

Calculation of Main Kinematic Characteristics of the Single-Shaft Vibrator with Aimed Fluctuations

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Abstract: The study gives the kinematic analysis of vibration mechanism with aimed fluctuations of the planetary type. Vibrators with aimed fluctuations are important in real technological processes. Planetary vibrators can be used to obtain aimed fluctuations. Planetary vibrators include planetary gear and unbalanced mass, located on level diameter dividing the circumference of a satellite. When the ratio of the diameter of the sun gears and satellite is 2:1, 3:1 or 4:1 one could receive different forms of aimed fluctuations. We have investigated and obtained values of the velocities and accelerations of the center of oscillating masses depending on the ratio of the diameter gear wheels of planetary vibrators.

Key words: Planetary vibrator with aimed fluctuations, equation of motion, velocity, acceleration, ratio

INTRODUCTION

Planetary vibrators are widely used in different technological fields of industry (Bauman, 1970). Technical solution (Gerasimov and Isaev, 2007; Gerasimov *et al.*, 2014; Gerasimov, 2014) of obtaining directed mechanical vibrations with planetary vibrators can further expand the scope of application of such mechanisms. In recent years, a team of researchers working in the Belgorod State Technological University (BSTU named after V.G. Shukhov) made a number of theoretical and experimental works on the analysis and design of planetary vibrators with aimed fluctuations (Gerasimov and Gerasimov, 2013; Gerasimov *et al.*, 2013a, b; Gerasimov and Stepanistchev, 2014; Uralskiy and Sevostyanov, 2010).

The aim of this study to conduct kinematic analysis of the planetary mechanism for getting straight aimed fluctuations with various ratio of the sizes of the planetary gear and different frequency of reference of the satellite.

METHODOLOGY

The study uses the classic methods of kinematic analysis of the mechanical system to calculate new design of planetary vibrator for generation of aimed fluctuations.

KINEMATIC ANALYSIS

As by Gerasimov and Mkrthychev (2014) guide discusses a circle of radius R which rolls without sliding circle of radius r , the respective diameters D and d (Fig. 1). If the ratio of the radii of circles $m = R/r$ can

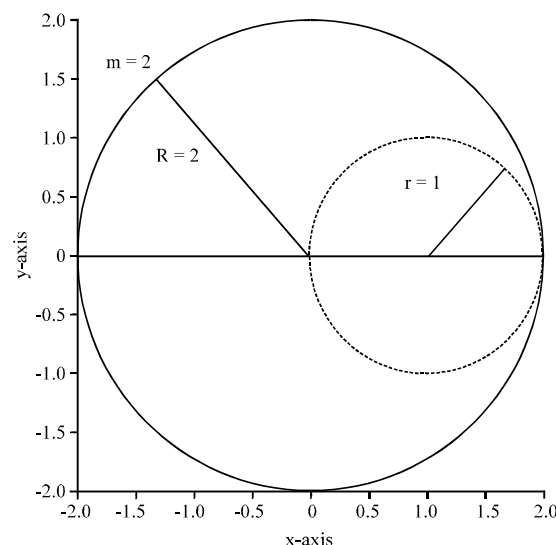


Fig. 1: A hypocycloid with a character number $R/r = 2$, degenerated in the segment of straight line (the fixed circle diameter $D = 4$)

be represented as an irreducible fraction b/a , hypocycloid will have b points and describes at a speed of the inner circle.

