

## Optimum Slope for Solar Insolation on a Surface Tilted Toward the Equator in Libya

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**Abstract:** The determination of optimum slope angles for solar collectors plays a significant role in the efficient utilization of solar energy. Monthly, seasonally and yearly slope and optimum slope angles for five locations, Tripoli, Shahat, Hun, Kufra and Ghat where determined using measured data of monthly mean daily global solar radiation on a horizontal surface. Over a period of 7 years, it was found that the annual optimum slope angle is approximately equal to the location latitude,  $\phi$  while for the winter season it is  $\phi+18$  for the coastal cities and  $\phi+21$  for cities in the desert and for summer season it is  $\phi-23$  for coastal cities and  $\phi-19$  for desert cities, respectively.

**Key words:** Solar radiation, optimum slope angle, coastal cities, utilization, Libya

### INTRODUCTION

The amount of insolation on a terrestrial surface at a given location depends on the slope of the surface. Although, it is, well known that a surface with always normal to the ray of the sun would receive maximum amount of solar radiation and the solar trackers have been developed for tracking the solar path and adjusting the device tilt angle accordingly for the purpose of maximizing solar radiation. Although, tracking is the most efficient means of solar radiation collection, it is costly, energy demanding for its operation and does not suit all solar devices (Markvart, 1994). Thus, emphasis is laid on the solar device optimal tilt angle to achieve maximum insolation on sloped surfaces. This solar device slope angle may be permanently fixed or adjusted monthly or seasonally (Liu and Jordan, 1962). For determination of the optimum slope angle, it is rather important to have the beam and diffuse components of total radiation incident on a horizontal surface. Once these components are determined, they can be transposed over sloped surfaces and hence, the short as well as the long term performances of tilted can be estimated.

Designers of solar systems that use photovoltaic devices (Elhassan *et al.*, 2011) and storage system need information about the solar radiation intercepted by a sloped surface. Unfortunately, insolation data on tilted surfaces in most parts of the world are not available. The generally available data are those of daily global radiation on horizontal surface. The estimation of insolation on a sloped surface and determination of optimum angle,

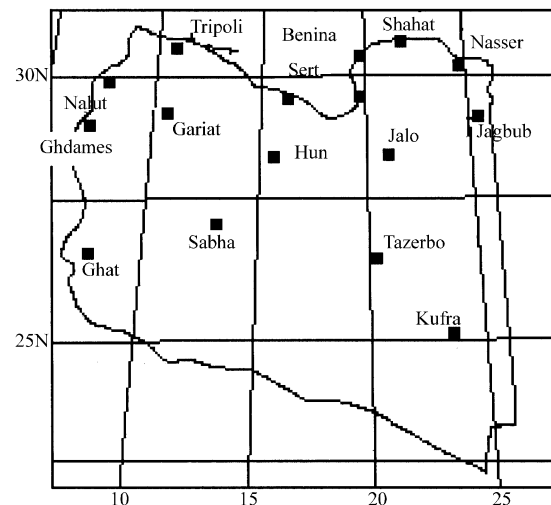


Fig. 1: The meteorological stations in Libya and the studied locations

need the separation of diffuse component from the horizontal radiation. A number of workers (Chang, 2010; Abdullah *et al.*, 1989; Liu and Jordan, 1962) have used this method. The most widely used method is that by Liu and Jordan (1962).

In this study, researchers present some results of estimation of solar radiation incident on tilted South facing surface in Tripoli (Lat. 32.97°), Shahat (Lat. 32.82°), Hun (Lat. 29.13°), Kufra (Lat. 24.6°), Ghat (Lat. 24.95°) Libya as shown on the map of Libya (Fig. 1). Researchers also present results of monthly, seasonal and annual

optimum tilt angle for each location. It is worth to the principle advantage of the work on optimizing the angle of slope is that the solar energy becomes controllable and uniform from period to period.

