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The Features of Resonance Stress Scatter in Turbine Wheels with a Weak Connectivity of Blade Vibrations

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Abstract: Forced vibrations of rotor wheels with nonidentical blades and a weak connectivity of vibrations are considered in the study. It is shown that in some frequency ranges such wheels lose the main characteristics peculiar to systems with a small deviation from rotation symmetry. It leads to a qualitative modification of character of their vibrations caused byinfluence of exciting harmonics. The mechanism of a resonance stressscatter in them becomes different.

Key words: Turbine wheel, disturbance of rotation symmetry, distortion of eigen vibrations, a connectivity of vibrations, mechanism

INTRODUCTION

Turbine wheels nominally represent the elasticsystems having rotation symmetry with an order S equal to number of blades. The symmetry is disturbed in the real constructions because of availability of manufacturing tolerances (Castanier and Pierre, 2006; Pierre and Murthy, 1992). It is commonly supposed that the noted disturbance is small even if the detuning of partial eigen frequencies of blades under the first bending mode makes the significant value (Klauke *et al*., 2009). In the case of a small deviation from rotation symmetry the resonance in rotor wheels is realized in the form of superposition of vibrations by paired modes with the segregated multiple frequencies (Ivanov, 1969; Tobias and Arnold, 1957; Feiner and Griffin, 2002). It leads to that the maximum dynamic stresses defined in the similar points of blades may essentially differ (a scattering of resonance stresses) and are achieved on various angular velocities of rotor spinning.

Computational studies have allowed to determine existence of the strong distortion of eigen modes in full-scale rotor wheels in the case of presence in the latter a weak connectivity of blade vibrations. The distorted eigen mode of the engine NK-12 compressor wheelis shown in Fig. 1. In Fig. 1, $\bar{q}_{\Sigma}$ is the summarized amplitude of the linear displacements in a direction of a model wheelaxis; N the blade number; digits on the diagram match to values of some characteristic partial frequencies of blades.

For the vibrations under the modeonly some blades of the model wheel have essential amplitudes. The maximum displacements of all remaining blades are in

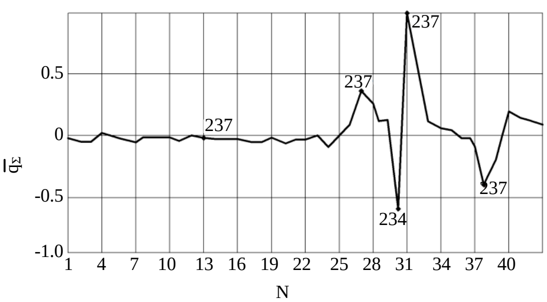

Fig. 1: The vibrations mode of the engine NK-12 compressor 4th stage rotor wheel

appreciable. The eigen modes of rotor wheels having such degree of distortion are named as localized. The localized modes can not be characterized by a number of nodal diameters. They are not orthogonal to one of exciting harmonics (Rivas-Guerra and Mignolet, 2003). Their presence leads to a qualitative modification of forced vibrations of rotor wheels in the frequency ranges where such modes exist. Investigation of the problem is the purpose of the present research.

MATERIALS AND METHODS

The procedure: Studies were performed on the model rotor wheel by means of the computer system ANSYS. The wheel has 16 blades and is made of a round plate with width of h = 7 mm. Its sizes are shown on Fig. 2.

In the computational and experimental studies the wheel was rigidly clamped at its centre by the area limited by a round in diameter of 42 mm. Figure 2 shows the fragment of a spectrum of eigen frequencies of the model wheel for modes without nodal rounds. The spectrum is defined by computation and experimentally.

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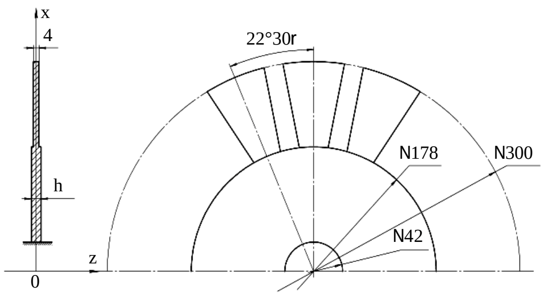