If $R/r = 2$, hypocycloid degenerates in the guide diameter of a circle (Fig. 1). It is this ratio can be used to obtain straight forward aimed fluctuations, if the center of mass oscillatory system is located at a distance of radius r from the center of the guide circle, i.e., in the point of contact under consideration circles. In this case the equations of hypocycloid's projections onto the axes of coordinates in a parametric form as well as the equations of the corresponding projections of velocities and accelerations will be:

$$\begin{aligned}x &= 2r \cos \varphi = R \cos \varphi \\ \dot{x} &= -2\omega r \sin \varphi = \varepsilon R \sin \varphi \\ \ddot{x} &= -2\varepsilon r \sin \varphi - 2\omega^2 r \cos \varphi = -\varepsilon R \sin \varphi - \omega^2 R \cos \varphi \\ y &= \dot{y} = \ddot{y} = 0\end{aligned}$$

where, the period of change of the parameter $0 \leq \varphi < 2\pi$ and $R/r = 2$ is the character number of hypocycloid, ω is angular speed of satellite's rolling (Gerasimov and Gerasimov, 2013), ε is angular acceleration of rolling.

Figure 2 shows the graphs of the projections of the velocity v_x and acceleration a_x onto the x-axis depending on time t and motion graph on the phase diagram $x-v_x$. Here, the rolling radius of the circle is $r = 1$ m, the guide radius of the circle is $R = 2$ m, angular acceleration of rolling is $\varepsilon = 0$, the start angle is $\varphi_0 = 0$, the angular velocity of the spinning is $\omega = 0.2 \text{ rad} \cdot \text{sec}^{-1}$ and the angle is $\varphi = \varphi_0 + \omega t = 0.2t \text{ rad}$.

If you change the input parameters of motion, then you can change the amplitude and frequency of oscillations describing points in the desired range. For example in Fig. 3 shows the graphs of the projections of the velocity v_x and acceleration a_x on the x-axis depending on time t for the following values: angular acceleration running is $\varepsilon = 0.2 \text{ rad} \cdot \text{sec}^{-2}$, the start angle is $\varphi_0 = 0$, the initial angular velocity spinning is $\omega_0 = 0.2 \text{ rad} \cdot \text{s}^{-1}$ and the angle of rotation is $\varphi = \varphi_0 + \omega_0 t + \varepsilon t^2 / 2 = 0.2t + 0.1t^2$.

Consider the rolling of planetary gear satellite of planetary vibrator with constant frequency, the value of which is taken from the sequences $n = 0:100:10000 \text{ rpm}$. Then, $\varepsilon = 0$, $\varphi_0 = 0$, $\omega = \pi n / 30 \text{ sec}^{-1}$ and $\varphi = \omega t = \pi n t / 30 \text{ rad}$ and:

$$\begin{aligned}x(r, n, t) &= 2r \cos \frac{\pi n}{30} t = R \cos \frac{\pi n}{30} t \\ \dot{x}(r, n, t) &= -2r \frac{\pi n}{30} \sin \frac{\pi n}{30} t = -\frac{\pi n}{30} R \sin \frac{\pi n}{30} t\end{aligned} \quad (1)$$

