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Modeling of an Energy Harvesting Piezoelectric Cantilever Beam

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Abstract: The study presented in this research targets the modeling and analysis of a 31 transverse mode type piezoelectric cantilever beam for voltage generation by transforming ambient fluid induced vibration energy into usable electrical energy. Piezoelectric materials have the ability to convert mechanical forces into an electric field in response to the application of mechanical stresses or vice versa. This property of the materials has found applications in sensor and actuator technologies and recently in the new field of energy harvesting. A mathematical model for energy harvesting by a piezoelectric cantilever beam device, based on classical beam analysis is presented. The optimization algorithm is implemented in Matlab, based on four physical dimension parameters of the energy harvesting cantilever. The optimal cantilever design from the theoretically derived algorithm determines four physical dimensions parameter to maximize output power. The output power is used to evaluate the performance of the energy harvester. Some interesting aspects that affect the generation of power are discussed. From this analysis, it is found that increasing the frequency of the vibration improve the output power while beyond a certain value further improvement can not be achieved by simply increasing the vibration frequency. Moreover, output power of the energy harvester is found as a function of external resistance. The model predicted anoptimized design with maximizes output power of 0.9 mW at a natural frequency of 200 Hz. Piezoelectric cantilever based energy harvester device can potentially replace the battery that supplies power in microwatt range necessary for operating wireless sensor devices.

Key words: Piezoelectric, energy, harvesting, modeling, Matlab

INTRODUCTION

Harvesting small amounts of electrical energy from ambient environmental sources like solar, fluid flow or vibrations using small scale energy harvesting devices, offers enormous prospect of powering various portable electronic devices such as bio-medical devices and sensing devices. These types of devices are frequently deployed in remote application areas for example wireless sensor nodes with limited use of batteries and in the body area networks for medical diagnostic (Gao *et al*., 2013). Numerous studies have shown that power densities of energy harvesting devices can be hundreds of μW (Sidek and Saadon, 2013). Meanwhile, recent advancements in the electronics industry dramatically reduced the total power consumptions of most electronic devices to a level that is analogous to the harvesting ability of a piezoelectric harvester device (Kim *et al*., 2013). Geometric designs (dimensions) are one of the major key factors that affect the power output of the energy harvester. Before prototyping an energy harvester device, modeling approach provides useful information to predict the effect of geometric dimensions in its power output that saves both time and cost (Romani *et al*., 2013). Theoretical modeling in the design process is to recognize various interrelated parameters in order to optimize key design parameters. While the feasibility of this application has been repeatedly demonstrated in the literature, challenges for accurate modeling of mechanical to electrical energy generation in real world applications still remain (Orfei *et al*., 2013).

Among many possibilities of harvesting energy, piezoelectric materials have been used extensively since its capability of converting ambient mechanical

