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Blade Wave Finite Element

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Abstract: This study considers the construction of an effective beam finite element for the blade as a the component of Cyclic Symmetric System. The resulting equations of the element are implemented as a computer program in Fortran. The comparison of the blades natural frequencies obtained by calculation and experimentally showed a good agreement.

Key words: Gas turbine engine, blade, finite element, modal analysis, efficiency

INTRODUCTION

The task of ensuring the vibration reliability concerning the rotor systems of turbomachines and their elements is accompanied by the implementation of a large amount of computational studies for the set of design models. There is a theoretical base and modern computational techniques that allow one to predict accurately the dynamic characteristics of complex structures. In this case the three-dimensional large-scale models are usually used. They consist of the universal high-order finite elements. However, the performance of blade wheel optimization work using such models is time consuming and the rotor system optimization for such models is an almost impossible task.

The reduction of time and expenses to ensure the reliability of rotor systems and their components, due to the development of the computer programs that have high rates of speed and accuracy is an important scientific and applied problem.

Currently, Samara State Aerospace University develops a specialized software for the turbomachinery rotor system dynamics study. The increase of computing performance with the due accuracy provision is achieved due to the mathematical apparatus which is based on the deep theoretical concepts about the dynamic phenomena accompanying the gas turbine engine operation in particular the properties of the Cyclic Symmetric System spectrum. Besides the use of two-level finite element models reduces the operation time. The first level models allow without too much geometric detailing to transform it into a variant close to an optimal one. The using of the second level models allows to perform the some final optimizing calculations. The final optimization stage which should be performed at full design detailing, demands the attraction of sofware calculation packages (ANSYS, NASTRAN, ABACUS and others) which are widely used in various fields of science and technology (Falaleev *et al*., 2006; Ulanov and Ponomarev, 2009; Polyakov *et al*., 2008; Klebanov *et al*., 2014; Igolkin *et al*., 2012; Vinogradov, 2014; Tikhonov *et al*., 2011; Nemov *et al*., 2014).

The purpose of this study is to develop an effective finite element of a blade (first level model) for the developed software package to study the turbomachinery rotor system dynamics.

MATHEMATICAL MODEL OF BLADE WAVE FINITE ELEMENT

Substantiation of blade beam model selection: The blade finite element is developed by applying the pre-twisted beam theory created by Vorobyov and Schorr (1983). The element has a variable cross-section and the coupled bending-torsion deformations.

The feasibility of using the Beam Model is related to the following circumstances. At first, the pre-twisted beam theory developed by Schorr-Vorobyov allows with a sufficient accuracy to determine the lowest natural frequencies of the blade. At second, the modes shape of blade wheel which have a practical interest the character of the force distribution along the disk rim width does not affect the values of natural frequencies (Vorobyov *et al*., 1989). At third, the beam models have the highest calculation performance due to their relative simplicity, compared with the shell and solid models.

The analysis of modern researchers (Al-Bedoor and Al-Qaisia, 2005; Lim *et al*., 2009; Sinha, 2005; Avramov *et al*., 2007; Temis and Karaban, 2001; Yao *et al*., 2014; Volovoi *et al*., 2001; Malcolm and Laird, 2007; Wang *et al*., 2014) publications shows that the beam model of a blade is widely used as it allows to obtain reliable results despite its simplicity. The Beam Model restriction is the inability to study the shell modes of blade vibrations. For these purposes, the shell or solid models are used.

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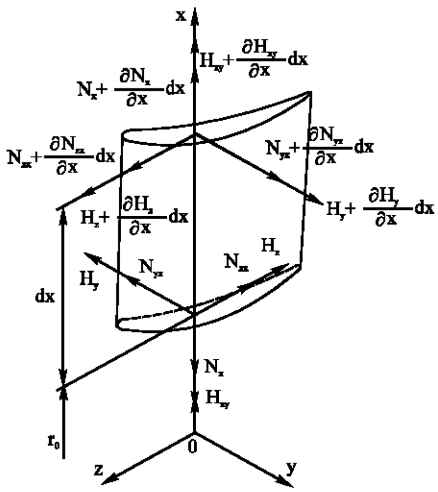


Fig. 1: Internal forces and the moments in an infinitely small blade element

Initial equations of an infinitely small blade element: Figure 1 shows the internal forces and the moments acting on an infinitely small blade element.

