Applying Computational Tools in Vector Calculus

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Abstract: Failure in Engineering Mathematics is a concern of the academician both locally and internationally. Various reason were identified for the poor performance in Engineering Mathematics and remedial steps were taken to overcome this issue. One of the step to improve the performance of students in Engineering Mathematics by including teaching via. computational tool which help the students to understand the subject better. Thus, the objective of this study is to investigate the recent usage of computational tools in the teaching and learning of vector calculus. The scope of this study is on the software's that are mostly used in the teaching and learning of vector calculus. MATLAB, Maple, Mathematica, GeoGebra and Sage are used in the teaching and learning of vector calculus. Students were excited learning vector calculus, motivated to learn the subjects, avoided wasting time in normal computation, agreed that software is a powerful tool for simulation and enhanced the comprehension of vector calculus.

Key words: Engineering Mathematics, vector calculus, computational tool, teaching and learning, performance, MATLAB

INTRODUCTION

Engineering is a branch of science and technology that combines knowledge, mathematics and experience to design an object or process. Most of the engineering problems involve mathematical equations and manipulation which require every engineering practitioner to master the mathematics knowledge and comprehension.

Some researchers have commented on poor achievement among Engineering Mathematics subjects (Felder and Brent, 2004; Gynnild *et al.*, 2005).

Lower performance by students not only because of the students itself but also due to the difficulty of the questions (Felder and Brent, 2004). Questions constructed not at par with what they have been taught. Questions should be constructed to meet the student's ability. Rather to embed higher-level of skills, instructors should include higher-level in learning objectives by giving some examples and further in assignments.

Evidence shown that mathematics performance of students in secondary school was related to the performance of basic calculus course in university (Gynnild *et al.*, 2005). Poor performers in mathematics in secondary school are also poor performers in the calculus course. This study further illustrates three major reasons for student's underachievement. The study approach, less effort and lacking of skills in the math entry are the factors for poor performance. This is classified as student's responsibility towards learning. On the other hand, lecturers and tutors should review the learning objectives for the target group. This includes the selection of the teaching content, estimated work load, ways of assessment and strategies for lecturing and tutoring.

There are many approaches used to enhance the process of teaching and learning in Engineering Mathematics, including the usage of computer and software. The use of mathematical software such as MATLAB, Maple and Mathematica, GeoGebra, Sage,

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Mathcad, C programming, Maxima softwares can stimulate student's interest in mathematical problems while reinforcing knowledge and understanding, especially, for engineering students. This study reviews the development on the usage of the software in the teaching of Engineering Mathematics.

MATERIALS AND METHODS

Application of computational tool in vector calculus: One of the fundamental subjects in every engineering course is vector calculus. It is an extension of calculus with the combination of vector properties. It is one of the most difficult subjects in Engineering Mathematics at the early years of engineering programme.

A study was conducted on the usage of MATLAB at the University of Queensland. In pre-2002, labs were conducted forth nightly at the University of Queensland where students underwent a self-study where they were provided with worksheets containing syntaxes and problems. In 2002, MATLAB labs were conducted weekly together with exercises and solutions. From 2003 onwards a learning model was launched in the University of Queensland for the first year syllabus using MATLAB. Eight hundred first year students participated in this project with a total of 50 lab classes. The learning model was revised for a 3 years time span. Courses in MATLAB include precalculus, calculus of one variable, calculus of many variables, Linear Algebra and ordinary differential equations. Students used a structured workbook which consists of several modules. Every module targets at one important concept. The module consists of mathematical concepts, examples and exercises based on MATLAB that was tested on 55 min duration. First of all, the students ran simple commands to familiarize and to gain confidence on using MATLAB. Later, they used GUIs from MATLAB to visualize advanced mathematical concepts in a computational manner. Finally, students developed their own coding before they ran the pre-written programs. Student survey reported that, students were positive about incorporating software in the mathematics curriculum. GUI experiments successfully visualize difficult concepts. Although, students worked in pairs during the implementation of the learning model, they preferred to work individually. The learning model has improved the learning outcomes over the years. However, the software packages faced some resistance from students due to three main reasons. Firstly, the software MATLAB was rather numerical than symbolic. Secondly, students could not see the link between laboratory exercises and course material. Lastly, it was difficult to write and execute the program.

Maple graphing tool was used in teaching three-dimensional calculus (Cook, 2006). It was very challenging even though lecturers spent ample time on their illustrations to get the students to understand the underlying concept. Yet, students still faced some difficulties to grasp the theory. Visulaization technique was used for understanding three dimensional calculus. To overcome this problem, example of eight function was written in calculus III plots using Maple (Cook, 2006). These examples were extended from classroom lessons to class projects. The examples include plotting Riemann sums for the functions of two variables, lagrange multipliers, line integrals and plotting secant vectors. The aim of these examples was not to make the students proficient in computational mathematics but rather to provide them with a better understanding on what was being computed.