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energy into electrical energy (Prashanthi *et al*., 2012). Lead Zirconate Titanate (PZT) is one of the most used piezoelectric material used because of its high electromechanical coupling characteristics in single crystals (Li *et al*., 2011). Umeda *et al*. (1996) were among the pioneers to study the piezoelectric harvesters and proposed an electrical equivalent model being converted from mechanical lumped models of a mass, a spring and a damper that describe a transformation of the mechanical impact energy into electrical energy in the piezoelectric material. Goldfarb and Jones (1999) presented a linear model of a piezoelectric stack and analyzed its power harvesting capabilities. Their study showed that the maximum efficiency of the energy harvesting system occurs in a low frequency region that is much lower than the structural resonance of the Piezoelectric Stack System. In addition, their research suggested that the efficiency of electrical power generation is related to the amplitude of the input forcing function due to the hysteresis behavior of the piezoelectric. Kasyap *et al*. (2002) developed a lumped element model to represent the dynamic behavior of piezoelectric in multiple energy domains using an equivalent circuit. They constructed fly back converter which allowed the impedance of the circuit to be matched to that of the piezoelectric material so as to maximize power transfer from the piezoelectric. Their model was experimentally verified using a one dimensional beam structure with peak power frequencies of around 20%. Roundy *et al*. (2003) studied the use of piezoelectric elements attached to vibrating beam to convert mechanical vibration energy for possible application in powering wireless sensor nodes. Their research demonstrated that when a piezoelectric energy harvester device was made to operate at frequency matching the vibrating frequency that is at resonance, the output electrical power generated was maximized. Meanwhile, Sodano *et al*. (2004) developed an analytical model of a beam with attached piezoelectric elements to provide an accurate estimate of the power generated by the piezoelectric effect. Their model was based on a more general model by Hagood *et al*. (1990), it incorporates the model by Crawley and Anderson (1990) in formulating the actuation equations for piezoelectric device and constitutive equations of bimorph actuators by Smits and Dalke (1989). The models by Crawley and Anderson (1990) and Smits and Dalke (1989) were developed for actuation. Sodano *et al*. (2004) adopted these models for application in energy harvesting. In addition, the researchers also introduced ‘material damping’ which was excluded from the previous models. Eggborn (2003) developed the analytical models to predict the energy harvesting from a cantilever beam and a plate using Bernoulli-Beam theory and made a comparison with the experimental result. Ajitsaria *et al*. (2007) developed modeling and analysis of a bimorph piezoelectric cantilever beam for voltage generation using on the analytical approach based on Euler-Bernoulli beam theory and Timoshenko beam equations which is then compared with two previously described models in the literature; the electrical equivalent circuit and energy method.

In the present study, a theoretical model of power generation by a piezoelectric element attached to a cantilever beam is developed. The modeling approach is fairly simpler since it uses basic classical beam analysis and basic piezoelectric equations with no in-depth piezoelectric material analysis. Moreover, the model is in corporating a computer-based optimization algorithm such that the output of the optimization process is a list of values for the device dimensions. The model results in an expression forthe power output of the model after optimization the geometrical parameters of the device.

MATERIALS AND METHODS

Description of the piezoelectric energy harvester and system modeling: The vibration to electricity piezoelectric energy harvester is a two-layered bending element mounted as a cantilever beam. The schematic view of the cantilever beam undertaken in this study is shown in Fig. 1. In the context of this model, it is assumed that the thin layer of the piezoelectric added to the surface of the cantilever does not change the deformed shape of the cantilever but increases the equivalent bending stiffness only.

For simplification, an elastic uniform rectangular cantilever beam of a single is entropic material clamped at the left end and a point load, P, acted upon at its free end is considered to develop the piezoelectric cantilever based energy harvester model. The elastic curve shape of the deflected beam is defined as $u = u(x)$. It is the deflection of the neutral axis with respect to its original condition. The schematic of the cantilever system is shown in Fig. 2.

The bending moment of the cantilever beam can be expressed as $M(x) = -P(L-x)$. Based on the moment-shear-load relations and the beam curvature, the governing differential equation for bending deflection of the cantilever beam modeled as a two-dimensional curve is shown in Eq. 1. The slope of deflection, $\theta(x)$ and the beam deflection $u(x)$ is expressed in Eq. 2 and 3, respectively:

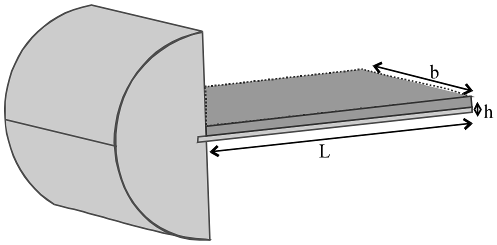

Fig. 1: Piezoelectric power generator with a unimorph-cantilever design (the cantilever length, L, cross-section of width, b and height is h)