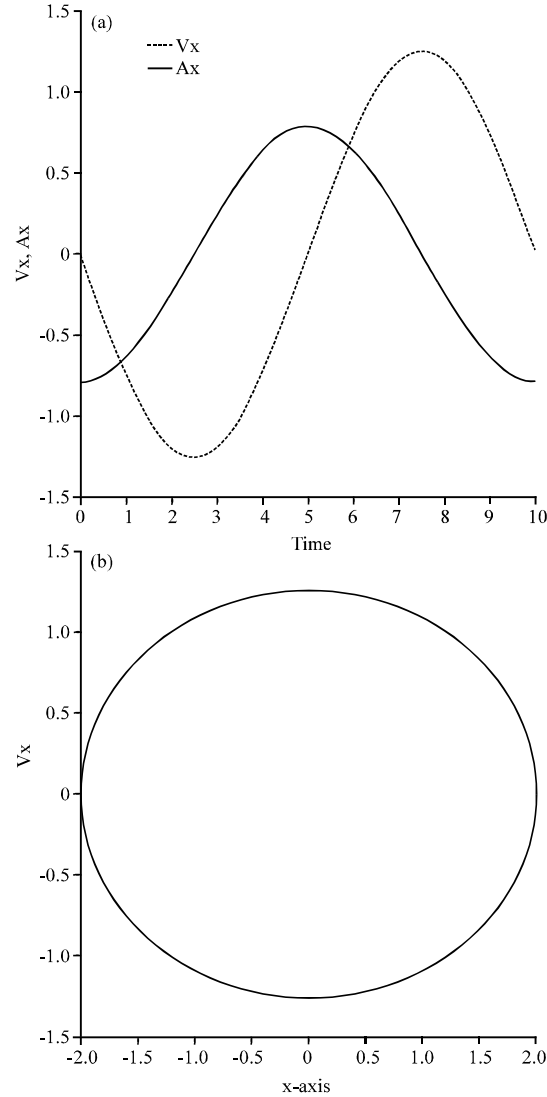


Fig. 2: a) The graphics of projections of velocity v_x (dashed) and acceleration a_x (solid) onto the x-axis depending on time t . b) the motion graph on the phase diagram $x-v_x$; $r = 1$, $R = 2$, $\varepsilon = 0$, $\omega = 0.2$ and $\varphi = \omega t$

$$\begin{aligned}\ddot{x}(r, n, t) &= -2\left(\frac{\pi n}{30}\right)^2 r \cos \frac{\pi n}{30} t = -\left(\frac{\pi n}{30}\right)^2 R \cos \frac{\pi n}{30} t \\ y &= \dot{y} = \ddot{y} = 0\end{aligned} \quad (2)$$

For $n = 500, 1000, 1500, 2000, 2500 \text{ rpm}$ graphs of coordinates, velocities and accelerations is shown in Fig. 4-6. Note that the time in Fig. 4-6 postponed for each schedule independently. The initial phase velocity and acceleration are different. The variations in the amplitude of the velocity and acceleration depending on frequency conversion are recorded in Table 1.

Figure 7 and 8 are built according to this table and show the dependence of amplitude of the projection of the velocity and acceleration onto the x-axis speed. Obtained graphic dependences can determine equations $v_{max} = v_{max}(n)$ and $a_{max} = a_{max}$. Covered in the case, you can get this dependence and analytically using Eq. 1 and 2:

$$v_{max}(n) = \frac{\pi n}{30} R \quad (3)$$

$$a_{max}(n) = \left(\frac{\pi n}{30} \right)^2 R \quad (4)$$

The same reasoning can be applied to the case when $m = R/r = 3$ (Fig. 9). Then have a case where hypocycloid

Table 1: The maximum value of the projection of the velocity and acceleration onto the x-axis at different frequencies

n (rpm)	$v_{max} (m \cdot sec^{-1})$	$a_{max} (m \cdot sec^{-2})$
500	15.708	2737.800
1000	31.416	10966.227
1500	47.124	24674.011
2000	62.488	43864.908
2500	75.864	68538.919

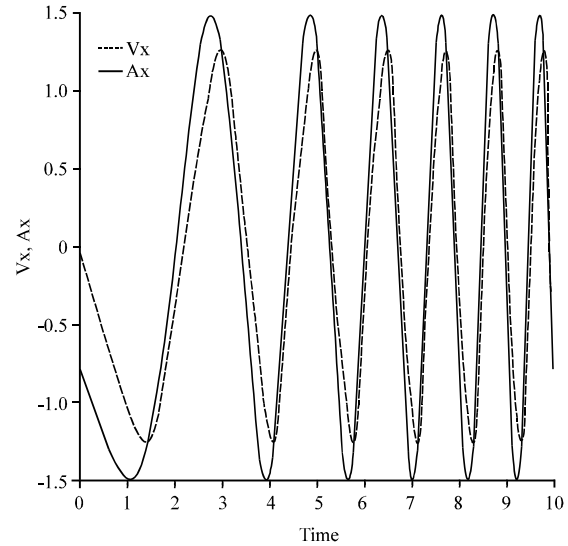


Fig. 3: The projections' graphs of velocity v_x (dashed) and acceleration a_x (solid) on the x-axis depending on time t . $r = 1$, $R = 2$, $\varepsilon = 0.2$, $\omega = 0.2$ and $\varphi = \omega t + \varepsilon t^2/2$