## MATERIALS AND METHODS

The insolation incident upon any horizontal surface consists of two components; the beam radiation,  $H_b$  and diffuse sky radiation,  $H_d$  including the part which is reflected by the ground and the surroundings. The monthly mean daily global radiation on a surface tilted at slope angle,  $\beta$  can be estimated from (Liu and Jordan, 1962):

$$H_t = (\bar{H} - \bar{H}_d)R_b + 0.5\bar{H}_d(1 + \cos\beta) + 0.5\rho\bar{H}(1 - \cos\beta) \quad (1)$$

Where:

$\bar{H}$  and  $\bar{H}_d$  = The monthly mean daily global and diffuse radiation on a horizontal surface

$\rho$  = Ground albedo taken as 0.20

The first term in Eq. 1 represents the beam radiation component and the factor  $R_b$  relates the beam radiation incident upon a tilted surface and that on horizontal surface. However for a surface facing towards the equator:

$$R_b = \frac{\cos\lambda \cos\delta \sin\omega_{st} + (\pi/180)\omega_{st} \sin\lambda \sin\delta}{\cos\phi \cos\delta \sin\omega_s + (\pi/180)\sin\phi \sin\delta} \quad (2)$$

For the Northern hemisphere:

$$\lambda = (\phi - \beta)$$

Where:

$\phi$  = Latitude and positive

$\beta$  = Inclination angle and negative

For the Southern hemisphere:

$$\lambda = (\phi + \beta)$$

Where:

$\phi$  = Latitude and negative

$\delta$  = Positive

$\omega_{st}$  is the sunset hour angle for the titled surface for mean day of the month which is given by:

$$\omega_{st} = \min \left[ \cos^{-1}(-\tan\phi \tan\beta), \cos^{-1}(-\tan\lambda \tan\delta) \right] \quad (3)$$

min here means the smaller of the two terms in the square bracket. The second term of Eq. 1 represents the diffuse radiation component on tilted surface while the third term represents the portion of ground reflection received by the surface.

Recently many models of varying complexity have been developed as the basis for calculating solar radiation received by tilted surfaces. The differences are largely in the way that the diffuse terms are treated. Some of researchers used isotropic models (Liu and Jordan, 1962; Badescu, 2002; Koronakis, 1986; Tian *et al.*, 2001) for estimating of diffusd component and others the anisotropic modles (Hay, 1979; Reindl *et al.*, 1990; Skartveit and Asle, 1986; Steven and Unsworth, 1980) however, it is inferred that the isotropic model is able to estimate  $H_t$  more accurate than the anisotropic one (El-Sebaai *et al.*, 2010). In the current research, the monthly average daily global irradiation on horizontal surface and sunshine duration for the five locations under study are taken from reference (Abughres, 1985). Since, measurements of monthly average daily diffuse radiation  $H_d$  are not available, it was estimated, using the regression developed by reference (Iqbal, 1978). A computer code was developed under Watfor and Excel Microsoft Software and was used for calculating the tilt and optimum tilt angles for the aforementioned locations using Eq. 1-3.

## RESULTS AND DISCUSSION

Figure 2 and 3 show the average daily total solar radiation at of the Tripoli which is a coastal station, on a

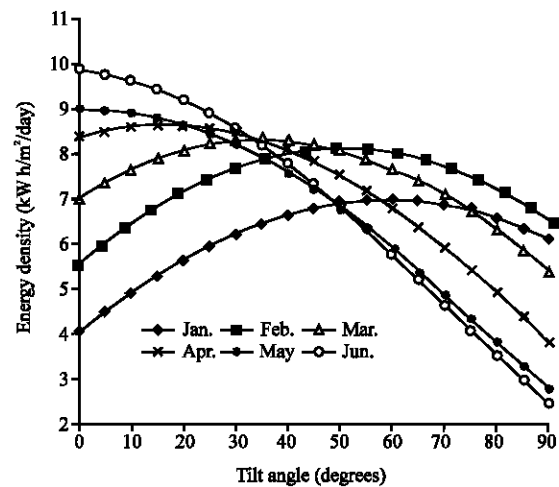


Fig. 2: Monthly mean daily global irradiation as a function of slope for the city of Tripoli from Jan. to Jun. ( $\phi = 32.97^\circ\text{N}$ )