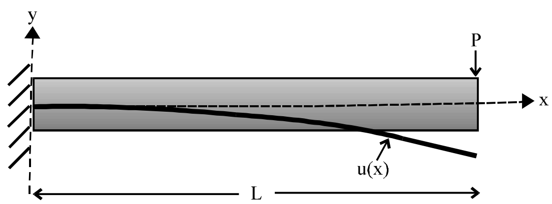

Fig. 2: Elastic clamped cantilever beam under a point load at the end

$$M(x)=EI\frac{d^2u}{dx^2}=P(x-L) \tag{1}$$

$$\theta(x)=\frac{du}{dx}=\frac{P}{2EI}(x-L)^2+C_1 \tag{2}$$

$$u(x)=\frac{P}{6EI}(x-L)^3+C_1x+C_2 \tag{3}$$

Boundary conditions, prescribed by the clamped endof the cantilever where the displacement, y(x = 0) = 0 and slope, $\theta(x = 0) = 0$; evaluate the constants of integration. Algebraic manipulation provide constants of integration: $C_1 = -PL^2/2EI$ and $C_2 = PL^3/6EI$. As a result, the Eq. 4 and 5 provide the angle of displacement and the displacement as functions of x, respectively:

$$\theta(x)=\frac{P}{2EI}(x-L)^2-\frac{PL^2}{2EI} \tag{4}$$

$$u(x)=\frac{P}{6EI}(x-L)^3-\frac{PL^2}{2EI}x+\frac{PL^3}{6EI} \tag{5}$$

Employing Linear Piezoelectric theory, charge in the x-direction and stress in the z-direction both zero within the piezoelectric layer. The electrical displacement in the z-direction can be expressed as a function of strain in the x-direction and the electric field within the piezoelectric layer as shown in Eq. 6:

$$D_z = e_{31}\varepsilon_{33}E_z \tag{6}$$

Where:
e_{31} = The piezoelectric constant in the 31 coupling direction
ε_{33} = The dielectric constant
E_z = The electric field in the z-direction within the piezoelectric layer

The charge collected on the electrode surface can be expressed as the electrical displacement integral on the area of the surface (Eq. 7):

$$Q=\int_A D_z dA = b\int_{l_0}^{l_1}(e_{31}\varepsilon_{33}+\varepsilon_{33}E_z)dx \tag{7}$$

where, E_z is the electric field within the piezoelectric layer. Because there is an electrode covered on the surface, the potential of the surface is the same. Assuming that the potential difference between the upper and lower surface of the piezoelectric layer denoted by ν, under the uniform electrical field assumption, the electric field canbe approximately expressed as shown in Eq. 8:

$$E_z=-\frac{\partial \nu}{\partial z}=-\frac{\nu}{\Delta} \tag{8}$$

Substituting Eq. 8 into Eq. 7, Q becomes as shown in Eq. 9:

$$Q=-b\int_{l_0}^{h} e_{31}\frac{h}{2}\frac{\partial^2\omega}{\partial x^2}dx + b\int_{l_0}^{h}\varepsilon_{33}E_z dz = \frac{bhe_{31}}{2}\left[\varphi(l_0)-\varphi(l_1)\right]-bL\varepsilon_{33}\frac{\nu}{\Delta} \tag{9}$$

where, $\varphi(x, t) = \partial w(x, t)/\partial x$ is the flexibility of the beam, $\varphi(l_0)$ and $\varphi(l_1)$ are the corresponding values at the beginning of the piezoelectric layer and that at the end of the piezoelectric layer. Current, charge and voltage are all functions of time. The frequency of these period functions is dependent upon the mechanical vibration. Because the differential of the charge on the electrode surface is the current flow out to the external impedance, the amplitude of the current is that of the charge times the frequency as shown in Eq. 10. The relationship between voltage and current for an electrical circuit with pure resistance is expressed as Eq. 11. Combining Eq. 9-11 the amplitude of the current can be determined as shown in Eq. 12:

$$I=\omega Q \tag{10}$$

$$I=\frac{\nu}{R} \tag{11}$$

$$I_m = \frac{\omega b h e_{31}\left[\varphi(l_0)-\varphi(l_1)\right]}{2\left(1+bL\varepsilon_{33}\frac{\omega R}{\Delta}\right)}R \quad (12)$$