Fig. 2: The model rotor wheel design

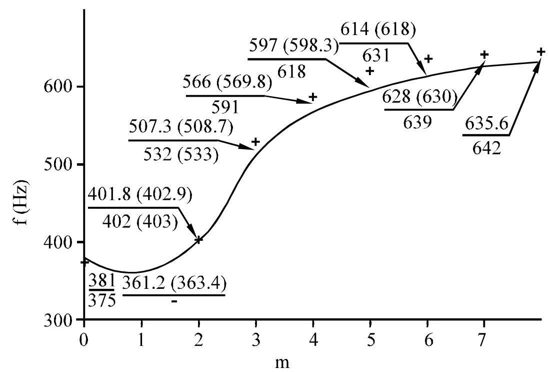

Fig. 3: The spectrum of eigen frequencies of an initial design: -: Computation, +: experiment; $f_m^{(1)}$ $(f_m^{(2)})/f_{me}^{(1)}$ $(f_{me}^{(2)})$ the eigen frequencies of paired modes defined by computation ($f_m^{(1)}$, $f_m^{(2)}$) and experimentally ($f_{me}^{(1)}$, $f_{me}^{(2)}$)

Table 1: Partial eigen frequencies of blades of the model rotor wheel

The blade number	f (Hz)
1	675
2	670
3	684
4	663
5	676
6	688
7	687
8	684
9	684
10	675
11	670
12	679
13	686
14	687
15	684
16	673

Table 1 shows partial frequencies of blades at vibrations under the first bending mode. They are defined at rigid fixation of the disk by the entire area by means of thick plates and large vices.

Non identity of blades was simulated by arranging of pointwise masses over their rim. It was possible to detect experimentally a separation of the multiple frequencies only for modes with two and three nodal diameters.

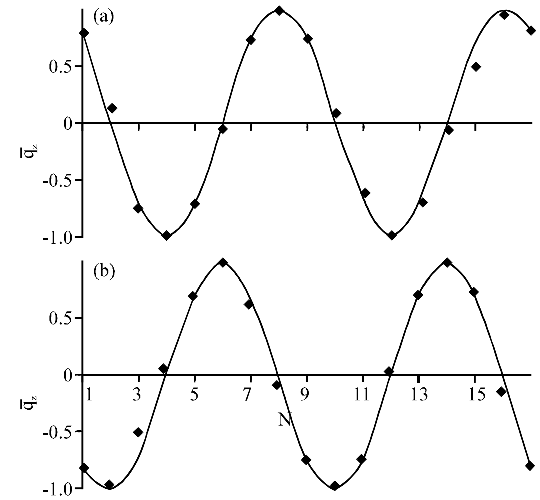

Fig. 4: Scatter of displacements of peripheral points of blades for vibrations of the initial model under paired modes with m = 2; —: Computation, ◆: experiment

Figure 3 shows computed and experimental scatter of displacements of blades over the rim for paired modes with two nodal diameters. Experimental displacements are defined by interferograms.

When holding studies the change of a connectivity of blade vibrations in the model wheel was carried out by means of increasing of the disk width h. Excitation of vibrations in the model wheel was executed by a loading in the form of concentrated forces by similar nodes making harmonic motions. Vibration forces amplitudes and phases were set such that the loading as a whole represents a train of back progressive waves (Fig. 4).

RESULTS

Change of the disk width has allowed to vary over a wide range a connectivity of blade vibrations down to its practical disappearance. As a result of the studies carried out it was defined that the connectivity of vibrations essentially influences distortion of eigen modes. The distortion increases with decreasing ina connectivity. Fragments of eigen mode distortion process for the model wheel are shown in Fig. 5 and 6. Number 1 on them means the initial mode of vibrations and numbers 2, 3 and 4 denote various stages of its distortion. Distortion of a spectrum for the nominal model wheel eigen frequencies in the case of increasing of its disk thickness is shown in Fig. 7.

Figures 8 and 9 the same eigen modes as in Fig. 5 and 6 but in the form expanded into a Fourier series. Strongly distorted modes contain in the capacity of components practically all the harmonics covered by a degree of symmetry which amplitudes are comparable by their value.