In accordance with the pre-twisted beam theory developed by Schorr-Vorobyov the equilibrium equations of the basic blade element in the coordinate system oxyz may be written as:

$$\frac{\partial N_x}{\partial x} - \rho \cdot F \cdot \frac{\partial^2 U}{\partial t^2} + \rho \cdot F \cdot \omega^2 \cdot \left(W + r_0\right) = 0 \qquad (1)$$

$$\frac{\partial N_{zx}}{\partial x} - \rho \cdot F \cdot \frac{\partial^2 W}{\partial t^2} = 0 \qquad (2)$$

$$\frac{\partial N_{yx}}{\partial x} - \rho \cdot F \cdot \frac{\partial^2 V}{\partial t^2} + \rho \cdot F \cdot \omega^2 \cdot V = 0 \qquad (3)$$

$$\frac{\partial H_y}{\partial x} + N_{zx} - N_x \cdot \alpha_y - \rho \cdot \left(J_y \cdot \frac{\partial^2 \alpha_y}{\partial t^2} + J_{zy} \cdot \frac{\partial^2 \alpha_z}{\partial t^2} \right) - \rho \cdot \omega^2 \cdot C \cdot \alpha_y - \rho \cdot J_y \cdot \frac{\partial^2 \alpha_y}{\partial t^2} = 0 \qquad (4)$$

$$\frac{\partial H_z}{\partial x} - N_{yx} - N_x \cdot \alpha_z - \rho \cdot \left(J_z \cdot \frac{\partial^2 \alpha_z}{\partial t^2} + J_{zy} \cdot \frac{\partial^2 \alpha_y}{\partial t^2} \right) - \rho \cdot \omega^2 \cdot C \cdot \alpha_z - \rho \cdot J_z \cdot \frac{\partial^2 \alpha_z}{\partial t^2} = 0 \qquad (5)$$

$$\frac{\partial H_{xy}}{\partial x} + N_{yx} \cdot \alpha_y - N_{zx} \cdot \alpha_z - \rho \cdot J_p \cdot \frac{\partial^2 \alpha_x}{\partial t^2} + \rho \cdot \omega^2 \cdot \left(J_y - J_z\right) \cdot \alpha_x = 0 \qquad (6)$$

Where:

N_x, N_{yx}, N_{zx}	=	Internal forces
H_y, H_z, H_{xy}	=	Internal moments
U, V, W	=	Translational movements along the axes x, y, z, respectively
$\alpha_{x, y, z}$	=	Angular displacements relative to the axes x, y, z, respectively
ρ	=	Blade material density
F	=	Blade cross section area
J_y, J_z, J_{zy}, J_p	=	Centrifugal and polar axial moments of inertia for the blade cross section
ω	=	Angular velocity of rotor rotation
t	=	Time

C parameter is determined by Eq. 7:

$$C = \int_{x}^{x_{CM}} F \cdot x \, dx \qquad (7)$$

x_{CM} gravity center coordinate of peripheral cross-section. According to the pre-twisted beam theory of Schorr-Vorobyov the internal forces and moments in a blade are associated with the deformations by the following relationships:

$$N_{yx} = \frac{k^2 \cdot E \cdot F}{2 \cdot (1+\mu)} \cdot \left(\frac{\partial V}{\partial x} + \alpha_z \right) \qquad (8)$$

$$N_{zx} = \frac{k^2 \cdot E \cdot F}{2 \cdot (1+\mu)} \cdot \left(\frac{\partial W}{\partial x} - \alpha_y \right) \qquad (9)$$

$$H_y = E \cdot J_y \cdot \frac{\partial \alpha_y}{\partial x} - E \cdot J_{yz} \cdot \frac{\partial \alpha_z}{\partial x} - E \cdot \dot{\alpha} \cdot J_{yr} \cdot \frac{\partial \alpha_x}{\partial x} \qquad (10)$$

$$H_z = -E \cdot J_{yz} \cdot \frac{\partial \alpha_y}{\partial x} + E \cdot J_z \cdot \frac{\partial \alpha_z}{\partial x} + E \cdot \dot{\alpha} \cdot J_{zr} \cdot \frac{\partial \alpha_x}{\partial x} \qquad (11)$$

$$H_{xy} = -E \cdot \dot{\alpha} \cdot J_{yr} \cdot \frac{\partial \alpha_y}{\partial x} + E \cdot \dot{\alpha} \cdot J_{zr} \cdot \frac{\partial \alpha_z}{\partial x} + \left(G \cdot J_k + E \cdot \dot{\alpha}^2 \cdot J_{4r} \right) \cdot \frac{\partial \alpha_x}{\partial x} \qquad (12)$$

Where:

E	=	Blade material elasticity modulus
G	=	Blade material shear modulus
μ	=	Blade material Poisson's ratio
k	=	Coefficient taking into account the law of shear stress distribution
$\dot{\alpha}$	=	Blade twist

$$J_{yr} = \int_F (y^2 + z^2) \cdot z\, dF,\ J_{zr} = \int_F (y^2 + z^2) \cdot y\, dF$$

polar-axial moments of inertia of the blade cross section; J_k geometric torsional stiffness of a non twisted beam with the same cross-sectional profile as that of the twisted one:

$$J_{4r} = \int_F (y^2 + z^2)^2\, dF$$

second polar moment of inertia. Let's exclude the longitudinal blade oscillations from further consideration due to their low practical significance.