As charge Q on the electrode surface is connected to the external impedance, if consider the external impedance is a pure resistance, the output voltage and the current have the same phase. The output power can be expressed as:

$$P = I\nu = I^2R \quad (13)$$

For an AC electrical circuit, substituting Eq. 12 into Eq. 13, the time-average of the output power, $\overline{P}$ of the piezoelectric layer can be expressed by multiplication of the voltage and current as shown in Eq. 14:

$$\overline{P} = \frac{\nu_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = \frac{\omega^2 b^2 h^2 e_{31}^2\left[\varphi(l_0)-\varphi(l_1)\right]^2}{8\left(1+bL\varepsilon_{33}\frac{\omega R}{\Delta}\right)^2}R \quad (14)$$

The output power is dependent on the frequency of the vibration ω and the external resistance R. The resonant frequency ω, combining Newton's Second Law and Hooke's Law is expressedas shown in Eq. 15:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{ma}{u}\times\frac{1}{m}} = \sqrt{\frac{a}{u}} = \sqrt{\frac{Ebh^3}{4\rho L^3}} \quad (15)$$

The resistance which gives the maximum output power is dependent on the variables of the system. Conversely, when the resistance requirement is determined by the application the corresponding parameters of the system can be designed to achieve the maximum output power. For simplification, the electrical load is modeled as a single resistor as described in Eq. 16:

$$R = \frac{t}{bL\varepsilon_{33}\omega} \quad (16)$$

Consider, L, b, t, h as the energy harvester design variables to be used in the maximum output power optimization process and substituting Eq. 4, 5, 15 and 16 into Eq. 14 the following expression for energy harvester power output as shown in Eq. 17 is derived in terms of only four energy harvester design variables and constants:

$$\overline{P} = \frac{9\rho^{3/2}e_{31}^2a^2}{8E^{3/2}\varepsilon_{33}}\times\frac{L^{3/2}t}{h^{3/2}b^{1/2}} \quad (17)$$

From the Eq. 17, the output power of the energy harvester is dependent upon several parameters. Firstly, increasing the length of the cantilever beam, L will increase total power output. Equation 17 also reveals that minimizing the beam thickness, b will help in increasing total power. As a result, during design optimization process small beam width is preferred. Another point to note is that power will increase if piezoelectric material thickness is large and metal thickness is small.

RESULTS AND DISCUSSION

Optimization simulation in Matlab: Coupled with the theoretical model provided in the previous study, the optimal design from the algorithm to achieve a maximum power while satisfying four design variables engineering constrains (L, b, t, h) are determined. Table 1 summarizes the results of the algorithm optimization to achieve maximum power.

According to Fig. 3, the energy harvester output power increases when mechanical vibration frequency increases and lower resistance. This result can be confirmed by examining Eq. 14, in which power is directly related to resonance frequency and inversely related to resistance. Considering automobile environment constraints imposed by the application environment that operate at a natural frequency range 200-600 Hz, therefore limiting the energy harvester power output as shown in Table 1. It also appears that the energy harvester design that operates with a natural frequency of 500 Hz would produce the most power. However, considering automotive application limiting the volume of the energy harvester, the optimal energy harvester design found by the optimization algorithm maintains a natural frequency of about 200 Hz.

Figure 4 shows power output as a function of frequency. The plot uses the external resistance value for maximum power output given by Eq. 16. From the equation of the power as a function of the frequency,

Table 1: Energy harvester property for maximize power found by the optimization algorithm

Design parameters	Values (mm)	Design to maximize output power	Values
Cantilever length (L)	13	Power	0.9 mW
Cantilever width (b)	18	Natural frequency	200 Hz
Thickness PZT (t)	1	Max. stress	5.7031 MPa
Metal beam height (h)	1	Matching resistance	20.408 kΩ
		Max. displacement	4.9581 μm