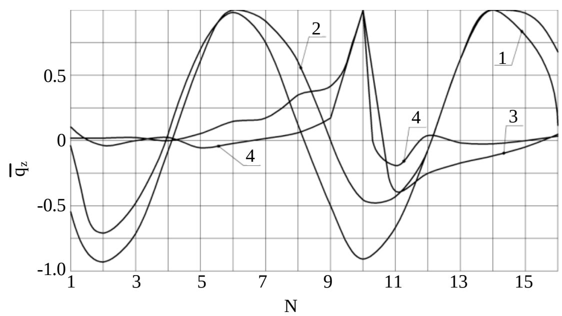

Fig. 5: Change of vibrations mode with m = 2 at increasing of disk thickness of the model rotor wheel

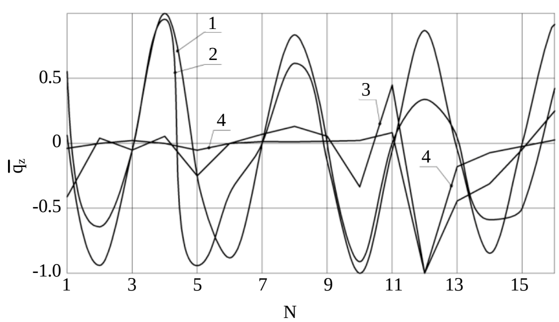

Fig. 6: Change of vibrations mode with m = 4 at increasing of disk thickness of the model rotor wheel

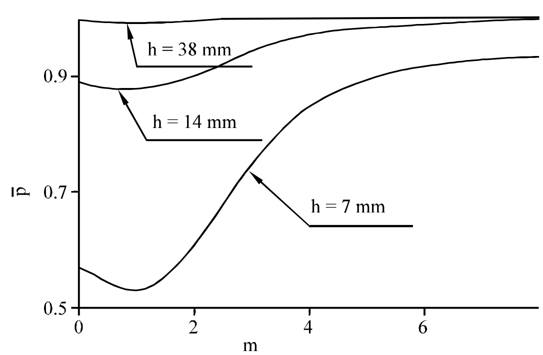

Fig. 7: Distortion of a spectrum of eigen frequencies of the model in the capacity of a rotation symmetrical system at increasing the disk thickness

Such modes are generally not orthogonal to any exciting harmonic. Therefore, vibrations of the rotor wheel on them can happen from effect of any such exciting harmonic.

Figure 10 shows scatter of the relative maximum ranges of stresses ($\bar{\sigma}$) under excitation of vibrations of an initial design by 3rd harmonic. Digits in the figure match to those frequencies on which each of blades achieve the maximum vibration amplitude.

The analysis of the data presented in the figure shows that the frequency range in which the maximum ranges of stress are achieved on each blade makes 1.4 Hz.

As one of the paired modes with three nodal diameters has the frequency $f_3^{(1)} = 507.3$ Hz and another has the frequency $f_3^{(2)} = 508.7$ Hz in this case it is completely defined by super position of paired modes. The dot line in the figure designates the scatter of resonance stresses obtained by means of dynamic factor which was used to define ranges of stresses at vibrations on each of paired modes. It was accepted that the distorted paired modes are excited with equal maximum amplitudes and with a phase shift in a time by a quarter of a period. It has allowed us to search for super position of vibrations of paired modes in the form of:

$$\tilde{\sigma}_{\Sigma k} = \lambda_0 \sqrt{\left[1-\left(\frac{p}{p_m^{(1)}}\right)^2\right]^2} + \frac{\delta^2}{\pi^2}\left(\frac{p}{p_m^{(1)}}\right)^2 \sigma_k^{(1)} \cos\left(pt-\gamma^{(1)}\right) + \lambda_0 \sqrt{\left[1-\left(\frac{p}{p_m^{(2)}}\right)^2\right]^2} + \frac{\delta^2}{\pi^2}\left(\frac{p}{p_m^{(2)}}\right)^2 \sigma_k^{(2)} \sin\left(pt-\gamma^{(2)}\right) \quad (1)$$

Where:

$$\gamma^{(1)} = \mathrm{arctg}\left(\frac{\delta}{\pi}\right)^2 \frac{1}{1-\left(p/p_m^{(1)}\right)^2},\ \gamma^{(2)} = \mathrm{arctg}\left(\frac{\delta}{\pi}\right)^2 \frac{1}{1-\left(p/p_m^{(2)}\right)^2}$$

Where:

- λ_0 = The coefficient defining an excitation level of paired modes. Its value was defined from a requirement that the maximum combined stresses $\tilde{\sigma}_{\Sigma k max}$ in ablade row are equal to unity
- $\sigma_k^{(1)}$, $\sigma_k^{(2)}$ = Ranges of stressesin k-th blade at vibrations under paired modes
- δ = A logarithmic decrement of vibrations
- $\sigma_m^{(1)}$, $\sigma_m^{(2)}$ = Eigenfrequencies of paired modes

Expression (Eq. 1) considers excitation by m_{ex}-th harmonics of only paired modes. Figure 10 shows good enough coincidence of resonance stress amplitudes scatter calculated by both methods.