Accounting for the cyclic symmetry properties and the transition to the row of blades: The accounting of the proper motion features spectra of the cyclically symmetric systems (Ivanov, 1983; Ermakov *et al.*, 1988) allows us to consider the elementary blade element as a discrete cyclically symmetrical set of isolated blade parts of the same type. That is to turn to the dynamics consideration of all impeller blades. In this case, at the vibrations according to any eigenmodes the circumferential distribution $\tilde{\chi}$ of the force and displacement waves will depends on the uniform discrete harmonic law:

$$\tilde{\chi} = \chi \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{13}$$

Where:

- χ = Complex amplitude of a component force or displacement
- I = Iimaginary unit
- S = No. of impeller blades
- m = No. of deformation waves in the circumferential direction
- K = Blade number
- p = Natural frequency

With the help of Eq. 13 forces and displacements are presented in the form of circumferential waves:

$$V = q_y \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{14}$$

$$W = q_z \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{15}$$

$$\alpha_x = \beta_x \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{16}$$

$$\alpha_y = \beta_y \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{17}$$

$$\alpha_z = \beta_z \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{18}$$

$$N_{yx} = Q_y \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{19}$$

$$N_{zx} = Q_z \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{20}$$

$$H_{xy} = M_x \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{21}$$

$$H_y = M_y \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{22}$$

$$H_z = M_z \cdot e^{i \cdot \frac{2 \cdot \pi}{S} \cdot m \cdot K} \cdot e^{i \cdot p \cdot t} \tag{23}$$

Where:

- q_y, q_z = Complex amplitudes of translational movement waves
- $\beta_x, \beta_y, \beta_z$ = Complex amplitudes of angular movement waves
- Q_y, Q_z = Complex amplitudes of internal forces waves
- M_x, M_y, M_z = Complex amplitudes of internal moment waves

The substitution of the expressions Eq. 14-23 in the Eq. 1-12, allows us to write these equations for the amplitudes of the force and displacement waves. This approach significantly reduces the amount of computations.

It should be noted that the record form Eq. 13 takes into account the relative circumferential displacement between the waves of the various components of forces and displacements.

System of differential equations: Turning to the amplitudes of the force and displacement waves in the Eq. 1-12 with the expressions (Eq. 14-23) the following system of differential equations is obtained:

$$\begin{Bmatrix} \frac{dQ_z}{dx} \\ \frac{dM_y}{dx} \\ \frac{dQ_y}{dx} \\ \frac{dM_z}{dx} \\ \frac{dM_x}{dx} \end{Bmatrix} = [A] \begin{Bmatrix} q_z \\ \beta_y \\ q_y \\ \beta_z \\ \beta_x \end{Bmatrix} + [B] \begin{Bmatrix} \frac{dq_z}{dx} \\ \frac{d\beta_y}{dx} \\ \frac{dq_y}{dx} \\ \frac{d\beta_z}{dx} \\ \frac{d\beta_x}{dx} \end{Bmatrix} \tag{24}$$

where, [A], [B] equation coefficient matrix. The system of differential Eq. 24 describes the dynamic properties of the impeller blade row elementary part.

Development of blade wave finite element: In order to develop the equation system of the blade wave finite element the Galerkin method is used (Norrie and de Vries, 1978):

$$\int_{x_1}^{x_2}[F]\begin{Bmatrix}\frac{dQ_z}{dx}\\ \frac{dM_y}{dx}\\ \frac{dQ_y}{dx}\\ \frac{dM_z}{dx}\\ \frac{dM_x}{dx}\end{Bmatrix}dx=\int_{x_1}^{x_2}[F]\left([A]\begin{Bmatrix}q_z\\ \beta_y\\ q_y\\ \beta_z\\ \beta_x\end{Bmatrix}+[B]\begin{Bmatrix}\frac{dq_z}{dx}\\ \frac{d\beta_y}{dx}\\ \frac{dq_y}{dx}\\ \frac{d\beta_z}{dx}\\ \frac{d\beta_x}{dx}\end{Bmatrix}\right)dx \tag{25}$$

where, x_1, x_2 finite element limits. The development of a shape functions matrix [F] is based on the assumption of a linear variation concerning the wave amplitudes of displacements along its length.

