continuum elements (C3D8R elements)are used in ABAQUS to model the beam. The element has 8-node and spring elements are used to model soil. A cube mesh is used for beam with size of 50 mm (Fig. 3).

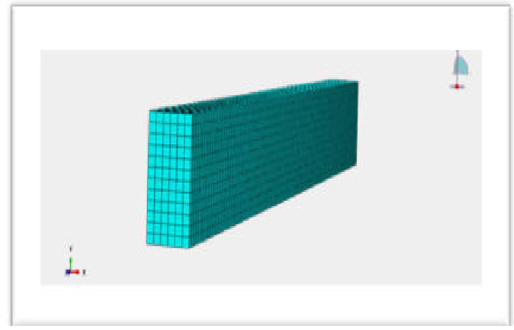

Fig. 3: Finate element mesh

RESULTS AND DISCUSSION

Case study: The numerical technique results acquired from the fortran computer program (finite difference analysis) and ABAQUS Software are to be validated with the closed form solutions. The following examples are considered.

Simply supported solid beam resting on springs and under a uniform load: A solid beam with properties indicated in Fig. 4 is considered herein. This problem was investigated by Bowles (1974), analytically. The results of vertical deformation, moment and shear are plotted with the exact solution as displayed in Fig. 5-7.

The figures demonstrate the acceptable congruence between them. The percentages of the divergence between the deformation, moment and shear for exact solution and the present study are 0.85, 0.85 and 1.7%, respectively.

Simply supported voided beam resting on springs and under a uniform load: A voided beam of properties shown in Fig. 8 is considered herein. The beam has a void dimension of width (b = 0.25 m) and depth (h = 0.4) giving a void percentage of 31% of cross sectional area. This problem was investigated by Bowles (1974), analytically. The results of vertical deformation, moment and shear are plotted with the exact solution as displayed in Fig. 9-11. The figures demonstrate the acceptable congruence between them. The percentages of the divergence between the deformation, moment and shear for exact solution and the present study are 1, 1.2 and 2%, respectively.

Parametric study: The study considers the effect of void percentage or void dimensions, void shape, vertical subgrade reaction, beam depth and type of loading, on the response of beams on springs. To study the influence of these factors or parameters, the same previous problem of beams will be considered in this study, under different types of loading and boundary conditions.