Instrumentation process of student's learning using the command driven Maple during a semester calculus course was studied. Assigments were aimed to improve the depth of student's conceptual understanding of calculus and usage of Maple as a problem-solving tool. A collective case study was implemented to examine student's proficiency in using the computer algebra system. Before distributing the assignments, two class sessions were facilitated in the computer lab at the beginning of the term to assist students in learning Maple. Individual tutorials assisted students in learning the commands and techniques of Maple. Maple was installed in all the computer labs on campus. Students were given one group assignment and five individual assignments every two weeks in order to help them learn Maple. The duration for each assignment was 1 week. All activities such as learning the software, interviews and completing assignments were video taped and analyzed. Since, the tutorial exercises were directed at the usage of each command, students barely had the chance to experience it themselves. Besides the fact that students did not spend most of their time to experiment this software they also did not have access to it in their laptops. To realize the benefit of the computer algebra system like Maple, it was suggested that Maple must be integrated into the course where Maple should be frequently used in class, regular homework assignments integrated with Maple and certain assessment from the software. Assessment includes verifying answers, correcting errors and analyzing results.

To improve the effectiveness of the study process, two components of educational technologies was applied (Kovacheva, 2007). Technologies used in the study process include the computer algebra system. Technologies of the study process include planning, organizing, carrying out and evaluating the entire process. Maple was incorporated in calculus, Linear Algebra, ordinary differential equations, numerical methods and statistics. Each subject consists of seven laboratory exercises. Although, the work method developed student's skills for team work, it is thought that

many students with one computer was a condition not deemed not ideal. Laboratory exercises using the Maple system was aimed to extend student's knowledge acquired in lectures and expose them to work with a ready-made system. Students were exposed to numerical methods of problem-solving using Maple. Laboratory exercises enhanced the depth comprehension of a subject and increased student's motivation.

Maple was employed as a multimedia tool in classrooms. Students who enrolled in calculus courses for engineering majors were required to learn and use Maple in their assignments. Maple's user friendly interface and the help sessions developed student's proficiency to operate commands and codes. The complex theory and concepts of calculus can be solved by visualizing the problems. Traditionally, it was difficult for students to learn Riemann sum for $y = f(x)$ on the interval $[a, b]$. Firstly, they need to sketch the region. Then by finding the area of the inscribed and circumscribed rectangles they can approximate the area under the curve. Only then, they will be exposed to the content of definite integral. By generating figures to plot graphs and calculate approximate area using various numbers of rectangles using Maple, students were able to visualize the insight of the Riemann sum concept. These multimedia tools excited the students and also prevented them from losing sight of the subjects. Maple turned to be significant in student's learning as it improves lecturers and student's problem solving skills via. technology.

The effects of using GeoGebra in differential calculus was explored (Dikovic, 2009). The lectures and exercises were conducted in the traditional method. Later, the experimental group which consisted of 31 students worked in a computer laboratory. The students were freely allowed to have group discussion. The teacher became a coordinator. Help was provided by the teacher on the student-to-student basis. The traditional teaching method was substituted by group work, individual research and investigations. Before the session began, a pre-test consisted of ten simple tasks was conducted and at the end of the session a post-test was carried out. The average score for the pre-test was M = 22.95 and for the post-test it was M = 51.64. The results proved that GeoGebra is a powerful tool for the visualization and simulation of important topics of differential calculus. Furthermore, the computing tool impacted positively on the understanding and knowledge of differential calculus among the experimental group members.

The development of procedural and conceptual knowledge in the teaching-learning double integral via. Maple 12 was investigated. A total of fourty four students from the second semester from Rajee Teacher Training University were chosen randomly for this study. They were divided into the experimental group and control group. Control group learnt using the traditional way

while the experimental group used Maple 12 as the mode of study. There were six sessions with one and half hours allocated for both groups. A pre-test and post-test were administered for both groups. The pre-test was aiming to identify student's previously acquired knowledge. There were no significant differences on the student's pre-test score for both groups. The post-test, however, showed that the experimental group had a better conceptual and procedural understanding than the control group. Ten students were interviewed from the experimental group to analyze student's depth of knowledge and to gather their opinion related to Maple 12 software. Most students agreed that Maple 12 was helpful in visualizing the basic concepts of multivariable calculus. Students demonstrated higher level of confidence, positive attitude towards mathematics and performed better result.

An efficient mathematics teaching-learning tool for integral calculus courses which included power point slides with maple animation and interactive Maplets with Maple worksheets were created. The teaching-learning tool aided the instructors to conduct lectures. It also helped the students in self-planned learning and self-assessment. Line integrals, surface integrals and volume integrals were taught using Maplets. The fundamental step by step concepts were learned through Maplets. Each Maplet covers three main functions which include some input functions to define a problem with help sections, a graphic visualization function and output functions for demonstrating the results. Maplets which supports mathematics concepts graphically, leads the students to achieve higher level of logical analytical thinking. In order to measure student's achievements, pre and post tests were conducted. Both pre and post-tests consist of basic concepts and application-based questions. Basic concepts were in the form of writing test. Maplets with Maple worksheets were used to check student's answers for homework. With a quick solution and enhanced visualization interactive Maplets and Maple worksheets, it reinforced student's conceptual understanding of integral calculus.