In the case of excitation by 3rd harmonic of a model wheel with a disk width h = 14 mm the maximum ranges of stress are achieved within wider resonance range than it is defined by eigen frequencies of matching paired modes. Scatter of resonance amplitudes of stresses for this case is shown in Fig. 11.

Eigen frequencies of paired modes in this case make accordingly $f_3^{(1)} = 645$ Hz and $f_3^{(2)} = 648$ Hz. The specified

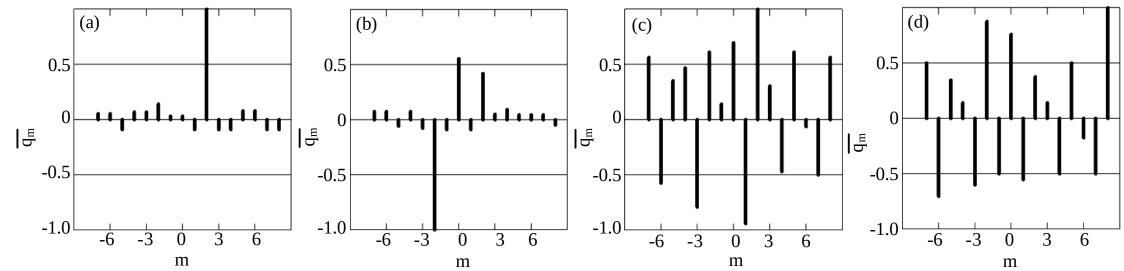

Fig. 8: Change of a harmonic composition of the vibration mode with m = 2 at increasing of a disk thickness of the model rotor wheel: a) h = 7 mm; b) h = 14 mm; c) h = 24 mm and d) h = 38 mm

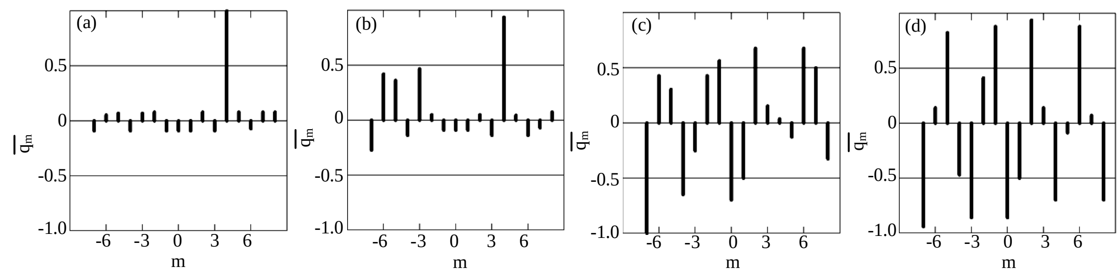

Fig. 9: Change of a harmonic composition of the vibration mode with m = 4 at increasing of a disk thickness of the model rotor wheel: a) h = 7 mm; b) h = 14 mm; c) h = 24 mm and d) h = 38 mm

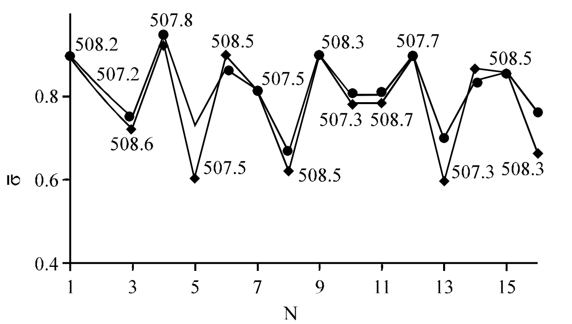

Fig. 10: The scatter of resonance stresses arising from effect on initial model of 3rd exciting harmonic