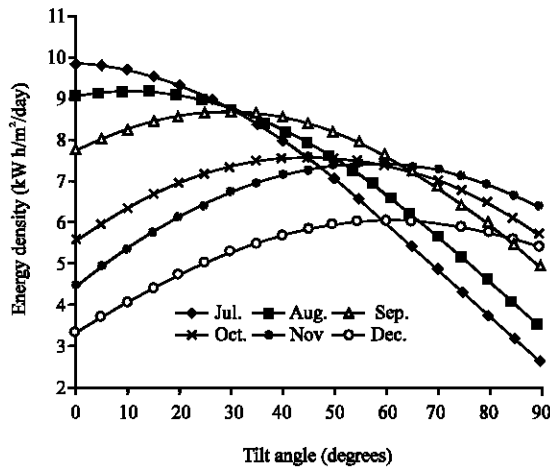


Fig. 3: Monthly average daily global irradiation as a function of slope from Jul. to Dec. for the city of Tripoli ( $\phi = 32.97^\circ\text{N}$ )

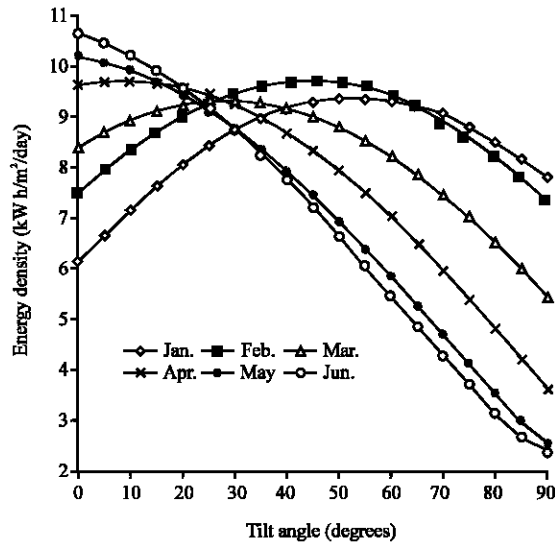


Fig. 4: Monthly mean daily global irradiation as a function of slope from Jan. to Jun. for the city of Kufra ( $\phi = 24.26^\circ\text{N}$ )

South facing surface as the angle of tilt is varied from 0-90° in steps of 10°. It is clear from these graphs that a unique  $\beta_{\text{opt}}$  exists for each month of the year for which the solar radiation is at a peak for the given month. Figure 4 and 5 show the average daily total solar radiation of the Kufra station which is in the desert region. Similar trend has been observed for all other stations selected under present study. Graphs for other stations have not been shown in order to avoid repetition. Table 1 summarizes the optimum tilt angles  $\beta_{\text{opt}}$  for 12 months of the year for the five stations under the present study.

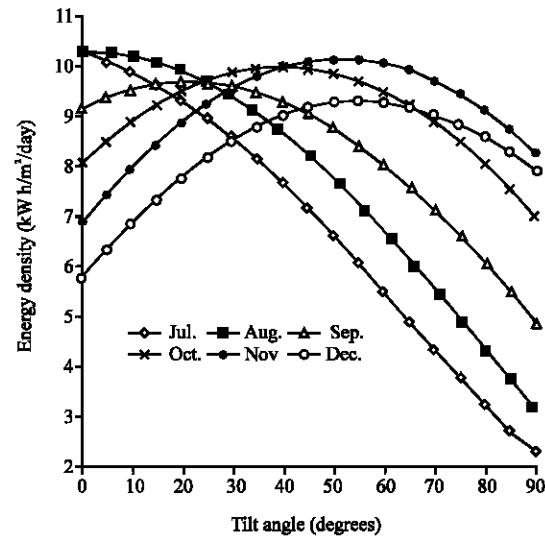


Fig. 5: Monthly mean daily global irradiation as a function of slope from Jul. to Dec. for the city of Kufra ( $\phi = 24.26^\circ\text{N}$ )