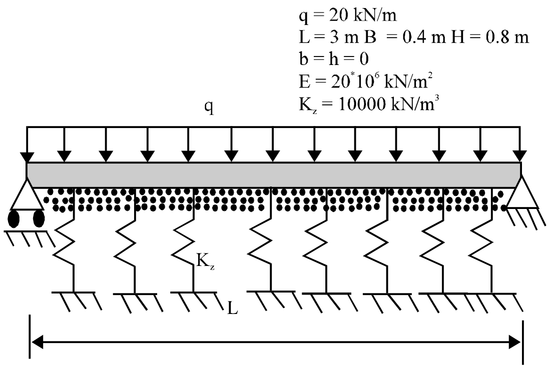

Fig. 4: Solid beam resting on springs with simple supports at ends

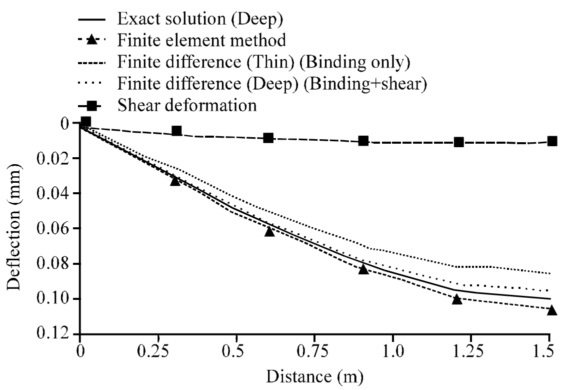

Fig. 5: Deflection variation for the solid beam resting on springs

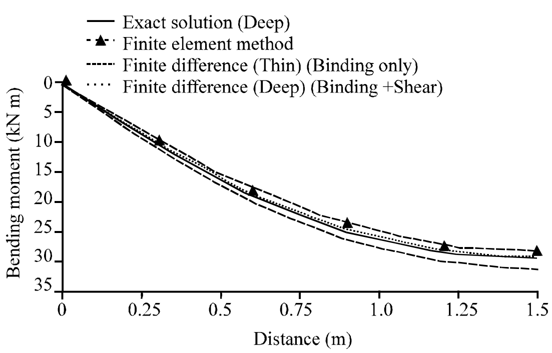

Fig. 6: Bending moment variation for the solid beam resting on springs

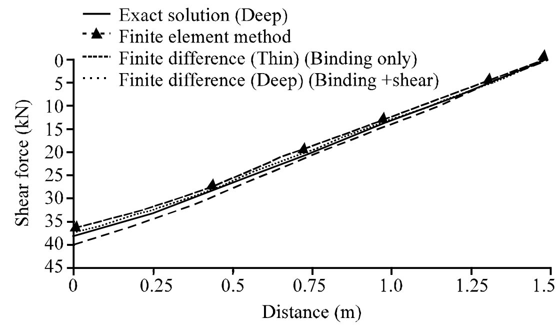

Fig. 7: Shear force variation for the solibeam resting on springs

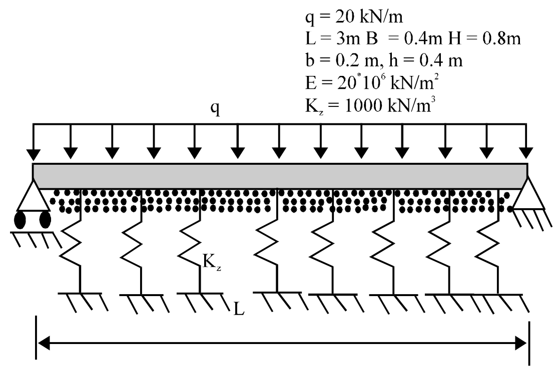

Fig. 8: Voided beam resting on springs with simple supports at ends

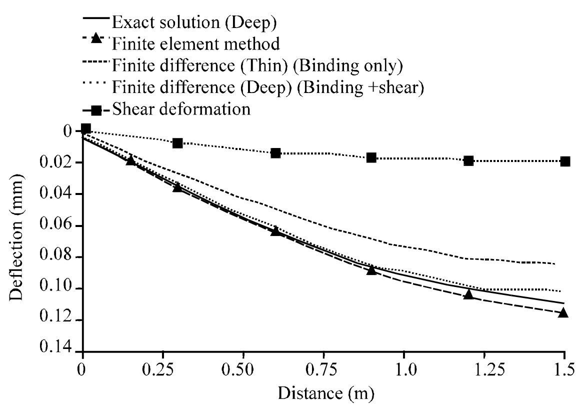

Fig. 9: Deflection variation for the voided beam resting on springs

Effect of void dimensions: The values of void dimensions 0.0-56% are considered for the same beam case. The 0% means b = h = 0.0 6% means b = 0.1 m and h = 0.2 m, 31% means b = 0.25 m and h = 0.4 m and 56 % means b = 0.3, h = 0.6 m (Fig. 12). Figure 13 shows the variation of void percentage with mid-span deflection of recognized that the enlargement of void percentage from beam. It was 0.0-56 % the displacement is increased by 22% for thin beam theory and 47% for deep beam theory because the shear deformation became larger. Figure 14 displays the variation of void percentage with beam moment. The beam moment will be reduced at a constant rate as the void percentage enlarges because the flexural rigidity of the beam decreased. Also, the difference