The integration of the Eq. 25 over the domain element, provides a system of algebraic equations for the complex amplitudes of internal forces and moments. In matrix form, this system is a matrix equation of the wave dynamic stiffness for the blade finite element:

$$\{N\}=[H_{10\times10}]\{q\} \tag{26}$$

Where:

$$[H]=[K]+\omega^2[C]-p^2[M]$$

the wave dynamic stiffness matrix which establishes the connection between the complex amplitude vector of nodal reactions waves {N}, that occur at the boundaries of the element and the vector of the complex amplitudes for the displacement waves {q} of these boundaries:

- [K] = Static stiffness matrix
- [C] = Matrix of rotor rotation influence on the blade element stiffness
- [M] = Mass matrix

In this study, the analytical expressions were obtained for the matrix coefficients [K], [C], [M]. These expressions allow to calculate the matrix coefficients with a high speed and accuracy without taking into account the numerical integration.

In order to develop a global matrix equation of wave dynamic stiffness for the blade the classic approach of finite element coupling is used. The conditions of equilibrium and displacement compatibility are considered in the nodes.

EVALUATION OF ACCURACY AND SPEED FOR BLADE WAVE FINITE ELEMENT

The WFEBlade program was developed on the basis of the wave finite element mathematical model to calculate the natural frequencies and the eigenmodes of unevenly heated rotating blades. The program was developed by using the Fortran programming language.

In order to assess the accuracy and performance of the developed element the comparative studies of the dynamic characteristics were carried out concerning the two compressor blades as shown in Fig. 2. The first had a cantilevered fixing, the second one had a hinged fixing.

The calculation results using WFEBlade program were compared with the results of a modal analysis in ANSYS and with experimental data.

In ANSYS the detailed three-dimensional models of the blades with SOLID185 elements were developed. To ensure an acceptable accuracy of calculations the model scale in ANSYS was about 210000 degrees of freedom. WFEBlade Model contained approximately 400 degrees of freedom. The task was run at the workstation with 16GB of RAM, using the one core of CPU Intel i7 2600K.

The results of blade oscillation studies according to different eigenmodes were presented in Table 1 and 2. The

Fig. 2: Explored compressor blades

Table 1: Natural frequencies of cantilever blade

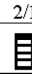
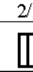
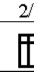
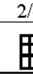
	Frequency (Hz)			
Eigen modes				
WFEBlade	1039	3312	6206	7615
ANSYS	1016	3375	6011	7354
Experiment	1028	3423	-	-
Difference from ANSYS/Exp. (%)	2/1	2/3	2/-	2/-
Eigen modes				
WFEBlade	12114	2711	5581	9628
ANSYS	11503	2655	5399	9200
Experiment	-	2680	5465	-
Difference from ANSYS/Exp. (%)	5/-	2/1	3/2	4/-

Table 2: Hinged blade natural frequencies

	Frequency (Hz)			
Eigen modes				
WFEBlade	332	921	1743	3013
ANSYS	328	927	1735	3087
Experiment	329	915	1763	2933
Difference from ANSYS/Exp. (%)	1/1	1/1	0.5/1	2/3

Eigen modes			
WFEBlade	1086	2282	3529
ANSYS	1121	2298	3582
Experiment	1101	2245	3428
Difference from ANSYS/Exp. (%)	3/1	1/2	1/3

difference of natural frequency values does not exceed 5% which confirm the high accuracy of the blade wave finite element.

The response speed was assessed at the calculation of 100 of eigenvalues and eigenvectors. The model consisting of wave finite elements showed >200 time gain in calculation time compared with the ANSYS. This is due to a much smaller number of elements necessary to ensure the specified accuracy as well as by their one-dimensionality.

It should be noted that even the reduction of the model dimension by half in WFEBlade leads to a slight change in the values of natural frequencies. At that the greatest difference makes from 2-8%.

CONCLUSION

The study results demonstrate the development of a blade wave finite element which has high speed and accuracy values. The calculated values of the natural frequencies are in good agreement with the experimental data and the analysis results in ANSYS. The difference makes <5%. At that, the calculation time gain is a multiple one, compared with ANSYS.

The blade wave finite element is primarily targeted at solving the problems of the blade dynamics as the part of an impeller, rotor and other parts of the engine. In this case due to its flexibility it may be successfully used for the dynamic analysis of a single blade.

Now the Samara State Aerospace University develops in the same way the wave finite elements for discs, shells, shroud shelf and other components of a gas turbine engine. The application of wave finite elements will significantly reduce the time and cost to solve the optimization problems for the development of a rotary engine system dynamic image.

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