Table 1: The monthly optimum slope angles (in degree) for the studied locations

| Months | Tripoli | Shahat | Hun | Ghat | Kufra |
|--------|---------|--------|-----|------|-------|
| Jan.   | 59      | 57     | 58  | 54   | 53    |
| Feb.   | 51      | 48     | 49  | 45   | 44    |
| Mar.   | 35      | 34     | 33  | 29   | 29    |
| Apr.   | 17      | 17     | 14  | 10   | 9     |
| May    | 1       | 1      | 0   | 0    | 0     |
| June   | 0       | 0      | 0   | 0    | 0     |
| July   | 0       | 0      | 0   | 0    | 0     |
| Aug.   | 11      | 11     | 8   | 4    | 3     |
| Sept.  | 29      | 28     | 27  | 22   | 22    |
| Oct.   | 46      | 45     | 45  | 41   | 40    |
| Nov.   | 57      | 55     | 56  | 52   | 52    |
| Dec.   | 61      | 59     | 60  | 56   | 55    |

It can be observed that the monthly optimum slope angles vary widely ( $0^\circ$ - $61^\circ$ ) while for the winter season (Nov. to Feb.) vary from ( $44^\circ$ - $61^\circ$ ) and almost horizontal ( $0^\circ$ - $11^\circ$ ) for summer season (May to August) for the other months of the year it varies from  $30^\circ$ - $46^\circ$ . It is clear that the monthly mean daily insolation reaches a maximum ( $10.82 \text{ kW h/m}^2/\text{day}$ ) in Jun. ( $\beta_{\text{opt}} = 0$ ) in Hun station (desert station) and a minimum ( $2.62 \text{ kW h/m}^2/\text{day}$ ) in December ( $\beta_{\text{opt}} = 0$ ) in Shahat station (coastal city). Figure 6 shows the variation of  $\beta_{\text{opt}}$  of the month of the year for the five locations.

The seasonal and annual optimum slope angles for the five stations were determined using Fig. 7. In Fig. 7, the irradiance received by a flat surface for various slopes is drawn and the area under each curve which represents the total amount of irradiation for the whole year is obtained and the maximum annual solar radiation was taken as the yearly optimum tilt angle.

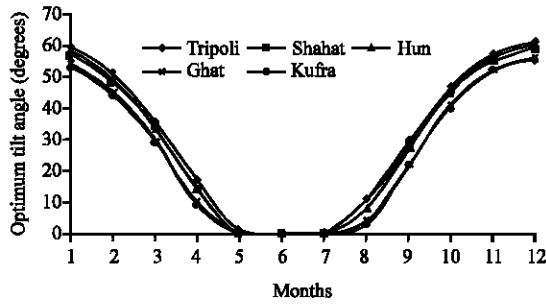


Fig. 6: Optimum average tilt angle for each month of the year at five different stations of Libya

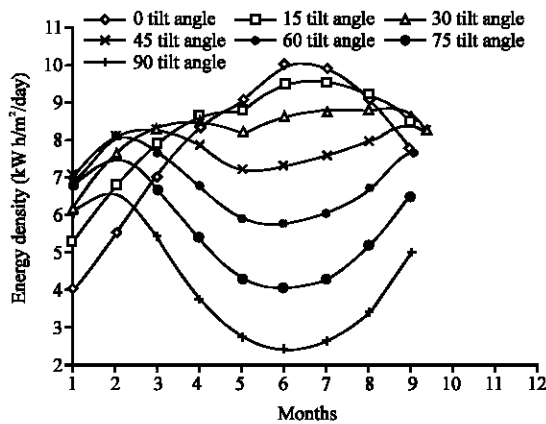


Fig. 7: Monthly mean daily global irradiation, for different slope angles for Tripoli