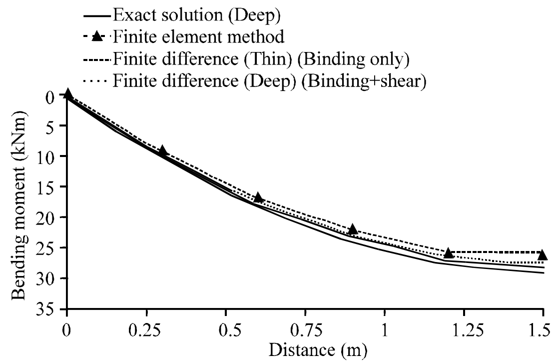

Fig. 10: Bending moment variation for the voided beam resting on springs

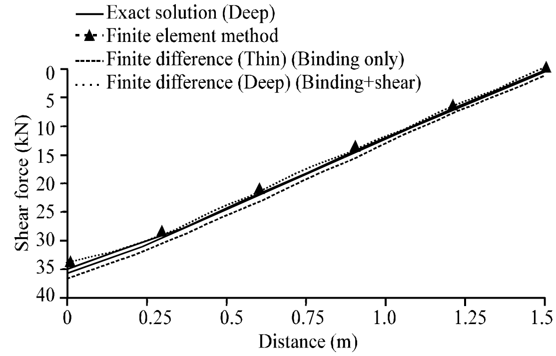

Fig. 11: Shear force variation for the voided beam reating on springs

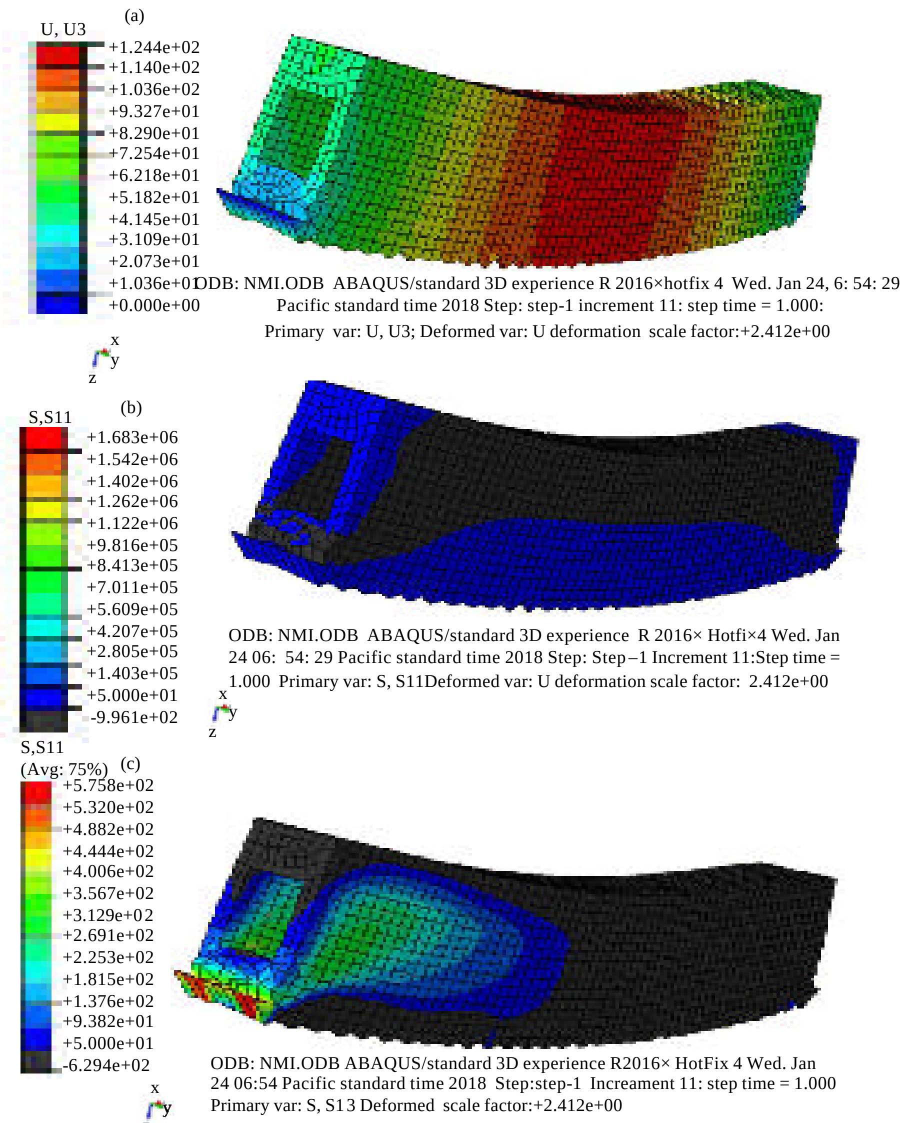

Fig. 12: Deflection, bending stress, shear stress obtained from finate element method