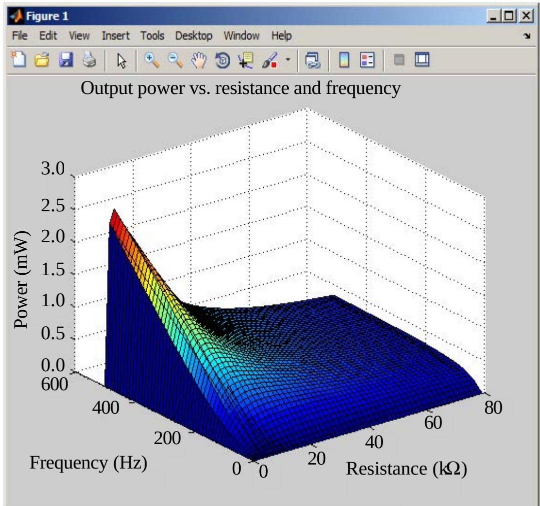

Fig. 3: Energy harvester for maximum output power as functions of resistance and frequency from the optimization algorithm in Matlab

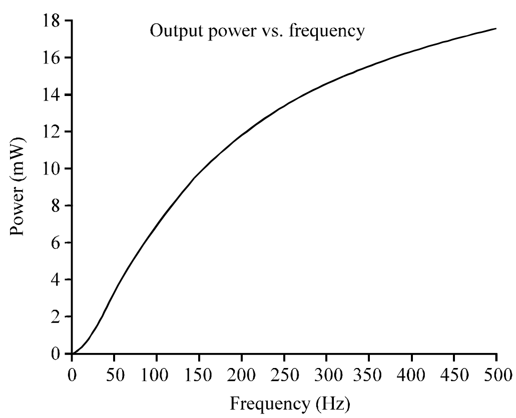

Fig. 4: Energy harvester output power as a function of operating frequency

increasing the mechanical vibration frequency can increase the output power. However, when the frequency reaches a certain value, the output power will increase very gradually as the frequency continues to increase further. The resistance which gives the maximum output power is dependent on the variables of the system. The output power as a function of the external resistance is shown in Fig. 5. The vibration amplitude is set as a constant. Maximum power occurs when the resistance is defined by Eq. 16 equal to 20.408 kΩ. Conversely, when the resistance requirement is determined by the application, the corresponding parameters of the system can be designed to achieve the maximum output power. As the frequency of the vibration increases, the corresponding optimal resistance is reduced.

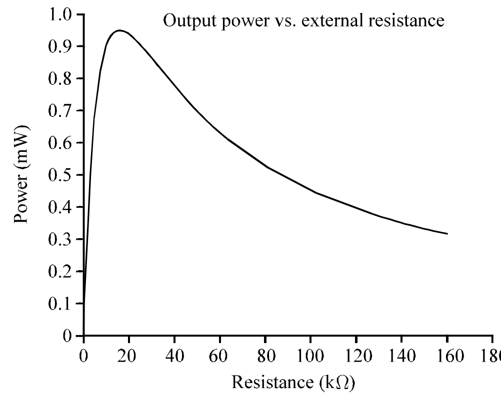

Fig. 5: Output power as a function of external resistance

CONCLUSION

This study gives a simple model for the analysis of piezoelectric power generator application in a low frequency environment. Basic Euler-Bernoulli cantilever beam bending analysis and electrical load modeled to in corporate the optimization algorithm for the energy harvester model. This model proves useful in identifying trends and design solutions for optimization of an energy harvester. Using four design variables of the energy harvester device (L, b, t, h) and necessary engineering and design constrains that can be utilized for different applications. Through this analysis, a design that maximizes output power to produce 0.9 mW at a natural frequency of 200 Hz. The output power is used to evaluate the performance of the generators. Some interesting aspects that affect the output power are discussed. From this analysis, it is found that there is an optimal external resistance that gives the maximum output power.

Furthermore, increasing the frequency of the vibration improve the output power while beyond a certain value, further improvement can not be achieved by simply increasing the vibration frequency. In future research, the model could be improved by incorporating the effects of damping and bonding layers on the voltage generated by the piezoelectric. Furthermore, the intended location of the piezoelectric on the host beam or structure needs to be studied so that its displacement can be optimized and the excitation range realized to allow for the tuning of power harvesting device and consequently increase the output power.

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