Sage, a free open source software used in the teaching of advanced calculus (Botana *et al.*, 2014). A live DVD of Sage was developed which contains thirty sage interacting as the complementary material for first year students from the School of Forestry Engineering from the University of Vigo, Spain. Students were encouraged to bring along their laptops in classes. Half of the students brought laptops while the other half shared with their friends. Sharing students were found to learn interface faster. Students were assigned tasks and they needed to solve some problems using interacts and returned as Sage worksheets. Students commented that these interacts avoided time-wasting in uninteresting mechanical computations. They preferred to concentrate in concepts instead.

The processes of learning in a computer algebra system environment for college students learning calculus through a case study involving three students was investigated (Meagher, 2005). The Rotman Model of mathematical reasoning was applied as the macro framework to view the place of technology in the computer algebra system classroom and to view the journey of each student across the quarter. The Pirie-Kieren Model for the growth of mathematical understanding was applied as the micro-framework to examine specific learning episodes as they occurred in the classroom. Students who took their first calculus class had the option either to attend the traditional lecture class or choose to take a computer-based class called calculus and mathematica. The calculus and mathematica class of 10 weeks per term used materials developed at The Ohio State University and The University of Illinois (Urbana-Champaign). Students from the calculus and mathematica course took up self-directed learning by using a CD as an interactive textbook. Lecturers and teaching assistants in the laboratory only responded to student's questions rather than to the lecture. One module was given for a week, whereby students worked in a group of three. The four sections were allocated in each module: basics, tutorial, give it a try and literacy. Students worked individually for the 1st two sections to learn and practice the basic concepts while in the other two sections the students worked in a group of three. The third section was done with the help of mathematica where students submitted their research to get grades. For the last section, the submission for grading was also required but without the aid of mathematica. The group's discussion audio tape and video capture of computer screens were the primary source in this study. Observations, interviews, grounded survey document analyses were undertaken in this qualitative research. Although, some students were very positive about the computer algebra system, many who were familiar with the graphic calculator claimed that the computer algebra system was too much for calculation. Case study revealed that mathematica was seen to be a burden in calculation and a student could not concentrate on the concept of calculus. Although, two other students agreed with the first part, they did see "true" mathematica as residing. Students developed resistance because there was no direct training given in using the software. Some students expressed considerable frustration to learn mathematica and calculus. However, students did have to take some time to get used to mathematica and eventually understand that mathematica was more powerful in calculation compared to graphics calculator.

RESULTS AND DISCUSSION

This research has identified the recent researchers from the period of 2005 till 2014 who has implemented

Table 1: Incorporating computational tool in vector calculus

Subject	Researchers	Computational tools
Vector calculus	Tonkes *et al.*	MATLAB
	Meagher (2005)	Mathematica
	Cook (2006)	Maple
	Synder in 2006	Maple
	Kovacheva (2007)	Maple
	Suanmali	Maple
	Dikovic (2009)	GeoGebra
	Godarzi *et al.*	Maple 12
	Noinang *et al.*	Maple
	Botana *et al.* (2014)	Sage

computational tools in teaching Engineering Mathematics. Table 1 shows the particular subject, researcher, respective computational tool used and the years the researchers conducted their studies.

MATLAB, Maple, GeoGebra, Sage and Mathematica are widely used in teaching vector calculus in recent years. Recent, Cook (2006), Kovacheva (2007), Dikovic (2009), Botana *et al.* (2014) and Meagher (2005) has successfully implemented computational tool in vector calculus subject.

Students commented positive on incorporating software in teaching vector calculus subject. Students were excited as computational tool engaged them in learning and motivated them (Kovacheva, 2007). Students avoided wasting time in normal computation (Botana *et al.*, 2014). Students also agreed that computational tool as a powerful source for visualization (Cook, 2006; Dikovic, 2009). Software tools are considered as powerful tool for simulation (Dikovic, 2009). It also helped students to enhance the comprehension of vector calculus subjects (Kovacheva, 2007; Meagher, 2005).

CONCLUSION

There are many applications of integrated computer software in the teaching and learning of Engineering Mathematics. Types of software used depend on the subject learning. For vector calculus, more researchers applied Maple in their teaching as compared to MATLAB. Mathematica, on the other hand is also suitable for all the engineering subjects. Some researchers combined both of the existing softwares with their own customized software. In some applications, problems were reported on the attitude level of students towards technology and time taken to acquire the knowledge and skills. For simple mathematical problems and calculations, the usage of software can be helpful in the understanding of complex concept and solving tedious calculation.

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