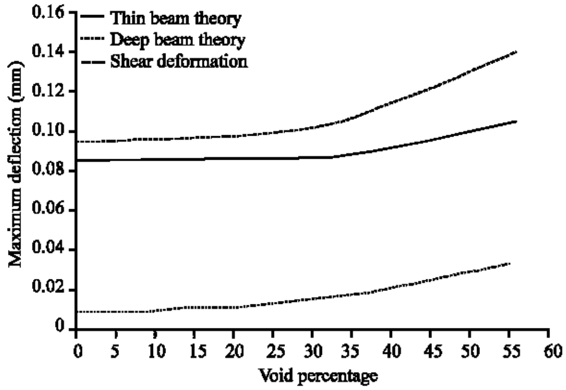

Fig. 13: Effects of void percentage on mid-span deflection

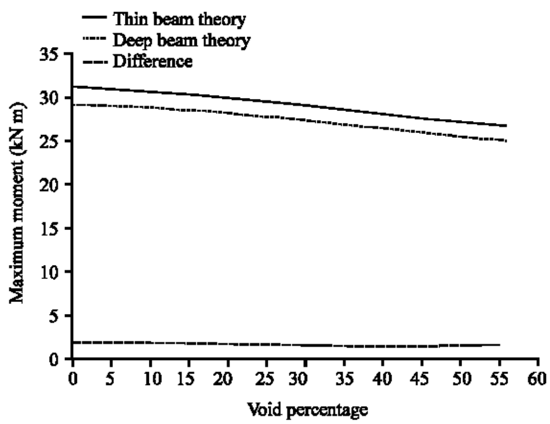

Fig. 14: Effects of void percentage on mind-apan moment

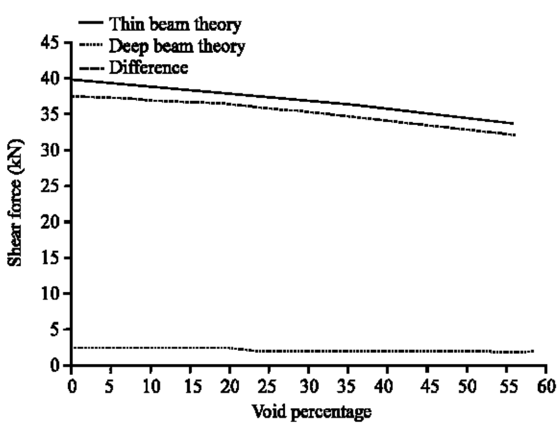

Fig. 15: Effects of void percentage on maximum shear

between deep beam and thin beam theories is decreased. It was recognized that the enlargement of the void percentage from 0.0-56% the moment is decreased by 15% for thin beam theory and 14% for deep beam theory and the difference between theories decreased by 21%. Figure 15 shows the variation of void percentage with maxi mum shearing force of the beam. The shear will be reduced at a constant rate as the void percentage enlarges because of reducing cross sectional area. Also, the difference between deep beam and thin beam theories is decreased. It was recognized that the enlargement of hollow percentage from 0.0-56% the beam shear force reduces by 15% for thin beam theory and 14% for deep beam theory and the difference between theories decreased by 23%.

Effect of void shape: To show the influence of void shape on the response of voided beam, circular and square shape are considered with constant void percentage of 31% as shown in Fig. 16. Table 1 shows the relationship between void shape and beam deflection, moment and shear. In this table, the deflection or vertical displacement will be reduced for the circular shape by 4% for thin beam theory and 5% for deep theory. The mid span moment will increased for circular void shape by 1.5% for thin beam theory and 5% for deep beam theory. The maxi mum shear force will increased for circular void shape 3% for thin beam theory and 2% for deep beam theory.