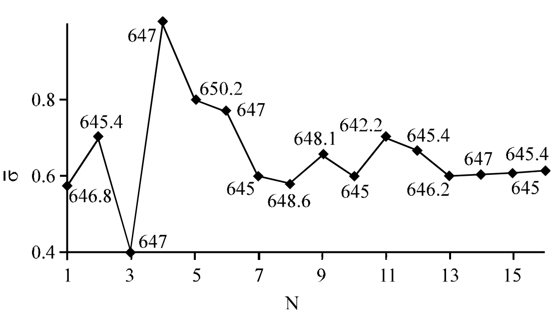

Fig. 11: The scatter of resonance stresses arising from effect on the model with a disk width h = 14 mm of 3rd exciting harmonic

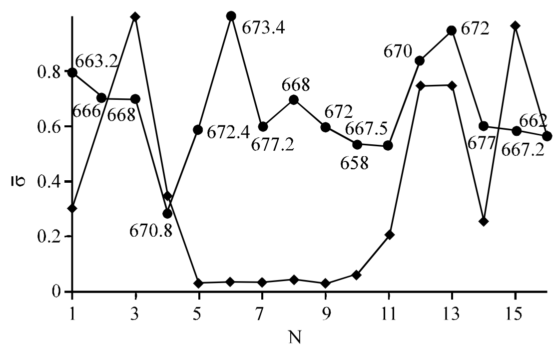

Fig. 12: The scatter of resonance stresses arising from effect on the model with a disk width of h = 14 mm of 6th exciting harmonic

expansion has happened because of a blade under the number 5 in which the maximum dynamic stresses have been reached at frequency of 650.2 Hz. It is possible to explain the obtained result after reviewing one of modes with number of the nodal diameters m = 4 having a lower eigen frequency. This mode is presented in Fig. 6 and 9. It is considerably distorted and is not orthogonal to 3rd harmonic. In particular, its Fourier-series expansion shows that the amplitude of a harmonic component with m = 3 in is essential. Besides, the mode with m = 4 has almost maximum vibration amplitude on 5th blade. All this has led to that decreasing in a vibration amplitude of 5th blade

Table 2: Eigen frequencies of the model with a disk width of 14 mm

m	0	1	2	3	4	5	6	7	8
$f_m^{(1)}$	606.0	592.4	612.0	645.0	657.1	663.5	667.4	672.2	677.0
$f_m^{(2)}$	-	597.0	615.8	648.0	660.0	665.6	669.9	676.4	-

after transiting ofthe resonance frequency $f_3^{(2)}$ was cancelled by its growth because of proximity to the resonance frequency $f_4^{(1)}$. As a result decreasing in amplitude of the summarized vibrations of 5th blade has begun only after transiting the frequency f = 650.2 Hz. Thus, expansion of a resonance range has happened due to excitation by 3rd harmonic of vibrations under the mode with m = 4.

In the case of influence on themodel wheel with a disk width of h = 14 mm of the sixth exciting harmonic the resonance frequency range in which the maximum dynamic stresses are achievedin blades, makes 19 Hz. Being positionedin frequencies 658-677 Hz, it is bound to superposition of 9 modes (m = 4-8). Their eigenfrequencies are shown in Table 2.

It is necessary to note that for strongly distorted modes the number of nodal diameters is the conventional concept. It is given for the name of the mode and specifies only that eigenvibrationof an initial design from which it was converted when decreasing in a connectivity of vibrations. Scatter of the maximum ranges of stresses is shown in Fig. 12. In the same figure, the dot line plots the scatter of the maximum stresses obtained by means of expression 1. Comparison of the presentedscatters showshow much strongly they differ.

If to reduce a connectivity even more for example to makewidth of a disk h = 28 mm, then how computational studies have shown, starting with influence on a wheel of 4th exciting harmonic, the resonance range is formed as a result of superposition of all 16 lowest modes with m from 0 up to 8.

DISCUSSION

Thus, in the case of a weak connectivity the resonance of the rotor wheel from influence of any exciting harmonic occurs in some frequency range in which this harmonic approximately equally gives rise to vibrations over the whole group of modes of one set.

CONCLUSION

The width of a resonance range depends on precision of blades manufacturing and can be significant. Also the mechanism of formation of resonance stresses scatter changes accordingly. The scatter in this case becomes a result of super position of vibrations of a large group of sequentially excited localized modes without dependence from proximity of their eigen frequencies.

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