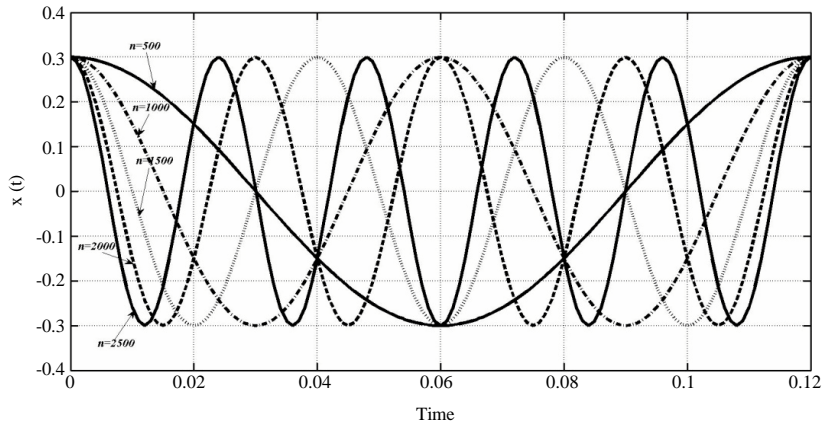


Fig. 4: The projections of the radius-vector onto the x-axis depending on time (sec) for different values of n (rpm)

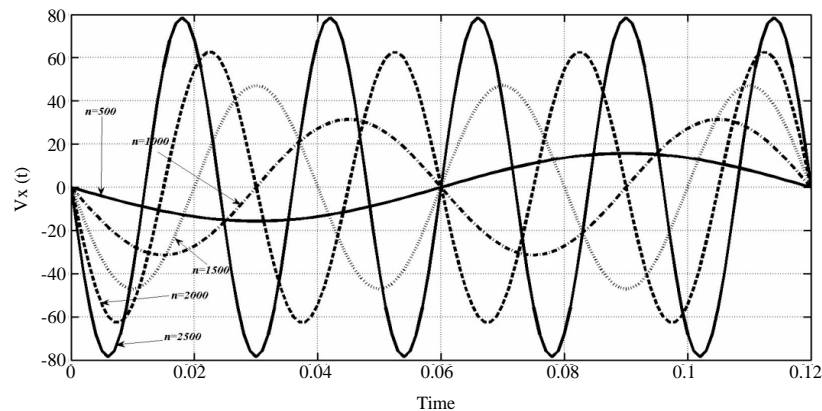


Fig. 5: The projections of the velocities v_x (m/sec) onto the x-axis depending on time (sec)

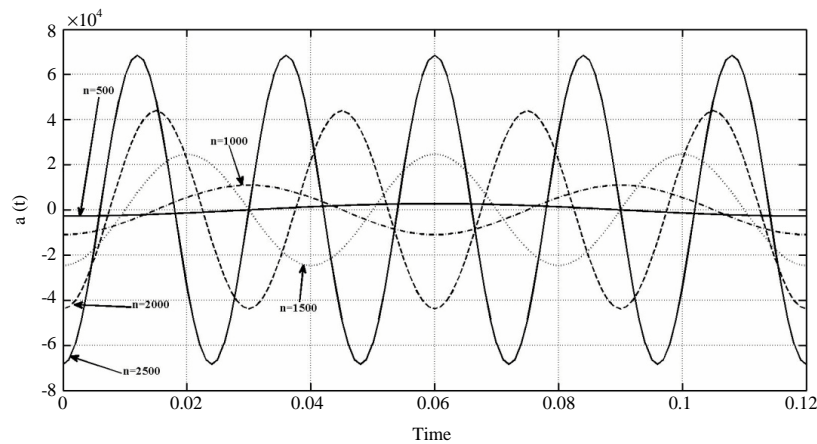


Fig. 6: The projections of acceleration a_x (m/sec²) onto the x-axis depending on time (sec)