Effect of beam depth: Different values of beam depth are considered (keeping void percentage constant and equal to 31% herein. The values of beam depth are 0.4, 0.8 and 1.2 m. Figure 17 shows the variation of beam thickness or depth with beam deflection. It was recognized that the enlargement of beam depth from 0.4-1.2 m cause a reduction in deflection by 95% for thin beam theory and 85% for deep beam theory and the shear deformation increased by 120%. Figure 18 shows the relationship between beam depth and mid-span moment of the beam. The difference between deep beam and thin beam theories is decreased. It was recognized that the enlargement of beam thickness or depth from 0.4-1.2 m, cause an increase in moment by 34% for thin beam theory, 93% for deep beam theory and the difference between theories decreased by 80%. Figure 19 shows the relationship between beam depth and 15 maximum shear force of the beam.The difference between deep beam and thin beam theories is decreased. It was recognized that the enlargement of beam thickness or depth from 0.4-1.2 m cause an increase in shear by 32% for thin beam theory, 78% for deep beam theory and the difference between theories decreased by 83%.

Effect of subgrade reaction (K_z): The selected beam has void of dimensions b = 0.25 m, h = 0.4 m. The selected values of subgrade reaction are 0.0, 10000 and 50000 kN/m^3. Figure 20 displays the relationship between

Table 1: Effect of void shape on the behaviour

	Deflection (mm)		Moment (kN m)		Maximum shear force (kN)	
Void shape	Thin	Deep	Thin	Deep	Thin	Deep
Square	0.097	0.115	27.77	26.05	35.24	33.53
Circular	0.093	0.109	28.20	27.40	36.30	34.30