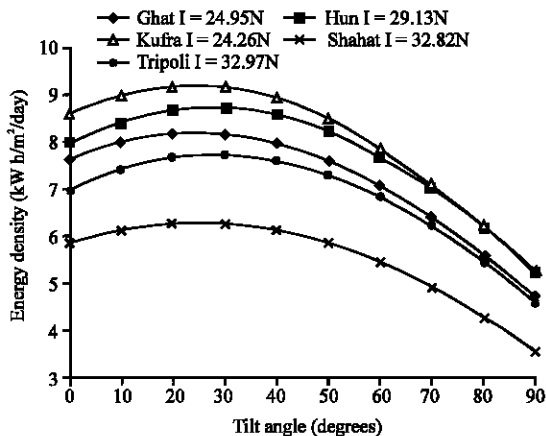


Fig. 8: Yearly global radiation on inclined surfaces for the five locations

Figure 8 shows the dependence of solar radiation on annual slope angle for the five locations and the maximum slope angles  $\beta_{opt}$  are corded in Table 2. It can be observed that annual  $\beta_{opt}$  for coastal cities (Tripoli and Shahat) is higher than that of desert cities (Kufra and Ghat).

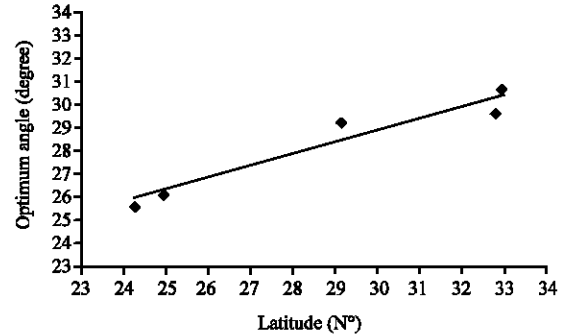


Fig. 9: Correlation between optimum angle and latitude for five studied locations

Table 2: The yearly optimum slope angles for the five stations in Libya

| City                 | Kufra | Ghat  | Hun   | Shahat | Tripoli |
|----------------------|-------|-------|-------|--------|---------|
| Lat.                 | 24.26 | 24.95 | 29.13 | 32.82  | 32.97   |
| Yearly optimum angle | 25.58 | 26.08 | 29.17 | 29.58  | 30.58   |

Mathematical expression correlating the yearly optimum slopes  $\beta_{opt}$  with latitudes  $\phi$  are presented in Table 2 for 5 station given by:

$$\beta_{opt} = 13.243 + 0.5189\phi \quad (4)$$

$$R^2 = 0.9367$$

This mathematical expression predicts the yearly optimum slope angles for any location within the range of latitudes covered in this research (Fig. 9).

## CONCLUSION

Using Liu and Jordan isotropic diffuse model, it was possible to estimate the solar radiation incident on a tilted surface facing South in five locations in Libya using the measured data over a period of 7 years. The tilt and optimum tilt angles for the locations were calculated on monthly seasonally and yearly basis. The results of the study concludes that:

- For maximum annual radiation collection, the tilt angle should be equal to latitude  $\phi$
- For maximum winter radiation energy collection, the tilt angle of  $\phi + 18$  for the coastal cities and  $\phi + 21$  is recommended for desert cities
- For maximum summer radiation energy collection the tilt angle of  $\phi - 23$  for the coastal cities and  $\phi - 19$  is recommended for desert cities

Finally, the availability of tilt and optimum and optimum tilt angles for any location enables designers of solar systems to make the at most utilization of solar

radiation at such location which makes the solar energy more economical. Such data can also be used as a controlling means of solar systems.

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