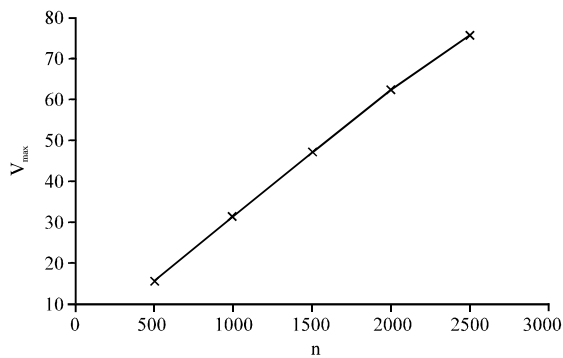


Fig. 7: The velocity's projection's amplitude v_x (m/sec) onto the x-axis depending on frequency n (rpm)

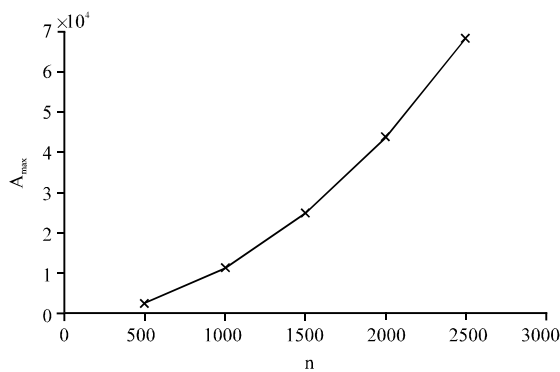


Fig. 8: The acceleration's projection's amplitude a_x (m/sec²) onto the x-axis depending on frequency n (rpm)

has three vertices located through 120°. This ratio can be applied for example to obtain aimed cyclical fluctuations during sifting on a flat horizontal sieve.

Equations projection hypocycloid on the coordinate axes in parametric form (Fig. 9) and the equations the corresponding projections of velocities and accelerations will be:

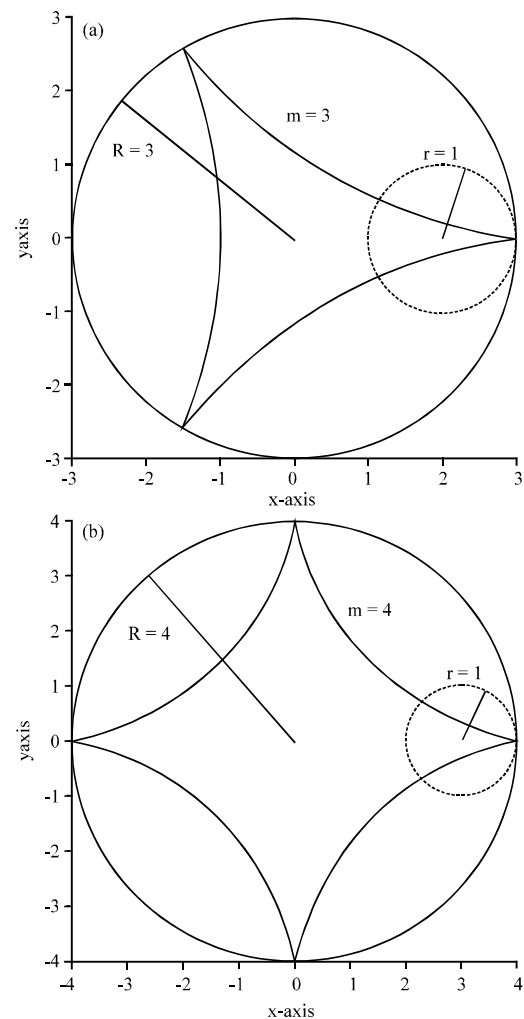


Fig. 9: a) Triangular hypocycloid with a characteristic number $m = R/r = 3$ and b) quadrangular hypocycloid with a characteristic number $m = R/r = 4$