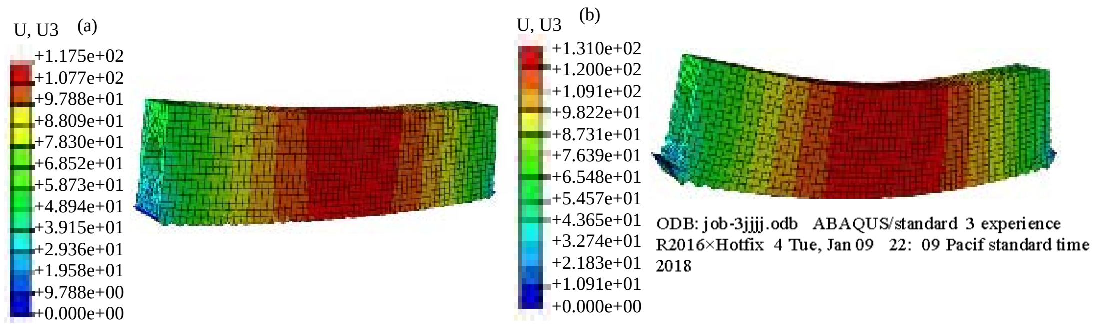

Fig. 16: Effect of void shape on deflection contours

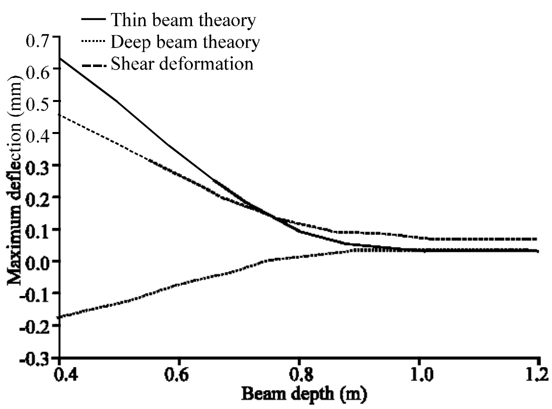

Fig. 17: Relationship between beam depth on mid-span deflection

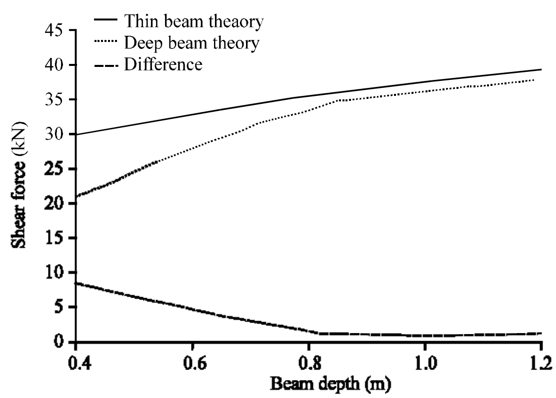

Fig. 19: Relationship between beam depth on maximum shear

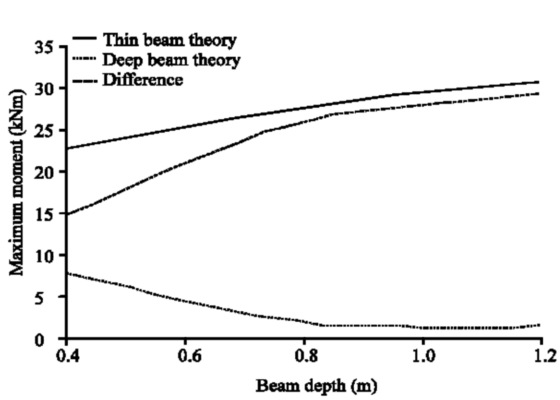

Fig. 18: Relationship between beam depth on mid-span moment

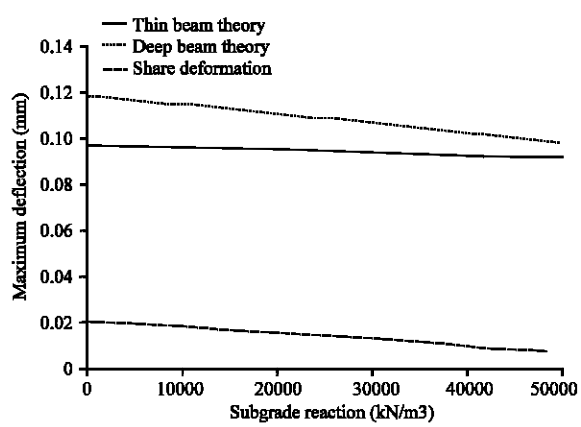

Fig. 20: Relationship between beam depth on maximum shear

the subgrade reactions and beam deflection. It was recognized that by the enlargement of subgrade reaction from 0.0-50000 kN/m^3cause a reduction in beam deflection by 6% for thin beam theory and 17% for deep beam theory

Table 2: Effect of loading type on behavior

	Deflection (mm)		Moment (kN m)		Maximum shear force (kN)	
Loading types	Thin	Deep	Thin	Deep	Thin	Deep
Uniform	0.097	0.115	27.77	26.05	35.24	33.53
Concentrated	0.143	0.174	50.00	44.26	36.57	33.20