$$\begin{aligned}x &= 2r \cos \varphi + r \cos (2\varphi), \quad y = 2r \sin \varphi - r \sin (2\varphi) \\ \dot{x} &= -2r\omega (\sin \varphi + \sin (2\varphi)), \quad \dot{y} = 2r\omega (\cos \varphi - \cos (2\varphi)) \\ \ddot{x} &= -2r\epsilon (\sin \varphi + \sin (2\varphi)) - 2r\omega^2 (\cos \varphi + 2 \cos (2\varphi)) \\ \ddot{y} &= 2r\epsilon (\cos \varphi - \cos (2\varphi)) - 2r\omega^2 (\sin \varphi - 2 \sin (2\varphi))\end{aligned}$$

Consider the rolling of planetary gear satellite of planetary vibrator with constant frequency, the value of which is taken from the sequences $n = 0:100:10000$ rpm. Then, $\epsilon = 0$, $\varphi_0 = 0$, $\omega = \pi n/30/\text{sec}$ and $\varphi = \omega t = \pi n t/30$ rad and:

$$\begin{aligned}x &= 2r \cos \left(\frac{\pi n}{30} t \right) + r \cos \left(2 \frac{\pi n}{30} t \right) \\ y &= 2r \sin \left(\frac{\pi n}{30} t \right) - r \sin \left(2 \frac{\pi n}{30} t \right) \\ \dot{x} &= -2r \frac{\pi n}{30} \left[\sin \left(\frac{\pi n}{30} t \right) + \sin \left(2 \frac{\pi n}{30} t \right) \right] \\ \dot{y} &= 2r \frac{\pi n}{30} \left[\cos \left(\frac{\pi n}{30} t \right) - \cos \left(2 \frac{\pi n}{30} t \right) \right] \\ \ddot{x} &= -2r \left(\frac{\pi n}{30} \right)^2 \left[\cos \left(\frac{\pi n}{30} t \right) + 2 \cos \left(2 \frac{\pi n}{30} t \right) \right] \\ \ddot{y} &= -2r \left(\frac{\pi n}{30} \right)^2 \left[\sin \left(\frac{\pi n}{30} t \right) - 2 \sin \left(2 \frac{\pi n}{30} t \right) \right]\end{aligned}$$

Let us put $r = 0.1$ m and then $R = 0.3$ m. Consider the character of changes of the amplitude of the velocity and acceleration depending on rolling frequency and record the results in Table 2, Fig. 10 and 11).

Table 2: The maximum value of magnitudes and projections of the velocity and acceleration onto the coordinate axes at different frequencies

n (rpm)	$V_{x, \max}$ (m·sec ⁻¹)	$V_{y, \max}$ (m·sec ⁻¹)	V_{\max} (m·sec ⁻¹)	$a_{x, \max}$ (m·sec ⁻²)	$a_{y, \max}$ (m·sec ⁻²)	a_{\max} (m·sec ⁻²)
500	1.894	1.124	2.110	99.670	115.950	197.392
1000	4.097	2.529	4.615	468.716	590.274	789.568
1500	6.304	3.944	7.075	1111.400	1425.300	1776.500
2000	8.441	5.339	9.632	2004.100	2620.200	3158.300
2500	10.727	6.744	12.039	3125.400	4147.500	4934.800

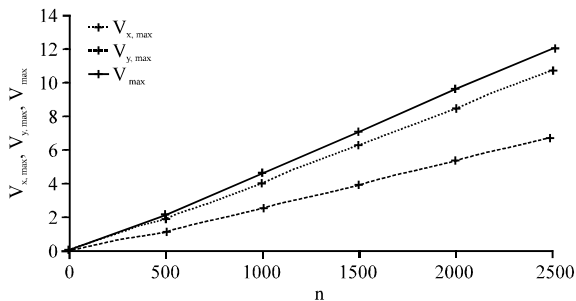


Fig. 10: The velocity's magnitude and projection's amplitude v (solid), v_x (dotted), v_y (dashed) (m/sec) onto the axes depending on frequency n (rpm)

Obtained graphic dependences allow to determine in this case the approximated equations $v_{\max} = v_{\max}(n)$ and $a_{\max} = a_{\max}(n)$. It will now more difficult, than in the first case to get these equations in an analytical form.

When $m = R/r = 4$ (Fig. 9), we have the case of the quadrangular hypocycloid with four vertices located through 90 degrees. This ratio can be used for obtaining aimed cyclical fluctuations in vibromills.

Equations of hypocycloid's projections onto the axes of coordinates in a parametric form as well as the equations of the corresponding projections of velocities and accelerations will be:

$$\begin{aligned}x &= 3r \cos \varphi + r \cos (3\varphi), \quad y = 3r \sin \varphi - r \sin (3\varphi) \\ \dot{x} &= -3r\omega (\sin \varphi + \sin (3\varphi)), \quad \dot{y} = 3r\omega (\cos \varphi - \cos (3\varphi)) \\ \ddot{x} &= -3r\epsilon (\sin \varphi + \sin (3\varphi)) - 3r\omega^2 (\cos \varphi + 3 \cos (3\varphi)) \\ \ddot{y} &= 3r\epsilon (\cos \varphi - \cos (3\varphi)) - 3r\omega^2 (\sin \varphi - 3 \sin (3\varphi))\end{aligned}$$

Under conditions considered in the previous examples, we get:

$$\begin{aligned}x &= 3r \cos \left(\frac{\pi n}{30} t \right) + r \cos \left(3 \frac{\pi n}{30} t \right) \\ y &= 3r \sin \left(\frac{\pi n}{30} t \right) - r \sin \left(3 \frac{\pi n}{30} t \right) \\ \dot{x} &= -3r \frac{\pi n}{30} \left[\sin \left(\frac{\pi n}{30} t \right) + \sin \left(3 \frac{\pi n}{30} t \right) \right] \\ \dot{y} &= 3r \frac{\pi n}{30} \left[\cos \left(\frac{\pi n}{30} t \right) - \cos \left(3 \frac{\pi n}{30} t \right) \right] \\ \ddot{x} &= -3r \left(\frac{\pi n}{30} \right)^2 \left[\cos \left(\frac{\pi n}{30} t \right) + 3 \cos \left(3 \frac{\pi n}{30} t \right) \right] \\ \ddot{y} &= -3r \left(\frac{\pi n}{30} \right)^2 \left[\sin \left(\frac{\pi n}{30} t \right) - 3 \sin \left(3 \frac{\pi n}{30} t \right) \right]\end{aligned}$$

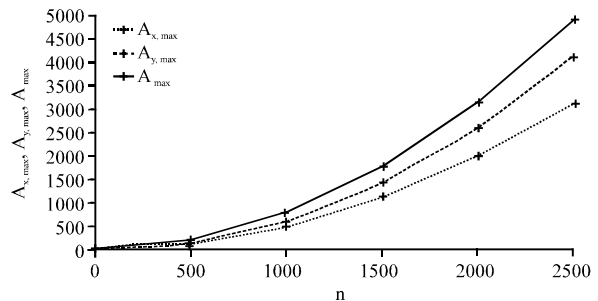


Fig. 11: The graph of the acceleration's magnitude and projection's amplitude a (solid), a_x (dotted), a_y (dashed) (m/sec²) onto the axes depending on frequency n (rpm)