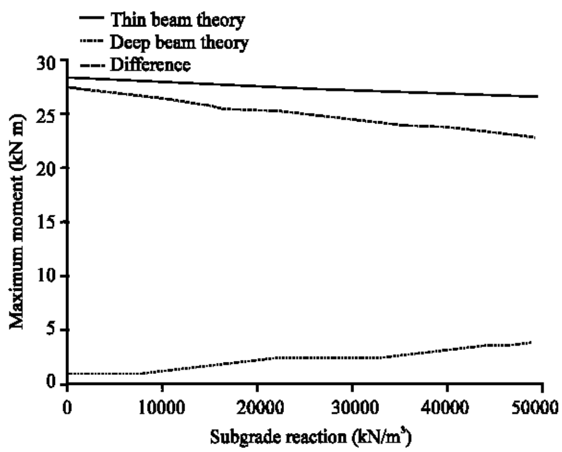

Fig. 21: The relationship between subgrade reaction and moment

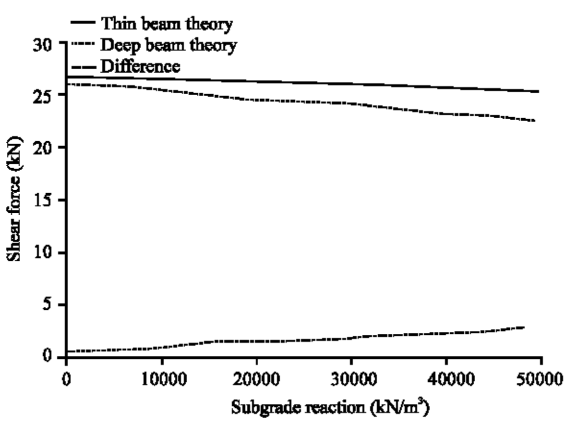

Fig. 22: The relationship between subgrade reaction and shear

and the shear deformation decreased by 67%. Figure 21 displays the relationship between the subgrade reactions and beam moment. The difference is increased between deep beam and thin beam theories. It was recognized that the enlargement of subgrade reaction 0.0-50000 kN/m^3 cause a reduction in beam moment by 6% for thin beam theory and 17% for deep beam theory and the difference between theories increased by 280%. Figure 22 displays the relationship between the subgrade reactions and beam shear. The difference is increased between deep beam and thin beam theories. It was recognized that the enlargement of subgrade reaction 0.0-50000 kN/m^3 cause a reduction in beam shear by 5% for thin beam theory, 14% for deep beam theory and the difference between theories increased by 297%.

Type of loading: To show the effect of type of loading, uniform and concentrated load are considered. The values of uniform load is (q = 20 kN/m) and the point load is P = 60 kN and voided beam is considered. Table 2 shows the effect of loading type on beam deflection, moment and shear. In this table, the mid-span deflection will increased for concentrated load case by 47% for thin beam theory and 51% for deep theory. The mid span moment will increased for concentrated load case by 80% for thin beam theory and 70% for deep beam theory. The maxi mum shear force will increased for concentrated load case by 4% for thin beam theory and 1% for deep beam theory.

CONCLUSION

Based on the obtained results the followings are the main conclusions: good corresponds are acquired by using numerical solutions for the voided beam on Winkler foundation and the divergence percentages with exact solution for displacements, moments and shears are about 5, 2 and 3%, respectively.

The effect of enlargement of depth in voided beam on deflections is recognized to be more considerable than internal stress resultant (moment and shear resistances). When thickness (h) changed from 0.4-1.2 m,the influence percentages are 95 and 33 for deflection and internal forces, respectively. The effect of void shape is found to be significant on deflection 5% but negligible on moment and shear 1.5% for voided beams.

The influence of increasing subgrade reaction on the response of the voided beam is found to be considerable.When Kz ranges from 0.0-50000 kN/m^3 the decrease percentage in deflection is 16%. The influence of increasing void dimension on vertical displacements is more. Considerable than on the internal forces and moments. When void percentage increased from (0-56%), the percentages are 22 15 and 14% for deflection, moment and shear, respectively. Also, the shear deformation effect increased with increasing void percentage. The effect of loading is found to be significant on deflection 45% and moment 75% but negligible on shear 4% for voided beams.

REFERENCES

Al-Azzawi, A., 2011. Analysis of Timoshenko beam resting on nonlinear compressional and frictional Winkler foundation. ARPN J. Eng. Appl. Sci., 6: 100-113.

Al-Azzawi, A.A., 2017. Free vibration of simply supported beam on elastic foundations. Alnahrain J. Eng. Sci., 20: 353-357.

Al-Jubory, A.A., 1992. Deep beams and thick plates under generalized loadings. MSc Thesis, Nahrian University, Baghdad, Iraq.

Amany, A. and D. Pasini, 2009. Material and shape selection for stiff beams under non-uniform flexure. Mater. Des., 30: 1110-1117.

Bowles, J.E., 1974. Analytical and Computer Methods in Foundation Engineering. McGraw-Hill, New York, ISBN: 9780070067530, Pages: 519.

Henry, F.D.C., 1986. The Design and Construction of Engineering Foundations. 2nd Edn., Chapman and Hall, London, New York, USA., ISBN:9780412125300, Pages: 1090.

Selvadurai, A.P.S., 1979. Elastic Analysis of Soil-Foundation Interaction. Elsevier, New York, USA., ISBN:9780444596284, Pages: 543.