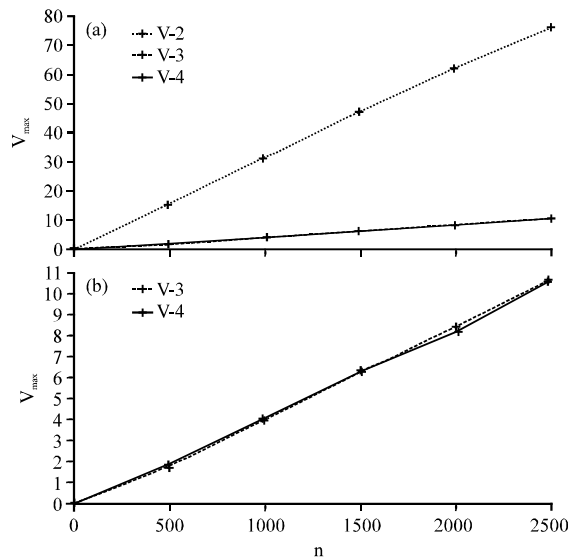


Fig. 12: At the top are the graphs of the amplitude of the projections of the velocity onto the x-axis v_x (m/sec) of the frequency n (rpm) for a degenerated in diameter (dotted), triangle (dashed) and rectangular (solid) hypocycloids. At the bottom is the top fragment of a drawing for a triangular (dashed) and rectangular (solid) hypocycloids

Table 3: The maximum value of magnitudes and projections of the velocity and acceleration onto the coordinate axes at different frequencies

n (rpm)	$V_{x, \max}$ (m·sec ⁻¹)	$V_{y, \max}$ (m·sec ⁻¹)	V_{\max} (m·sec ⁻¹)	$a_{x, \max}$ (m·sec ⁻²)	$a_{y, \max}$ (m·sec ⁻²)	a_{\max} (m·sec ⁻²)
500	1.966	1.849	2.483	154.353	222.067	296.088
1000	4.133	3.980	5.305	888.264	1027.900	1184.400
1500	6.286	6.200	8.127	2181.300	2417.800	2664.800
2000	8.201	8.295	10.949	3941.100	4391.800	4737.400
2500	10.656	10.551	13.176	6662.000	7032.100	7402.200

Let us put $r = 0.075$ m and then $R = 0.3$ m. Again we consider the variations in the amplitude of the velocity and acceleration depending on frequency and record the results in Table 3.

Comparable data Table 1-3 values of the amplitudes of projections of velocities and accelerations on to the x-axis for all the above types of hypocycloid. The result is shown in Fig. 12 and 13.

Thus, if we consider the rolling of a planetary gear satellite ring of planetary vibrator with aimed fluctuations with constant frequency and under the conditions specified above, the Eq. 3 and 4 allow for a given rolling frequency to determine the amplitude value of speed and acceleration. The equations allows for the design stage set given values of the velocity and acceleration of the planetary mechanism. Use any standard application

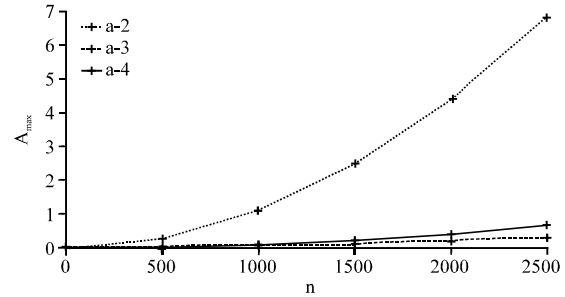


Fig. 13: The amplitude of the projections of the acceleration onto the x-axis a_x (m/sec²) of the frequency n (rpm) for a degenerated in diameter (dotted), triangle (dashed) and rectangular (solid) hypocycloids

software package will allow you to get the required values in any range of parameters of the problem (Mkrtychev, 2012, 2013).

CONCLUSION

Kinematic analysis and numerical simulations allow using the data obtained to the real design of vibration mechanisms with the given geometrical parameters of vibration exciter of planetary type.

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